Design of A Fuzzy Logic Control System and Its Stability Analysis

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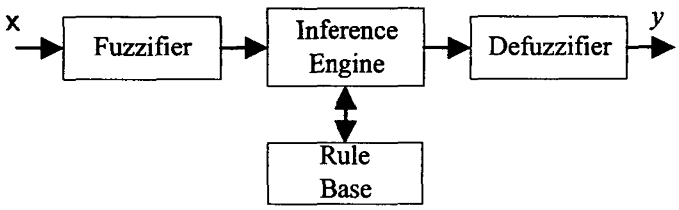
Introduction

- Fuzzy Logic Controller (FLC)
  - ✓ employs fuzzy logic and fuzzy inference in control systems
  - ✓ is superior to their corresponding linear controllers to control linear and nonlinear processes
  - ✓ successfully applied to many industrial and commercial applications



Introduction

◆ Fuzzy Control Rule

IF <process states> THEN <control variable>

$R^{(j)}$: IF x_1 is LX_1^j and ⋯ and x_n is LX_n^j
THEN y is LY^j

◆ Input Variables of FLCs

✓ Representing the contents of the rule antecedent

✓ ex) Error, Change-of-error, Sum-of-errors, and etc

◆ Output Variables of FLCs

✓ Representing the contents of the rule consequent

✓ ex) Control Input or Incremental Control Input

New Design of FLC

◆ n-th Order Process (linear or nonlinear)

$$\dot{\mathrm{x}} = \mathrm{F}(\mathrm{x}, u) \qquad y = x$$

$$\mathrm{x} = [x_1, x_2, \cdots, x_n]^T = [x, \dot{x}, \cdots, x^{(n-1)}]^T$$

▶ $\mathrm{F}(\mathrm{x}, u)$: partially known continuous functions

▶ $\mathrm{x}(t) \in R^n$: the process state vector

▶ $y(t) \in R$: the output of the system

◆ Tracking Error Vector

$$\mathrm{e}(t) = \mathrm{x}(t) - \mathrm{x}_d(t) = [e, \dot{e}, \cdots, e^{(n-1)}]^T$$

New Design of FLC

◆ Various Rule Tables

✓ Kickert and Mamdani

<table>
<tr><th>$\dot{e}$ \ e</th><th>NB</th><th>NM</th><th>NS</th><th>NO</th><th>PS</th><th>PM</th><th>PB</th></tr>
<tr><td>PB</td><td colspan="2" rowspan="2"></td><td rowspan="2">NM</td><td colspan="4" rowspan="2">NB</td></tr>
<tr><td>PM</td></tr>
<tr><td>PS</td><td colspan="2">PS</td><td>NO</td><td colspan="2">NM</td><td colspan="2"></td></tr>
<tr><td>PO</td><td colspan="2" rowspan="2">PM</td><td rowspan="2">PS</td><td rowspan="2">NO</td><td rowspan="2">NS</td><td colspan="2" rowspan="2">NM</td></tr>
<tr><td>NO</td></tr>
<tr><td>NS</td><td colspan="2"></td><td colspan="2">PM</td><td>NO</td><td colspan="2">NS</td></tr>
<tr><td>NM</td><td colspan="4" rowspan="2">PB</td><td rowspan="2">PM</td><td colspan="2" rowspan="2"></td></tr>
<tr><td>NB</td></tr>
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(O: zerO, N: Negative, P: Positive
S: Small, M: Medium, B: Big)

Li and Gatland

e \ $\dot{e}$	NL	NM	NS	ZR	PS	PM	PL
PL	zr	ps	pm	pl	pl	pl	pl
PM	ns	zr	ps	pm	pl	pl	pl
PS	nm	ns	zr	ps	pm	pl	pl
ZR	nl	nm	ns	zr	ps	pm	pl
NS	nl	nl	nm	ns	zr	ps	pm
NM	nl	nl	nl	nm	ns	zr	ps
NL	nl	nl	nl	nl	nm	ns	zr

New Design of FLC

◆ Most Conventional FLCs: PD-type or PI-type

✓ Input Variables : Error and Change-of-error

✓ Output Variable : Control Input or Incremental Control Input

→ Suitable for Simple Second Order Processes

◆ Common Properties of 2-dim. Rule Tables

✓ Zero band is built around the neighborhood of the main diagonal line in the normalized space

✓ Negative (Positive) control actions are exerted above (below) the line

✓ Absolute magnitude of control actions is strengthened proportional to the distance from the zero band

