종속형제어기의 영점의 영향을 고려한 저차제어기의 설계: 특성비지정 접근법

A Design Method Reducing the Effect of Zeros of a Cascaded Three-Parameters Controller: The Characteristic Ratio Assignment Approach

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Abstract -This paper presents a new approach to the problem of designing a cascaded three-parameters controller for a given linear time invariant (LTI) plant in unity feedback system. We consider a proportional-integral-derivative (PID) and a first-order controller with specified overshoot and settling time. This problem is difficult to solve because there may be no analytical solution due to the use of low-order controller and furthermore, the zeros of controller just appear in the zeros of feedback system. The key idea of our method is to impose a constraint on the controller parameters so that the zeros of resulting controller are distant from the dominant pole of closed-loop system to the left as far as the given interval. Two methods realizing the idea are suggested. We have employed the characteristic ratio assignment (CRA) in order to deal with the time response specifications. It is noted that the proposed methods are accomplished only in parameter space. Several illustrative examples are given.

Key Words : Three-parameters controller, Time response, Characteristic ratio, Generalized time constant, Pseudo break frequency.

1. Introduction

Under the structure of controller cascaded with a LTI plant in unit feedback system, we consider a problem of designing a three-parameters controller that meets the given time response specifications such as overshoot and settling time, if any. This is a simple but not easy to tackle the problem. The reason is that the existence of such controller can not be analytically solved for the case where the order of controller is lower than $n-2$, where n is the order of plant. Furthermore, the other difficulty comes from the fact that the zeros of the closed-loop system must include the zeros of controller. These zeros generally affect the overall system in its damping. For the similar problems with the two parameter configuration, many results based on the partial model matching concept[1] have been represented so far. Note that zeros of controller in this structure do not appear in the numerator of the overall system.

In this paper, we present a new design method that will be able to reduce the effect of zeros of controller on the step response. We begin with finding all stabilizing PID/first-order controllers by means of Datta[2] and Tantaris[3]. Let the set be S Then we will investigate a way that extracts a subset of controllers from S which satisfies the time response specifications. The key idea of this approach is to impose a constraint on the controller parameters so that zeros of controller are distant from the dominant pole of closed-loop system to the left in the s-plane as far as the given interval. Both dominant pole and the constraint can be approximately represented in terms of plant parameters and some design parameters, characteristic ratios α_i and a generalized time constant τ We will give several examples.

2. Definitions and Preliminaries

Consider a polynomial

$$\delta(s)=a_n s^n+\cdots+a_2 s^2+a_1 s+a_0. \quad (1)$$

The *characteristic ratios* α_i and the *generalized time constant* τ are defined as [1,4]

$$\alpha_1:=\frac{a_1^2}{a_2 a_0},\ \alpha_2:=\frac{a_2^2}{a_3 a_1},\ \cdots,\ \alpha_{n-1}:=\frac{a_{n-1}^2}{a_n a_{n-2}}; \quad (2)$$

$$\tau=\frac{a_1}{a_0}. \quad (3)$$

It was shown in [1,4] that α_i's of the denominator polynomial of rational model closely relate to the damping and the settling time of the system can be controlled by τ These two parameters will be used when we make a proper target polynomial for time response requirements.

The *characteristic pulsatances* β_i and the *pseudo break*

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※ This research is supported in part by grant R01-2003-000-11738-0 of KOSEF.

frequencies ω_i^p are defined by [4,5]

$$\beta_0^* = \frac{a_0}{a_1}, \beta_1^* = \frac{a_1}{a_2}, \cdots, \beta_{n-1}^* = \frac{a_{n-1}}{a_n}. \tag{4}$$

$$\omega_i^p = \sqrt{\frac{\eta_i}{\eta_{i+1}} \frac{\triangle_1^i}{\tau}}, i=0,1,\cdots,n-1. \tag{5}$$

where $\eta_k^* = 1 - \frac{2}{\alpha_k} + \frac{2}{\alpha_k \triangle_{k-1}^{k+1}} - \cdots + (-1)^k \frac{2}{\alpha_k \Pi_{j=1}^{k-1} \triangle_{k-j}^{k+j}}$, and

$$\triangle_i^j = \begin{cases} \Pi_{k=i, i<j}^j \alpha_k, & \text{if } 0<i<j \\ \alpha_i, & \text{if } 0<i=j \end{cases}. \quad \eta_0=1,\ \triangle_1^0=1.$$

