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# 이동 로봇의 연동 제어를 위한 동기화 기법

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## A study on Mutual Cooperative Control in the Chaos Mobile Robot

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### Abstract

In this paper, we propose that the synchronization method for mutual cooperative control in the mobile robot. In order to achieve the synchronization for mutual cooperative control in the mobile robot, we apply coupled synchronization technique and driven synchronization technique in the mobile robot with obstacle.

### 1. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into the real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1-2], chaos synchronization and secure/crypto communication [3-7], Chemistry [8], Biology [9], and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods with mutual cooperative control.

In this paper, we propose a chaotic mobile robots that have unstable limit cycles in a chaos trajectory surface with Lorenz equation, n-double scroll equation. We assume that all obstacles in the chaos trajectory surface have a Van der Pol

equation with an unstable limit cycle. When chaos robots meet obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Lorenz equation or hyper chaos equation, the obstacles reflect the chaos robots.

Computer simulations also show multiple obstacles can be avoided by using mutual cooperative control with an Lorenz equation or hyper chaos equation.

### 2. Chaotic Mobile Robot embedding Chaos Equation

#### 2.1. Mobile Robot

As the mathematical model of mobile robots, we assume a two-wheeled mobile robot as shown in Fig. 1.

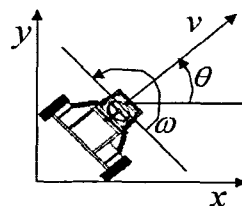


Fig. 1 Two-wheeled mobile robot

Let the linear velocity of the robot  $v$  [m/s] and angular velocity [rad/s] be the input to the system. The state equation of the four-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where  $(x,y)$  is the position of the robot and  $\theta$  is the angle of the robot..

### 2.2 Some Chaos Equations

In order to generate chaotic motions for the mobile robot, we apply some chaos equations such as an Lorenz equation or hyper chaos equation

#### 1) Lorenz equation

The Lorenz equation describes the famous chaotic phenomenon. We define the Lorenz equation as follows:

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= \gamma x - y - xz \\ \dot{z} &= xy - bz \end{aligned} \quad (2)$$

where  $\sigma = 10, \gamma = 28, b = 8/3$ .

#### 2) Hyper-chaos equation

Hyper-chaos equation is one of the simplest physical models that have been widely investigated by mathematical, numerical and experimental methods for complex chaotic dynamic. We can easily make hyper-chaotic equation by using some of connected N-double scroll. We can derive the state equation of N-double scroll equation as followings.

$$\begin{aligned} \dot{x} &= \alpha[y - h(x)] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \quad (3)$$

Where,

$$h(x) = m_{2n-1}x + \frac{1}{2} \sum_{i=1}^{2n-1} (m_{i-1} - m_i)|x + c_i| - |x - c_i| \quad (8)$$

In order to make a hyper-chaos, we have compose to 1 dimensional CNN(Cellular Neural Network) which are identical two N-double scroll circuits and then we have to connected each cell by using unidirectional coupling or diffusive

coupling. In this paper, we used to diffusive coupling method. We represent the state equation of x-diffusive coupling and y-diffusive coupling as follows.

#### x-diffusive coupling

$$\begin{aligned} \bar{x}^{(j)} &= \alpha[y^{(j)} - h(x)^{(j)}] + D_x(x^{(j-1)} - 2x^{(j)} + x) \\ \bar{y}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} \\ \bar{z}^{(j)} &= -\beta y^{(j)}, \quad j=1,2,\dots,L \end{aligned} \quad (4)$$

#### y-diffusive coupling

$$\begin{aligned} \bar{x}^{(j)} &= \alpha[y^{(j)} - h(x)^{(j)}] \\ \bar{y}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x) \\ \bar{z}^{(j)} &= -\beta y^{(j)}, \quad j=1,2,\dots,L \end{aligned} \quad (5)$$

where, L is number of cell.

### 2.3 Embedding of Chaos circuit in the Robot

In order to embed the chaos equation into the mobile robot, we define and use an Lorenz equation or hyper chaos equation as follows.

