

# Hybrid Multi-layer Perceptron with Fuzzy Set-based PNs with the Aid of Symbolic Coding Genetic Algorithms

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**Abstract** - We propose a new category of hybrid multi-layer neural networks with hetero nodes such as Fuzzy Set based Polynomial Neurons (FSPNs) and Polynomial Neurons (PNs). These networks are based on a genetically optimized multi-layer perceptron. We develop a comprehensive design methodology involving mechanisms of genetic optimization and genetic algorithms, in particular. The augmented genetically optimized HFPNN (namely gHFPNN) results in a structurally optimized structure and comes with a higher level of flexibility in comparison to the one we encounter in the conventional HFPNN. The GA-based design procedure being applied at each layer of HFPNN leads to the selection of preferred nodes (FPNs or PNs) available within the HFPNN. In the sequel, two general optimization mechanisms are explored. First, the structural optimization is realized via GAs whereas the ensuing detailed parametric optimization is carried out in the setting of a standard least square method-based learning. The performance of the gHFPNNs quantified through experimentation where we use a number of modeling benchmarks—synthetic and experimental data already experimented with in fuzzy or neurofuzzy modeling.

**Key Words** : HFPNN(Hybrid Fuzzy Polynomial Neural Networks), FSPN(Fuzzy Set based Polynomial Neuron), Symbolic coding GAs(Genetic Algorithms)

## 1. Introduction

A lot of researchers on system modeling have been interested in the multitude of challenging and conflicting objectives such as compactness, approximation ability, generalization capability and so on which they wish to satisfy. It is common practice to use various forms of neural networks and fuzzy systems in designing nonlinear system with good predictive abilities as well as approximation capabilities. In particular, when dealing with high-order nonlinear and multi-variable equations of the model, we require a vast amount of data for estimating all its parameters that is an important key to determine the model performance. The Group Method of Data Handling (GMDH)[1] introduced by A.G. Ivakhnenko is one of the approaches that help alleviate the problem. But, GMDH has some drawbacks. First, it tends to generate quite complex polynomial for relatively simple systems. Second, owing to its limited generic structure, GMDH also tends to produce an overly complex network(model) when it comes to highly nonlinear systems. In alleviating the problems of the GMDH algorithms, Polynomial Neural Networks(PNN)[2-3] was introduced as a new class of networks. Combination of neural networks and fuzzy systems (or neurofuzzy systems for short) has been recognized as a powerful alternative approach to develop fuzzy systems. We have investigated a new category of neuro-fuzzy networks, Fuzzy Set

based Polynomial Neural Networks (FSPNN) and developed Hybrid Fuzzy Set based Polynomial Neural Networks (HFSPNN) composed of multi-layer with two kinds of heterogeneous neurons that are fuzzy set based polynomial neurons (FSPNs) and polynomial neurons (PNs). Although the HFSPNN has flexible architecture whose potential can be fully utilized through a systematic design, it is difficult to obtain the structurally and parametrically optimized network because of the limited design of the nodes (viz. FSPNs and PNs) located in each layer of the network. In this paper, we study a genetic optimization-driven new neurofuzzy topology, called genetically optimized Hybrid Fuzzy Set based Polynomial Neural Networks (gHFSPNN) and discuss a comprehensive design methodology supporting their development. gHFSPNN is a network resulting from the combination of fuzzy inference system and PNN algorithm driven to genetic optimization. Each node of the first layer of gHFSPNN, that is a fuzzy polynomial neuron (FSPN) operates as a compact fuzzy inference system. The networks of the second and higher layers of the gHFSPNN come with a high level of flexibility as each node (processing element forming a PN). The determination of the optimal values of the parameters available within an individual PN and FSPN (viz. the number of input variables, the order of the polynomial, a collection of preferred nodes, and the number of membership functions (MFs)) leads to a structurally and parametrically optimized network.