Both definitions are used as approximate break points in Bode plot. It has been observed in [5] that the pseudo-break point is better approximation comparing with pulsatance. The lowest break frequencies ω_0^p and β_0 correspond to the equivalent real poles which are placed nearest from origin in complex plane. Here, the negative real pole nearest from origin is defined as the dominant pole. Therefore, we take the ω_0^p of a characteristic polynomial as its dominant pole.

The design objectives considered on the time response are: (i) slow response acceptable but strictly small overshoot, and (ii) small overshoot admissible but strict settling time. In the next section we will address how we design a PID/first-order controller satisfying the above objectives.

3. Controller design with fixed zeros

Fig.1 shows a cascaded feedback configuration. It is possible to consider a general transfer function model. However, in order to explain the effect of zeros of controller more explicitly, we consider an all-pole plant:

$$G(s) = \frac{N(s)}{D(s)} = \frac{n_0}{d_n s^n + \cdots + d_1 s + d_0}. \tag{6}$$

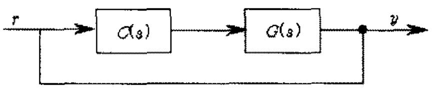

Fig.1. A unit feedback system with cascaded controller.

We first obtain a set of all stabilizing PID or first order controllers using the algorithms by Datta[2] and Tantaris[3]. The details about the algorithms are omitted here. Let the set be S Now, we present design approaches for PID and first order controllers respectively.

To reduce the effect of the zeros of controller, the following inequality are strongly required.

$$\gamma - \omega_o^p > 0 \tag{7}$$

where γ denotes a fixed zero of controller to be selected. From (5) and (7), the bound of α_1 can be calculated.

$$\alpha_1 > \frac{2\gamma^2\tau^2}{\gamma^2\tau^2 - 1}. \tag{8}$$

3.1 PID controller design

The transfer function of PID controller is

$$C(s) = \frac{B(s)}{A(s)} = \frac{k_d s^2 + k_p s + k_i}{s}. \tag{9}$$

The closed-loop transfer function is described by

$$T(s) = \frac{B(s)N(s)}{A(s)D(s) + B(s)N(s)} = \frac{n_0(k_d s^2 + k_p s + k_i)}{\delta(s)}. \tag{10}$$

where

$$\delta(s) = d_n s^{n+1} + \cdots + d_2 s^3 + (d_1 + n_0 k_d) s^2 + (d_0 + n_0 k_p) s + n_0 k_i. \tag{11}$$

It is clear that the design parameter is only α_1, τ since the controller parameters are only included in $\delta_0, \delta_1, \delta_2$ From (11), we have

$$\alpha_1 = \frac{(d_0 + n_0 k_p)^2}{n_0 k_i (d_1 + n_0 k_d)}, \quad \tau = \frac{d_0 + n_0 k_p}{n_0 k_i}. \tag{12}$$

The zeros of closed-loop system are identical to the roots of numerator of (9). Here we impose the following constraint on controller parameters so that the nearest zero from origin is placed at γ, where γ is properly chosen.