#### 1) Lorenz equation

Combination of equation (1) and (2), we define and use the following state variables:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \sigma(y-x) \\ \gamma x - y - xz \\ xy - bz \\ v \cos x_3 \\ u \sin x_3 \end{pmatrix} \quad (6)$$

Eq. (6) is including Lorenz equation. The behavior of Lorenz equation is chaos. We can get chaotic mobile robot trajectory.

#### 2) Hyper-chaos equation

Combination of equation (1) and (4) or (5), we define and use the following state variables (7) or

$$\begin{pmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \\ \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{pmatrix} = \begin{pmatrix} a[y^{(j)} - h(x^{(j)})] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ x^{(j)} - y^{(j)} + z^{(j)} \\ -\beta y^{(j)} \\ v \cos x_3 \\ v \sin x_3 \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \\ \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{pmatrix} = \begin{pmatrix} a[y^{(j)} - h(x^{(j)})] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ -\beta y^{(j)} \\ v \cos x_3 \\ v \sin x_3 \end{pmatrix} \quad (8)$$

Using equation (7) and (8), we obtain the embedding chaos robot trajectories with Hyper-chaos equation.

### 3. Mutual cooperative control by using synchronization methods

To achieve mutual cooperative control in the mobile robot, we applied the chaotic synchronization technique from the several mobile robot trajectories. Firstly, we applied coupled synchronization method and then we also applied driven synchronization method for mutual cooperative control between the several robots.

#### 3.1 Coupled synchronization method

In order to accomplish mutual cooperative control in the several chaos mobile robots, we applied a coupled synchronization method proposed by Cuomo [11] in the Lorenz chaos mobile robots.

To applied coupled synchronization method in the Lorenz circuit, transmitter-receive state equations are following:

Transmitter state equation

$$\begin{aligned} \dot{x} &= \sigma(y - x) + k(x - y) \\ \dot{y} &= \gamma x - y - xz \\ \dot{z} &= xy - bz \end{aligned} \quad (9)$$

Receiver state equation

$$\begin{aligned} \dot{x} &= \sigma(y - x) + k'(y - x) \\ \dot{y} &= \gamma x - y - xz \\ \dot{z} &= xy - bz \end{aligned} \quad (10)$$

In order to accomplish synchronization of the Eq. (9), (10), we need to find stable coupled-register  $R_x$  value between the transmitter and the receiver.

#### 3.2 Coupled mutual cooperative control in the Hypers chaos robot by using coupled synchronization

To accomplish synchronization of the two chaos robot embedding hyper chaos circuit, first we formed each state equation for Eq. (11), (12). Then found coupled coefficient k and k' by using stability criteria. After that, we applied k and k' within stable area to perform computer simulation.

Main chaos robot's state equation

$$\begin{aligned} \dot{x}_1 &= a[y^{(j)} - h(x^{(j)})] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) + k \\ \dot{x}_2 &= x^{(j)} - y^{(j)} + z^{(j)} \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= v \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \quad (11)$$

Sub chaos robot's state equation

$$\begin{aligned} \dot{x}_1 &= a[y^{(j)} - h(x^{(j)})] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) + k' \\ \dot{x}_2 &= x^{(j)} - y^{(j)} + z^{(j)} \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= v \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \quad (12)$$

#### 3.3 Driven mutual cooperative control in the hyper chaos robot by using driven synchronization

To accomplish synchronization of the two chaos robot embedding hyper chaos circuit, first we formed each state equation for Eq. (13), (14). Then found driven coefficient k and k' by using stability criteria. After that, we applied k and k' within stable area to perform computer simulation.

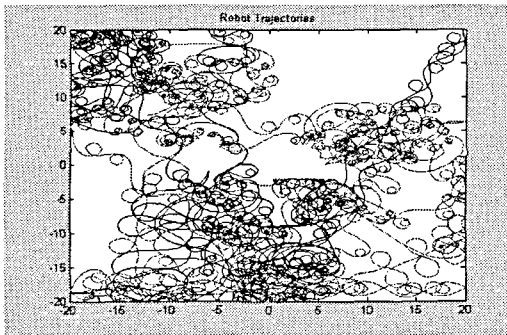
Main chaos robot's state equation

$$\begin{aligned}
 \dot{x}_1 &= \alpha[y^{(j)} - h(x^{(j)})] \\
 \dot{x}_2 &= x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) + k \\
 \dot{x}_3 &= -\beta y^{(j)} \\
 \dot{x} &= v \cos x_3 \\
 \dot{y} &= v \sin x_3
 \end{aligned}
 \tag{13}$$

