# Power Series를 이용한 불확실성을 포함된 비선형 시스템의 지능형 디지털 재설계

# Intelligent Digital Redesign of Uncertain Nonlinear Systems Using Power Series

성화창*, 주영훈**, 박진배*, 김도완*
Sung Hwa Chang, Joo Young Hoon, Park Jin Bae, Kim Do Wan

**Abstract** - This paper presents intelligent digital redesign method of global approach for hybrid state space fuzzy-model-based controllers. For effectiveness and stabilization of continuous-time uncertain nonlinear systems under discrete-time controller, Takagi-Sugeno(TS) fuzzy model is used to represent the complex system. And global approach design problems viewed as a convex optimization problem that we minimize the error of the norm bounds between nonlinearly interpolated linear operators to be matched. Also by using the power series, we analyzed nonlinear system's uncertain parts more precisely. When a sampling period is sufficiently small, the conversion of a continuous-time structured uncertain nonlinear system to an equivalent discrete-time system have proper reason. Sufficiently conditions for the global state-matching of the digitally controlled system are formulated in terms of linear matrix inequalities (LMIs).

**Key Words** : uncertain nonlinear systems, intelligent digital redesign, T-S fuzzy model, power series

## 1. Introduction

Many complex dynamical systems comprise uncertain plants, so we have many technical problem to control the whole systems. The uncertainty about the plant arises from unmodelled dynamics, sensor noises, parameter variations, etc. Commonly, complex dynamic systems should be described by a continuous-time and/or discrete-time uncertain framework. As advanced digital implements, represented computer and microprocessor, many analog systems are converted to digital systems. Because digital device has more merit, better performance, more flexibility, and lower cost, than analog parts. For digital simulation, digital control and digital implementation of a continuous-time uncertain linear system, it is necessary to find an equivalent discrete-time uncertain model. A digital implementation of the continuous-time controller is indeed very desirable when the designed continuous-time controller uses some recent and advanced control algorithm.

저자 소개
*연세大學 전기전자공學科
**군산大學 전기전자공學科

So digital control of continuous-time systems have been more interest.

The efficient approach to design digital controller is called digital redesign, which was first proposed by Kuo. And Joo et la first apply digital redesign technique to complex nonlinear systems, and we call this new approach to intelligent digital redesign. This intelligent digital redesign technique is that complex nonlinear system has analyzed by using Takagi-Sugeno (T-S) fuzzy model which combines the fuzzy inference rules with some local linear state-space models for a global representation of the system dynamics [ ]. Lee et la proposed new intelligent digital redesign of global approach for the TS fuzzy systems which are represented as a convex optimization problem of the norm distance between nonlinearly interpolated linear operators to be matched, and thus can be cast into LMI framework,

But there has unsolved problem, which represented by uncertainty. Because of uncertain exponential parts, it is too difficult to apply intelligent digital redesign technique which includes uncertain parts. In this brief, we further develop a systematic method for the intelligent digital redesign of a hybrid state space TS fuzzy-model-based controller for sampled-data control of continuous-time complex dynamical systems by using power series.

## 2. Preliminaries

Consider a class of continuous-time nonlinear systems that contain parametric uncertainties, in the following form:

$$x(t) = f(x(t)) + \Delta f(x(t)) + (g(x(t)) + \Delta g(x(t)))u(t) \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $f(x(t)) \in R^n$ and $g(x(t)) \in R^n$ are nonlinear vector functions, and $\Delta f(x(t)), \Delta g(x(t))$ are uncertain vector functions. The $i$ th rule of T-S fuzzy system is formulated in the following form:

IF-THEN Form:

$R^i$ : *IF* $x_1(t)$ *is about* $\Gamma_1^i$ *and ... and* $x_n(t)$ *is about* $\Gamma_n^i$
*THEN* $\dot{x}_c(t) = (A_i + \Delta A_i)x_c(t) + (B_i + \Delta B_i)u_c(t), \quad i = 1, 2, ..., q,$

Defuzzified Form:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)),$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma^i{}_j(x_j(t)), \qquad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q}\omega_i(x(t))}, \quad (2)$$

where $\Gamma_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in $\Gamma_j^i$. We use the following fuzzy-model-based controller structure[6], and the resulting continuous-time closed-loop T-S fuzzy system becomes

$$\dot{x}_c(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))\{(A_i + \Delta A_i) + (B_i + \Delta B_i)K_{ci}\}x_c(t). \quad (6)$$