$$\gamma = \frac{k_p - \sqrt{k_p^2 - 4k_d k_i}}{2k_d}. \tag{13}$$

From (12) and (13), we can derive that

$$k_p = f_1(\alpha_1, \tau, \gamma) = \frac{\gamma^2 \tau \alpha_1 d_1 - \gamma^2 \tau^2 d_0 - \alpha_1 d_0}{(\gamma^2\tau^2 + \alpha_1 - \gamma\tau\alpha_1) n_0}. \tag{14a}$$

$$k_i = f_2(\alpha_1, \tau, \gamma) = \frac{\gamma^2 \alpha_1 d_1 - \gamma\alpha_1 d_0}{(\gamma^2\tau^2 + \alpha_1 - \gamma\tau\alpha_1) n_0}. \tag{14b}$$

$$k_d = f_3(\alpha_1, \tau, \gamma) = \frac{\gamma\tau\alpha_1 d_1 - \gamma\tau^2 d_0 - \alpha_1 d_1}{(\gamma^2\tau^2 + \alpha_1 - \gamma\tau) n_0}. \tag{14c}$$

According to (14), we see that the set (k_p, k_i, k_d) is obtained by design parameters (α_1, τ, γ). However, this set can not guarantee the stability. Thus, we have to check whether the set is in the stabilizing set S

Using the necessary condition of *Hurwitz* stability $\delta_i(s) > 0$, for $i = 0, 1, \cdots, n+1$, and substituting (14) into (12), the following inequality should be held.

$$\alpha_1 < \frac{\gamma^2\tau^2}{\gamma\tau - 1}. \tag{15}$$

Combining (8) and (15), the admissible range of α_1 becomes

$$\frac{2\gamma^2\tau^2}{\gamma^2\tau^2 - 1} < \alpha_1 < \frac{\gamma^2\tau^2}{\gamma\tau - 1}. \tag{16}$$

3.2 First-order controller design

Consider a first-order controller

$$C(s) = \frac{B(s)}{A(s)} = \frac{k_1 s + k_0}{s + l_0}. \tag{17}$$

The closed-loop transfer function can be described by

$$T(s) = \frac{B(s)N(s)}{A(s)D(s) + B(s)N(s)} = \frac{n_0(k_1 s + k_0)}{\delta(s)}. \tag{18}$$

where

$$\delta(s) = d_n s^{n+1} + \cdots + (d_1 + d_2 l_0) s^2 + (d_0 + d_1 l_0 + n_0 k_1) s + (d_0 l_0 + n_0 k_0). \tag{19}$$

α_1 and τ of (19) become

$$\alpha_1 = \frac{(d_0 + d_1 l_0 + n_0 k_1)^2}{(d_1 + d_2 l_0)(d_0 l_0 + n_0 k_0)}, \quad \tau = \frac{d_0 + d_1 l_0 + n_0 k_1}{d_0 l_0 + n_0 k_0}. \tag{20}$$

The zero of closed-loop system is equal to the roots of numerator of (17). We impose the following constraint so that the zero of controller is identical to the given zero γ

$$\gamma=\frac{k_0}{k_1}. \tag{21}$$

From (20) and (21), l_0, k_1, k_0 can be derived as

$$l_0=f_1(\alpha_1,\tau,\gamma)=\frac{\gamma\tau^2 d_0-(\gamma\tau-1)\alpha_1 d_1}{\tau^2(d_0-\gamma d_1)+(\gamma\tau-1)\alpha_1 d_2}. \tag{22a}$$

$$k_1=f_2(\alpha_1,\tau,\gamma)=\frac{(\gamma\tau^2 d_1-\tau^2 d_0+\alpha_1 d_2-\gamma\tau\alpha_1 d_2)n_0}{(\tau^2 d_0^2-\tau\alpha_1 d_1 d_0+\alpha_1 d_1^2-\alpha_1 d_2 d_0)}. \tag{22b}$$

$$k_0=f_3(\alpha_1,\tau,\gamma)=\frac{\gamma(\gamma\tau^2 d_1-\tau^2 d_0+\alpha_1 d_2-\gamma\tau\alpha_1 d_2)n_0}{(\tau^2 d_0^2-\tau\alpha_1 d_1 d_0+\alpha_1 d_1^2-\alpha_1 d_2 d_0)}. \tag{22c}$$