New Design of FLC

◆ Consideration of Conventional FLC: PD-type FLC

R^l_{PD}: IF e is LE^l and $\dot{e}$ is LDE^l THEN u is LU^l

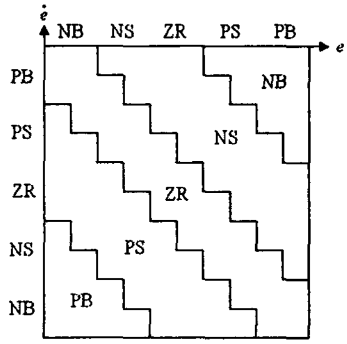
R^1_{PD}: IF e is NB and $\dot{e}$ is PB THEN u is ZR

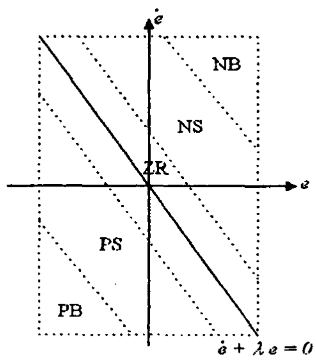
$\dot{e}$ \ e	NB	NS	ZR	PS	PB
PB	ZR	NS	NS	NB	NB
PS	PS	ZR	NS	NS	NB
ZR	PS	PS	ZR	NS	NS
NS	PB	PS	PS	ZR	NS
NB	PB	PB	PS	PS	ZR

New Design of FLC

◆ Consideration of Conventional FLC

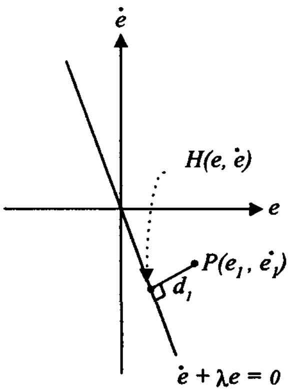
→ Multilevel Relay Controller with Five Bands





New Design of FLC

◆ Derivation of New Variable called Signed Distance



▶ **d_1: Distance between $H(e, \dot{e})$ and $P(e_1, \dot{e}_1)$**

$$d_1 = [(e - e_1)^2 + (\dot{e} - \dot{e}_1)^2]^{1/2}$$

$$= \frac{|\dot{e}_1 + \lambda e_1|}{\sqrt{1 + \lambda^2}}$$

▶ **Definition of Signed Distance d_s**

$$d_s = sgn(s_1) \cdot \frac{|\dot{e} + \lambda e|}{\sqrt{1 + \lambda^2}}$$

$$= \frac{\dot{e} + \lambda e}{\sqrt{1 + \lambda^2}}$$

New Design of FLC

◆ Design of Simple-structured FLC

✓ Single-input FLC (SFLC)

$$u = -\phi(d_s)$$

$$IF \ \ d_s \ is \ LDL^{(k)} \ \ THEN \ \ u \ is \ LU^{(k)}$$

d_s	NB	NS	ZR	PS	PB
u	PB	PS	ZR	NS	NB

New Design of FLC

- Extension to General Case
  - ✓ Input Variables : Error and its time derivative terms
  - ✓ Rule Form

$R_{G2}^{(k)}$: *If* e_1 *is* $LE_1^{(k)}$, e_2 *is* $LE_2^{(k)}$, $\cdots$, and e_n *is* $LE_n^{(k)}$ *then* u *is* $LU^{(k)}$

  - ✓ Switching line is changed to switching hyperplane on n-dim. Space
  - ✓ Switching Hyperplane

$$S_l:\ e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \cdots + \lambda_2\dot{e} + \lambda_1 e = 0$$

New Design of FLC

- Extension to General Case
  - ✓ Distance with Sign between Switching Hyperplane and Operating Point

$$D_s = \frac{e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \cdots \lambda_2\dot{e} + \lambda_1 e}{\sqrt{1 + \lambda_{n-1}^2 + \cdots \lambda_2^2 + \lambda_1^2}}$$

  - ✓ Information about all process states as well as the error and the change-of-error
  - ✓ Rule Form for SFLC in n-dim. Case

$R_{GSI}^{(k)}$: *IF* D_s *is* $LGDL^{(k)}$ *THEN* u *is* $LU^{(k)}$

D_s	NB	NS	ZR	PS	PB
u	PB	PS	ZR	NS	NB

Stability Analysis

◆ Perturbed Lure System [3]

✓ Expanding the system into a Taylor series about (x_0, u_0)

$$\dot{x} = Ax + Bu + g(x, u),$$
$$y = x,$$

$$A = \frac{\partial F}{\partial x}\bigg|_{(x_0,\, u_0)}, \quad B = \frac{\partial F}{\partial u}\bigg|_{(x_0,\, u_0)}$$