Next we discuss the discretization of the continuous-time T-S fuzzy system. Consider a class of T-S fuzzy system governed by

$$\dot{x}_d(t) = \sum_{i=1}^{q} \mu_i(z(t))\{(A_i + \Delta A_i)x_d(t) + (B_i + \Delta B_i)u_d(t)\}. \quad (7)$$

where $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined in the time interval $[kT, kT + T)$, where $T > 0$ is a sampling period. For the digital control, the digital fuzzy-model-based controller is employed following form:

$R^i$ : IF $z_1(kT)$ is about $\Gamma_1^i$ and ... and $z_n(kT)$ is about $\Gamma_1^i$
THEN $u_d(t) = K_d{}^i x_d(kT) \quad (8)$

for $t \in [kT, kT + T)$, where $K_d{}^i$ is the digital control gain matrix to be redesigned for the $i$ th rule, and the overall control law is given by

$$u_d(t) = \sum_{i=1}^{q} \mu_i(z(kT))K_d{}^i x_d(kT) \quad (9)$$

for $t \in [kT, kT + T)$.

For state matching, we need appropriate assumption because of time-varing polytopic system and approximated discretization.

*Assumption 1[3]:* Assume that the firing strength of the $i$ th rule, $\mu_i(z(t))$ is approximated by its value at time $kT$, that is

$$\mu_i(z(t)) \approx \mu_i(z(kT))$$

for $t \in [kT, kT + T)$. If a sufficiently small sampling period $T$ is chosen, Assumption 1 is reasonable.

Thanks to Assumption 1, we efficiently derive the discretization of T-S fuzzy system(7),

$$x_d(kT + T) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(kT))\mu_j(z(kT))(\hat{G} + \hat{H}_i k_{dj})x_d(kT) \quad (10)$$

where

$$\hat{G} = \exp(A_i + \Delta A)T,\ \hat{H} = \int_0^T e^{(A+\Delta A)\tau}(B + \Delta B)d\tau = (\hat{G} - I_n)(A_0 + \Delta A)^{-1}(B + \Delta B)$$

The pointwise dynamical behavior of the continuous-time closed-loop T-S fuzzy system(6) cam also be approximately discretized as

$$x_c(KT + T) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))\Phi_{ij}x_c(kT) \quad (11)$$

where $\Phi_{ij} = \exp\{((A_i + \Delta A) + (B_i + \Delta B)K_c{}^j)T\}$

And $\Delta A_i, \Delta B_i$ are the unknown and possibly time-varing matrices representing the uncertainties of the system. In order to find these gain matrices, $K_i$, we should remove the uncertain matrices under some reasonable assumptions.

*Assumption 2:* The uncertainty matrices $\Delta A_i$ and $\Delta B_i$ are norm bounded and have the following structures:

$$[\Delta A_i \quad \Delta B_i] = D_i F_i(t)[E_{i1} \quad E_{i2}]$$

where $D_i$, $E_{i1}$, and $E_{i2}$ are predetermined constant real matrices of appropriate dimensions, which represent the structures of the system uncertainties, and $F_i(t) \in R^{i \times j}$ is an unknown matrix function with Lebesque-measurable elements and satisfies

$$F_i^{\mathrm{T}}(t)F_i(t) \le I.$$

## 3. Main Result

Sate matching problem are linked the discretization of digitally redesigned T-S fuzzy model. To design an equivalent digital fuzzy-model-based controller from the continuous−time counterpart, so we take a global digital redesign approach. That is, a digital redesign algorithm for the uncertain nonlinear system is proposed, which is used for the design of the local digital control gains of the digital fuzzy−model−based controller.

*Problem 1(γ − Suboptimal Global Intelligent Digital Redesign Problem) [3]* : Given a well−constructed gain matrices $K_{ci}$ for the stabilizing analog fuzzy−model−based controller (13), find gain matrices $K_{di}$ for the digital fuzzy−model−based control law (11) such that the following constraints are satisfied.

Minimize $\gamma$ subject to

$\|\Phi_{ij} - G_i - H_i K_d^j\| < \gamma,\ i, j = 1, 2, \ldots, q,$ in the sense of the induced 2−norm distance measure.

$$\begin{bmatrix} -\gamma Q & * \\ \Phi_{ij}Q - \hat{G}Q - \hat{H}U_j & -\gamma I \end{bmatrix} < 0 \qquad (13)$$

where $\Phi_{ij} = \exp\{((A_i + \Delta A) + (B_i + \Delta B)K_c^{\ j})T\}$.