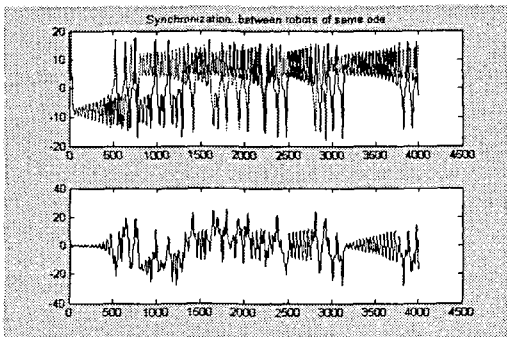
Sub chaos robot's state equation

$$\begin{aligned}
 \dot{x}_1 &= \alpha[y^{(j)} - h(x^{(j)})] + k \\
 \dot{x}_2 &= x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) + k' \\
 \dot{x}_3 &= -\beta y^{(j)} \\
 \dot{x} &= v \cos x_3 \\
 \dot{y} &= v \sin x_3
 \end{aligned}
 \tag{14}$$

The Fig. 2 and 3 showing synchronization of two hyper chaos robot after using Eq.(11) and (12). Fig. 2 is showing the result of synchronization at fixed obstacle.



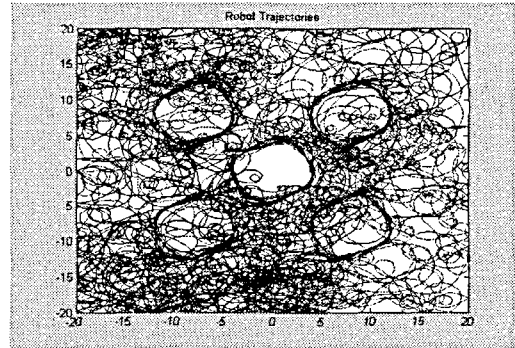
(a) Robot trajectory



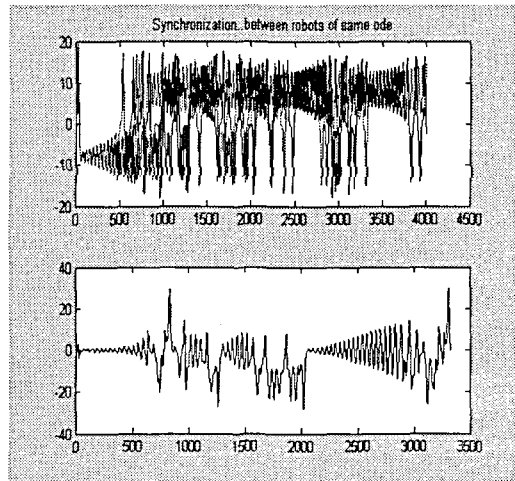
(b) The result of synchronization

Fig.2 The result of synchronization in the Lorenz robot with fixed obstacles by using coupled mutual cooperative control

The Fig. 3 showing synchronization of two Lorenz chaos robot after using Eq.(13) and (14). Fig. 3 is showing the result of the synchronization after applying hidden obstacle, VDP.



(a) Robot trajectory



(b) The result of synchronization

Fig. 3 The result of synchronization in the hyper chaos robot with hidden obstacles using driven mutual cooperative control

#### 4. Conclusion

In this paper, we proposed a chaotic robots, which employs a robots with Lorenz or hyper chaos equation trajectories, and also proposed a robot synchronization methods in which coupled-synchronization and driven synchronization.

We designed chaotic robot trajectories such that

the total dynamics of the robots was characterized by a Lorenz or hyper chaos equation and we also designed the chaotic robot trajectories to include an obstacle avoidance method. As a result, we realized that the result of synchronization is generalized synchronization.

#### ACKNOWLEDGEMENT

This work has been carried out under University Research Program supported by Ministry of Information & Communication in Republic of Korea

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