적응 퍼지 슬라이딩 모드 제어기설계를 위한 새로운 해석

An Analysis of Adaptive Fuzzy Sliding Mode Controller of Nonlinear System

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Abstract - This paper is concerned with an Adaptive Fuzzy Sliding Mode Control(AFSMC) that the fuzzy systems are used to approximate the unknown functions of nonlinear system. In the adaptive fuzzy system, we adopt the adaptive law to approximate the dynamics of the nonlinear plant and to adjust the parameters of AFSMC. The stability of the suggested control system is proved via Lyapunov stability theorem, and convergence and robustness properties are demonstrated. The simulation results demonstrate that the performance is improved and the system also exhibits stability.

Key Words : adaptive sliding control, adaptive law, Lyapunov theorem

1. Introduction

Since Zadeh introduced the fuzzy set theory in 1965 [1], it has received much attention from various fields and has also demonstrsted nice performance in various applications. One of those successful fuzzy application is to model unknown nonlinear systems by a set of fuzzy rules.

Fuzzy systems can be used to model virtually any nonlinear systems within a required accuracy provided that enough rules are given. Based on the universal approximation theorem and by incorporating fuzzy systems into FSMC control schemes, many fuzzy sliding mode control approaches for nonlinear systems have been developed[2-4].

In this paper, the Fuzzy sliding mode control problem for high order nonlinear systems is studied. To control a nonlinear plant by the proposed FSMC, we adopt the adaptive control algorithm to approximate the nonlinear plant and tuning the parameters. The suggested controller can accommodate to the changed plants well and will possess the robust property against the dynamic uncertainties in the control system.

2. Fuzzy System

Construct the fuzzy system $\hat{f}(x\,\theta_f)$ from the $\prod_{i=1}^{n} p_i$ rules:

$$IF\ x_1\ is\ A_1^{l_1}\ and \cdots and\ x_n\ is\ A_1^{l_n},THEN\ y\ is\ E^{l_1 \cdots l_n} \tag{2-1}$$

where $l_i = 1, 2, \cdots, p_i$, $i = 1, 2, \cdots, n$.

Specifically, using the product inference engine, singleton fuzzifier and center average defuzzifier, we obtain

$$\hat{f}(x|\theta_f) = \frac{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} \bar{y}_f^{l_1 \cdots l_n} \left(\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i) \right)}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} \left(\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i) \right)} \tag{2-2}$$

$$\hat{g}(x|\theta_g) = \frac{\sum_{l_1=1}^{q_1} \cdots \sum_{l_n=1}^{q_n} \bar{y}_g^{l_1 \cdots l_n} \left(\prod_{i=1}^{n} \mu_{B_i^{l_i}}(x_i) \right)}{\sum_{l_1=1}^{q_1} \cdots \sum_{l_n=1}^{q_n} \left(\prod_{i=1}^{n} \mu_{B_i^{l_i}}(x_i) \right)} \tag{2-3}$$

Let $\bar{y}_f^{l_1 \cdots l_n}$ and $\bar{y}_g^{l_1 \cdots l_n}$ be the free parameters that are collected into $\theta_f \in R \prod_{i=1}^{n} p_i$ and $\theta_g \in R \prod_{i=1}^{n} q_i$, respectively, so we can rewrite (2-2), (2-3) as

$$\hat{f}(x|\theta_f) = \theta_f^T \xi_f(x) \tag{2-4}$$

$$\hat{g}(x|\theta_g) = \theta_g^T \xi_g(x) \tag{2-5}$$

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where $\xi(x)$ is a $\prod_{i=1}^{n} p_i (\text{or } q_i)$ - dimensional vector

$$\xi_{f_{l_1 \cdots l_n}}(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^{l}}(x_i)}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} \left(\prod_{i=1}^{n} \mu_{A_i^{l}}(x_i) \right)} \quad (2\text{-}6)$$

$$\xi_{g_{l_1 \cdots l_n}}(x) = \frac{\prod_{i=1}^{n} \mu_{B_i^{l}}(x_i)}{\sum_{l_1=1}^{q_1} \cdots \sum_{l_n=1}^{q_n} \left(\prod_{i=1}^{n} \mu_{B_i^{l}}(x_i) \right)} \quad (2\text{-}7)$$

We see that some parameters in θ_f and θ_g are chosen according to the rule (2-1), and the remaining parameters in θ_f and θ_g are chosen some structure.