## 2. The architecture of the hybrid fuzzy set based polynomial neural networks

### 2.1 The architecture of fuzzy set-based polynomial neurons

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**(FSPN) based layer of gHFSPNN**

The FSPN encapsulates a family of nonlinear "if-then" rules. When put together, FSPNs results in a self-organizing Fuzzy Set-based Polynomial Neural Networks (FSPNN). The FSPN consists of two basic functional modules. The first one, labeled by F, is a collection of fuzzy sets (here denoted by  $\{A_k\}$  and  $\{B_k\}$ ) that form an interface between the input numeric variables and the processing part realized by the neuron. The second module (denoted here by P) refers to the function -based nonlinear (polynomial) processing that involves some input variables This nonlinear processing involves some input variables ( $x_i$  and  $x_j$ ), which are capable of being the input variables (Here,  $x_p$  and  $x_q$ ), or entire system input variables. Each rule reads in the form.

$$\begin{aligned} &\text{if } x_p \text{ is } A_k \text{ then } z \text{ is } P_{pk}(x_i, x_j, a_{pk}) \\ &\text{if } x_q \text{ is } B_k \text{ then } z \text{ is } P_{qk}(x_i, x_j, a_{qk}) \end{aligned} \quad (1)$$

where  $a_{qk}$  is a vector of the parameters of the conclusion part of the rule while  $P(x_i, x_j, a)$  denoted the regression polynomial forming the consequence part of the fuzzy rule which uses several type of the high order polynomials besides the constant function forming the simplest version of the consequence.

The activation levels of the rules contribute to the output of the FSPN being computed as a weighted average of the individual condition parts (functional transformations)  $P_{(l,k)}$ .

$$\begin{aligned} z &= \frac{\sum_{l=1}^{\text{total input}} \left( \sum_{k=1}^{\text{total rule}_l} \mu_{(l,k)} P_{(l,k)}(x_i, x_j, a_{(l,k)}) \right)}{\sum_{l=1}^{\text{total input}} \mu_{(l,k)}} \\ &= \frac{\sum_{l=1}^{\text{total input}} \left( \sum_{k=1}^{\text{total rule}_l} \mu_{(l,k)} P_{(l,k)}(x_i, x_j, a_{(l,k)}) \right)}{\sum_{l=1}^{\text{total input}} \mu_{(l,k)}} \end{aligned} \quad (2)$$

**2.2 The architecture of polynomial neurons (PN) based layer of gHFSPNN**

As underlined, the PNN algorithm in the PN based layer of gHFSPNN is based on the GMDH method and utilizes a class of polynomials such as linear, quadratic, modified quadratic, etc. to describe basic processing realized there. The estimated output  $\hat{y}$  reads as

$$\hat{y} = c_0 + \sum_{i=1}^N c_i x_i + \sum_{i=1}^N \sum_{j=1}^N c_{ij} x_i x_j + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N c_{ijk} x_i x_j x_k \quad (3)$$

**3. Symbolic coding Genetic optimization of gHFSPNN**

Genetic algorithms (GAs) are optimization techniques based on the principles of natural evolution [5]. In essence, they are search algorithms that use operations found in natural genetics to guide a comprehensive search over the parameter space. In this study, for the optimization of the gHFSPNN model, GA

uses the serial method of symbolic coding, roulette-wheel used in the selection process, one-point crossover in the crossover operation, and a binary inversion (complementation) operation in the mutation operator. To retain the best individual and carry it over to the next generation, we use elitist strategy [6].

As mentioned, when we construct PNs and FSPNs of each layer in the conventional HFSPNN, such parameters as the number of input variables (nodes), the order of polynomial, and input variables available within a PN and a FSPN are fixed (selected) in advance by the designer.

**4. The algorithm and design procedure of genetically optimized HFSPNN (gHFSPNN)**

The framework of the design procedure of the HFSPNN based on genetically optimized multi-layer perceptron architecture comprises the following steps.

- [Step 1] Determine system's input variables.
- [Step 2] Form a training and testing data.
- [Step 3] Decide initial information for constructing the gHFSPNN structure.
- [Step 4] Decide a structure of the PN and FSPN based layer of gHFSPNN using genetic design.