To solve this state matching, we have following Theorem which comprise Power Series:

*Theorem 1* : The exponential uncertainty terms which included the equation (13) are solved by Power Series,

$$\hat{G}_i \approx I_n + (A_i + \Delta A_i)T \qquad (14)$$

$$\hat{H}_i \approx (B_i + \Delta B_i)T \qquad (15)$$

$$\Phi_{ij} \approx I_n + (A_i + \Delta A_i)T + (B_i + \Delta B_i)K_c^{\ j}T \qquad (16)$$

*Proof)* The general power series are these form:

$$\exp(A_i T) = I_n + A_i T + A_i^{\ 2}\frac{T^2}{2} + \cdots \qquad (17)$$

As the above power series, the equation (14) and (16) is easily defined, but the control of second and the bigger terms are very difficult. In this brief, we assume that these terms are approximate 0, when sampling time should be sufficiently small. And equation (15) is that,

$$\begin{aligned} \hat{H}_i &= (\hat{G}_i - I_n)(A_i + \Delta A_i)^{-1}(B_i + \Delta B_i) \\ &\approx (I_n + (A_i + \Delta A_i)T - I_n)(A_i + \Delta A_i)^{-1}(B_i + \Delta B_i) \\ &\approx (A_i + \Delta A_i)(A_i + \Delta A_i)^{-1}(B_i + \Delta B_i)T \\ &= (B_i + \Delta B_i)T \end{aligned}$$

The exponential terms are easily defined by power series, but the uncertainty term $\Delta A_i, \Delta B_i$ are still discussed. Following Lemma can help us to solve these difficulties.

*Lemma 1 [2]* : Given constant symmetric matrices $N, O,$ and $L$ of appropriate dimensions, the following two inequalities are equivalent:

(a) $O > 0,\quad N + L^T O L < 0,$

(b) $\begin{bmatrix} N & L^T \\ L & -O^{-1} \end{bmatrix} < 0$ *or* $\begin{bmatrix} -O^{-1} & L \\ L^T & O \end{bmatrix} < 0.$

*Lemma 2 [2]* : Given constant matrices $D$ and $E$, and a symmetric constant matrix $S$ of appropriate dimensions, the following inequality holds:

$$S + DFE + E^T F^T D^T < 0,$$

where $F$ satisfies $F^T F \le I$, if and only if for some $\varepsilon > 0$,

$$S + [\varepsilon^{-1}E^T \quad \varepsilon D]\begin{bmatrix} \varepsilon^{-1}E \\ \varepsilon D^T \end{bmatrix} < 0.$$

Applying above Lemma 1 and 2 to (13), we have another Theorem, which contain the LMI approach.

*Theorem 2(Globally state matching):* If there exist symmetric positive definite matrix $Q$, symmetric positive-semidefinite matrix $O$, constant matrices $F_i$ and a possibly small positive scalar such that the following generalized eigenvalue problem (GEVP) has solutions:

$$\underset{Q,\,O,\,F_i}{\text{Minimize}}\ \gamma \ \text{subject to}$$

$$\begin{bmatrix} -\gamma Q & * & * & * \\ B_i T(K_c^{\ j} - F_j) & -\gamma I & * & * \\ (E_{2i}T(K_c^{\ j} - F_j))^T & 0 & -\varepsilon_{ii} I & * \\ 0 & D_i^T & 0 & -\varepsilon_{ii} I \end{bmatrix} < 0 \qquad (18)$$

## 참 고 문 헌


[1] KUO, B.C, "Digital control systems", Saunders College publishing, 1992, 2nd edn.

[2] L.Xie, "Output feedback H∞ control of systems with parameter uncertainties," Int. J. Contr., Vol. 63, No.4, pp. 741−750, 1996.

[3] Ho Jae Lee, Hagbae Kim, "A New Intelligent Digital Redesign for T−S fuzzy systems: Global Approach", IEEE Trans. Vol. 12, No2, April 2004.

[4] A. J. Viterbi and J. K. Omura, Principles of Digital communication and coding, New York: McGraw−Hill, 1979.

[5] CHEN, T, and FRANCIS, B, "Optimal sampled−data control systems", Springer, 1995.

[6] W. Chang, J.B. Park, H.J. LEE and Y.H. Joo, "LMI approach to digital redesign of linear time−invariant systems", IEE Proc−Control Theory, Appl., Vol. 149, No. 4, July 2002.