Let the optimal parameter vectors of fuzzy logic systems θ_f^*, θ_g^*, we can put the approximation errors,

$$f_{\Delta} = f(\underline{x}, t) - \hat{f}(\underline{x}|\theta_f^*), \quad g_{\Delta} = g(\underline{x}, t) - \hat{g}(\underline{x}|\theta_g^*) \quad (2\text{-}8)$$

3. Sliding Mode Controller Design

Consider the nth-order nonlinear system:

$$x^{(n)} = f(\underline{x}) + g(\underline{x}) u, \quad y = x \quad (3\text{-}1)$$

where $f(x)$ and $g(x)$ are unknown continuous functions, $u, y \in R$ are the control input and the output of the system, respectively. $\underline{x} = (x, \dot{x}, \cdots, x^{(n-1)})^T \in R^n$ is the state vector of the system.

The object is to design a controller u such that state vector $\underline{x}$ will converge to the desired state $\underline{x}_d$

Let $e = x - x_d$ be the tracking error, and let

$$\underline{e} = \underline{x}(t) - \underline{x}_d(t) = (e, \dot{e}, \cdots, e^{(n-1)})^T \quad (3\text{-}2)$$

be the error vector.

The traditional sliding surface is chosen in the following form:

$$s(e) = \underline{c}\,\underline{e} = 0 \quad (3\text{-}3)$$

where $\underline{c} = [c_1 c_2 \cdots c_{n-1} 1]$.

Differentiating $s(e)$ is following form

$$\begin{aligned} \dot{s} &= \underline{c}\,\dot{\underline{e}} \\ &= c_1 \dot{e} + c_2 \ddot{e} + \cdots + c_{n-1} e^{(n-1)} + x^{(n)} - x_d^{(n)} \\ &= \sum_{i=1}^{n-1} c_i e^{(i)} + x^{(n)} - x_d^{(n)} \\ &= \sum_{i=1}^{n-1} c_i e^{(i)} + f(x) + g(x) u - x_d^{(n)} \end{aligned} \quad (3\text{-}4)$$

4. Adaptive Fuzzy Sliding Mode Control

Our control objective is to design a suitable control law based on an adaptive control algorithm for adjusting the parameter θ such that the plant output y follows the desired output x_d that is smooth and bounded.

Since $f(\underline{x}, t)$ and $g(\underline{x}, t)$ are unknown, we apply the fuzzy logic systems $\hat{f}(\underline{x}|\theta_f)$ and $\hat{g}(\underline{x}|\theta_g)$. Therefore, the controller becomes

$$u = u_a + u_s \quad (4\text{-}1)$$

where

$$u_a = \hat{g}(x)^{-1} \left(-\hat{f}(x) - \sum_{i=1}^{n-1} c_i e^{(i)} + x_d^{(n)} - k\,sgn(s) \right)$$

$$u_s = -\Gamma |u_a|, \quad \Gamma \geq \frac{g_{\Delta \max}}{\hat{g}(\underline{x}, \theta_g)}$$

So, (3-4) is rewritten

$$\begin{aligned} \dot{s} &= f(\underline{x}, t) - \hat{f}(\underline{x}|\theta_f) + (g(\underline{x}, t) - \hat{g}(\underline{x}|\theta_g)) u_a \\ &\quad + g(\underline{x}, t) u_s - k\,sgn(s) \\ &= f_{\Delta} + \phi_f^T \xi_f(x) + (g_{\Delta} + \phi_g^T \xi_g(x)) u_a \\ &\quad + g(\underline{x}, u) u_s - k\,sgn(s) \end{aligned} \quad (4\text{-}2)$$

where $\phi_f = \theta_f^* - \theta_f$, $\phi_g = \theta_g^* - \theta_g$.