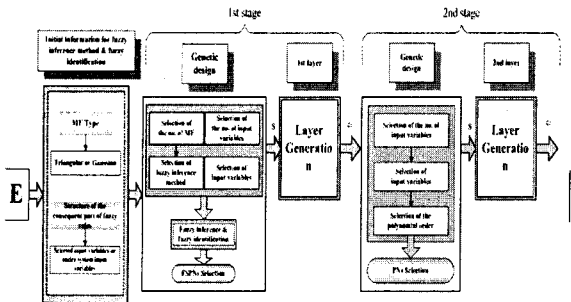


Fig 1. Overall scheme of the genetically-driven structural optimization process of gHFSPNN

- [Step 5] Estimate the coefficient parameters of the polynomial in the selected node (PN or FSPN).
- [Step 6] Select nodes (PNs or FSPNs) with the best predictive capability and construct their corresponding layer.
- [Step 7] Check the termination criterion.
- [Step 8] Determine new input variables for the next layer.

**5. Simulation study**

We demonstrate how the gHFSPNN can be utilized to predict future values of a chaotic Mackey-Glass time series [4,7]. The time series is generated by the chaotic Mackey-Glass differential delay equation comes in the form

$$\dot{x}(t) = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.1x(t) \quad (4)$$

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To obtain the time series value at each integer point, we applied the fourth-order Runge-Kutta method to find the numerical solution to (4). From the Mackey-Glass time series  $x(t)$ , we extracted 1000 input-output data pairs in the following format:

$$[x(t-24), x(t-18), x(t-12), x(t-6), x(t); x(t+6)]$$

where,  $t=118$  to  $1117$ . The first 500 pairs were used as the training data set while the remaining 500 pairs formed the testing data set. Table 1 summarizes the list of parameters used in the genetic optimization of the network.

Table 1. The list of parameters of GA

	Parameters	1 layer	2 to 3 layer
GA	Maximum gen	150	150
	Total population size	100	100
	Crossover rate	0.8	0.8
	Mutation rate	0.1	0.1
	String length	90	
HFSPNN	Maximal no. of inputs to be selected(Max)	$1 \leq I \leq \text{Max}$ (2~5)	$1 \leq I \leq \text{Max}$ (2~5)
	Polynomial Type(Type T) of the consequent part of rules	$1 \leq T \leq 4$	$1 \leq T \leq 4$
	Membership Function(MFs) type	Gaussian Triangular	
	No. of MFs per each input	$2 \leq T \leq 5$	

Table 2 gives a comparative summary of the network with other models.

Table 2. Comparative analysis of the performance of the network; considered are models reported in the literature

Model	PI	PIs	EPIs
Wang's model[5]	0.010		
ANFIS[6]		0.0016	0.0015
FNN model[7]		0.014	0.009
Recurrent neural networks[10]	0.0138		
SONN[11]	triangular	7.0e-4	6.0e-4
	gaussian	4.8e-5	7.1e-5
Our model	triangular	2.51e-4	3.09e-4
	gaussian	1.45e-4	2.12e-4

## 5. Concluding remarks

In this study, the GA-based design procedure of Hybrid Fuzzy Set based Polynomial Neural Networks (HFSPNN) along with its architectural considerations has been investigated. Through the consecutive generation of a layer through a growth process (iteration) of the gHFSPNN, the depth (layer size) and width (node size of each layer) of the network could be flexibly selected based on a diversity of local characteristics of these preferred FSPNs and PNs available within HFSPNN. The design methodology comes as a hybrid structural optimization (based on GMDH method and genetic optimization) and parametric

learning being viewed as two fundamental phases of the design process. The comprehensive experimental study involving well-known datasets quantify a superb performance of the network in comparison to the existing fuzzy and neuro-fuzzy models. Most importantly, through the proposed framework of genetic optimization we can efficiently search for the optimal network architecture (structurally and parametrically optimized network) and this becomes crucial in improving the performance of the resulting model.

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