Equation (22) shows that the set (l_0, k_1, k_0) is obtained by (α_1,τ,γ). However, this set can not guarantee the stability. Thus, we have to check whether the set is included in S To satisfy the necessary condition of *Hurwitz* stability, substituting (22) into (20), we obtain

$$\alpha_1<\frac{\tau^2(\gamma d_1-d_0)}{(\gamma\tau-1)d_2}. \tag{23}$$

Combing (8) and (23), we have

$$\frac{2\gamma^2\tau^2}{\gamma^2\tau^2-1}<\alpha_1<\frac{\tau^2(\gamma d_1-d_0)}{(\gamma\tau-1)d_2}. \tag{24}$$

4. Illustrative examples and simulation results

In this section, several illustrative examples are given.

Example 1 (PID controller): Consider a plant shown in $G(s)=\frac{1}{s^2+7s+12}$.

According to (14), the admissible region of (k_p, k_i, k_d) to $\tau\in[0.5, 1.5]$ and for $\alpha_1=2.8, \gamma=4$, and to $\gamma\in[4, 20]$ for $\alpha_1=2.8, \tau=1$ are depicted in Fig.2.

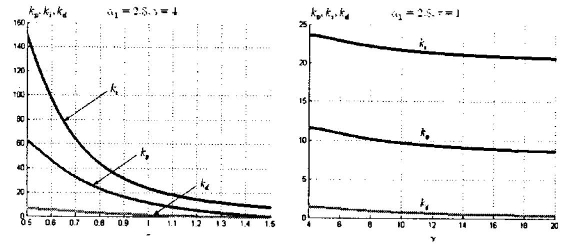

Fig.2. The admissible regions of (k_p, k_i, k_d) vs τ and γ

Table 1. and Table 2. show the time response results for the fixed α_1, γ and α_1, τ respectively.

Table 1. Overshoot and settling time to different τ

$\alpha_1=2.8, \gamma=4$	$\tau=0.5$	$\tau=1.0$	$\tau=1.5$
overshoot	7.5430%	0.1363%	0.0966%
settling time	0.8282s	1.6486s	3.4272s

Table 2. Overshoot and settling time to different γ

$\alpha_1=2.8, \tau=1$	$\gamma=4$	$\gamma=10$	$\gamma=20$
overshoot	0.1363%	0.2356%	0.2362%
settling time	1.6486s	3.4287s	3.4313s

Suppose that step response of a certain system requires the 1% overshoot and the 2% settling time of 2s.

The poles of plant are $-3, -4$. Put $\gamma=4$, $\tau=1$. From (16), we have $2.133<\alpha_1<5.333$. Select $\alpha_1=2.8$, then PID controller results in $C(s)=\frac{1.421s^2+11.58s+23.58}{s}$. The closed-loop poles are $-4, -2.2105\pm j1.004$ and zeros are $-4.1481, -4$. The overshoot and settling time are 0.1363% and 1.6486s. Therefore, the design is achieved successfully.

Example 2 (First-order controller): Consider a plant

$$G(s)=\frac{30}{0.01s^3+0.25s^2+s}.$$

Design objective: (i) overshoot< 1% (ii) settling time<2s.

The poles of plant are $0, -5, -20$. Let $\gamma=20$ and $\tau=0.8$. From (24), we obtain $2.008<\alpha_1<3.41$. When we select $\alpha_1=2.8$, then the designed first-order controller is $C(s)=\frac{0.04058s+0.8116}{s+18.26}$. The closed-loop poles and zero are $-20, -18.734, -2.2634\pm j1.1727$ and -20 respectively, the overshoot and settling time are each 0.1610%, 1.8540s. So, we conclude that the design is successfully achieved.

5. Concluding remarks

Subject to a unit feedback structure, a new approach for reducing the effect of controller's zeros has been proposed. We have considered a PID and first-order controller with the specified overshoot and settling time. The main idea of our method was to impose a constraint on the controller parameters so that the zeros of resulting controller are distant from the dominant pole of closed-loop system to the left as far as the given interval. We have employed the CRA in order to deal with the time response specifications. As illustrated in examples, we conclude that the proposed method works well.

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