Consider the Lyapunov candidate

$$V = \frac{1}{2} \left(s^2 + \frac{1}{\gamma_1} \phi_f^T \phi_f + \frac{1}{\gamma_2} \phi_g^T \phi_g \right) \quad (4\text{-}3)$$

where γ_1, γ_2 are positive constsnts. We obtain the time derivative of V as

$$\begin{aligned} \dot{V} &= s \dot{s} + \frac{1}{\gamma_1} \phi_f^T \dot{\phi}_f + \frac{1}{\gamma_2} \phi_g^T \dot{\phi}_g \\ &= s f_{\Delta} + s \phi_f^T \xi_f(x) + s(g_{\Delta} + \phi_g^T \xi_g(x)) u_a \\ &\quad + g(\underline{x}, u) s u_s - s k\,sgn(s) \\ &\quad + \frac{1}{\gamma_1} \phi_f^T \dot{\phi}_f + \frac{1}{\gamma_2} \phi_g^T \dot{\phi}_g \end{aligned}$$

$$\dot{V} \le s f_{\Delta} - k|s| + \frac{1}{\gamma_1} \phi_f^T(\dot{\phi}_f + \gamma_1 s \xi_f(x)) + \frac{1}{\gamma_2} \phi_g^T(\dot{\phi}_g + \gamma_2 s \xi_g(x) u_a) \quad (4\text{-}4)$$

Therefore, if we choose the following adaptive law

$$\dot{\theta}_f = \gamma_1 s \xi_f(x), \quad \dot{\theta}_g = \gamma_g s \xi_g(x) u_a \quad (4\text{-}5)$$

then $\dot{V}$ is negative definite.

5. Simulation Results

The control law is tested on the following inverted pendulum system. The dynamics of the inverted pendulum system arefollowing form:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{g\sin x_1 - amlx_2^2 \sin x_1 + a\cos x_1 u(t)}{\frac{4}{3} l - aml\cos^2(x_1)} \quad (5\text{-}1)$$

where g is the acceleration due to gravity, l is the half length of the pole, $a = \frac{1}{m+M}$, m is the mass of the pole, M is the mass of cart, x_1 and x_2 are the angular and velocity of the pole. we put the values of the variable as $m = 2.0kg$, $M = 8.0kg$, $2l = 1.0m$, $g = 9.8 m/s^2$.

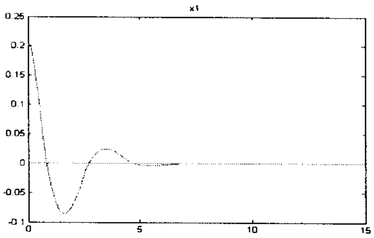

Fig. 1 The angular of the pole

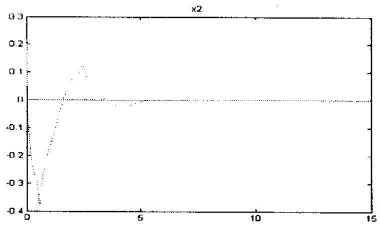

Fig. 2 The angular velocity of the pole

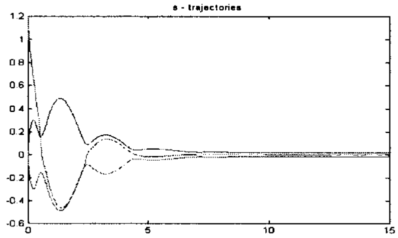

Fig. 3 s - trajectories

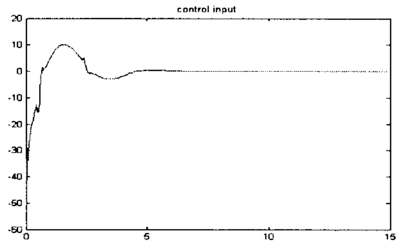

Fig. 4 control input of the system

6. Conclusions

In this paper, we proposed an adaptive fuzzy sliding mode controller for nonlinear systems. The prosed controller design is based on FSMC method and adaptive control technique.

We use adaptive control method to model the nonlinear plant and utilize the adaptive technique to adjust the parameter of the system. The stability of the proposed algorithm is proved via Lyapunov stability theorm.

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