- g(x,u) stands for higher-order terms in x and u

Stability Analysis

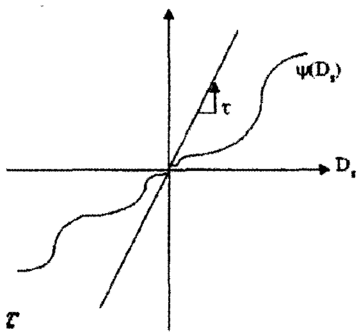
◆ From General Case of Proposed FLC

✓ Output of SFLC is symmetric with respect to zero and bounded by a linear gain

✓ Control input u

$$u = -\phi(D_s)$$

✓ $\phi(\cdot)$ is a nonlinear function that belongs to a sector $[0, \tau]$, where τ … positive constant



Stability Analysis

✓ As $x_d = 0$

$$D_s = \frac{e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \cdots + \lambda_2\dot{e} + \lambda_1 e}{\sqrt{1 + \lambda_{n-1}^2 + \cdots \lambda_2^2 + \lambda_1^2}}$$

$$= \frac{1}{\sqrt{\sum_{i=1}^{n}\lambda_i^2}}\left(x^{(n-1)} + \lambda_{n-1}x^{(n-2)} + \cdots + \lambda_2\dot{x} + \lambda_1 x\right)$$

$$= C_d x,$$

$$C_d = \frac{1}{\sqrt{\sum_{i=1}^{n}\lambda_i^2}}[\lambda_1, \lambda_2, \cdots, \lambda_{n-1}, 1], \quad \lambda_n = 1.$$

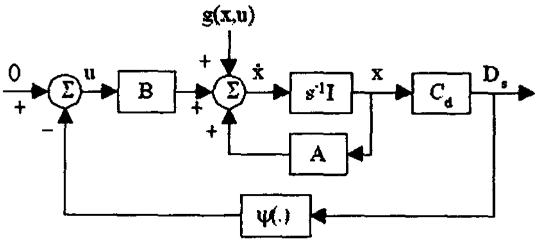
Stability Analysis

✓ **The system with SFLC**

$$\dot{x} = Ax + Bu + g(x,u)$$

$$u = -\phi(D_s) \qquad \phi(D_s)[\phi(D_s) - \tau D_s] \le 0$$

$$D_s = C_d x$$



Stability Analysis

◆ ***Theorem*** [5]

Consider the above system, where A is Hurwitz, (A,B,Cd) is a minimal realization of $G(s) = C_d(sI - A)^{-1}B$, and the nonlinearity g(x,u) is bounded as follows:

$$\|g(x,u)\|_2 \le \nu\|x\|_2 \le \frac{\varepsilon_g}{2\|P\|_{i2} + 2\eta\tau^2\|C_d\|_2^2}\|x\|_2 ,$$

$$\|P\|_{i2} = [\lambda_{max}(P^*P)]^{1/2},\ \nu > 0,\ \varepsilon_g > 0, ,$$

and $\phi(\cdot)$s a time-invariant nonlinearity that satisfies the sector condition globally. Then the system is absolutely stable if there is $\eta \ge 0$vith $-\frac{1}{\eta}$ not an eigenvalue of A such that

$$\mathrm{Re}[1 + (1 + j\eta\omega)\tau G(j\omega)] > 0, \quad \forall\omega \in R ,$$

$$G(j\omega) = \mathrm{Re}[G(j\omega)] + j\,\mathrm{Im}[G(j\omega)] .$$

Simulation Example

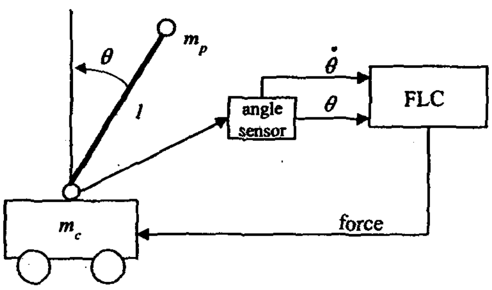
◆ Inverted Pendulum System

✓ System Configuration

$$\ddot{\theta} = \frac{g\sin\theta + a\cos\theta - \mu_p w^2 l\cos\theta\sin\theta}{l(4/3 - \mu_p\cos^2\theta)} .$$

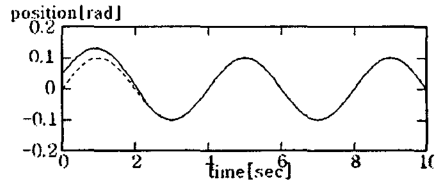
$$\mu_p = \frac{m_p}{m_p + m_c} ,$$

$$a = \frac{F}{m_p + m_c} .$$

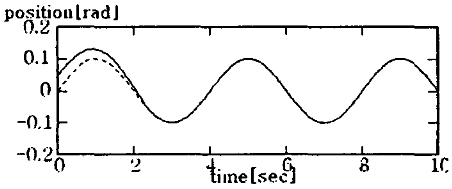


Simulation Example

✓ Control Performance



(a) Conventional FLC



(b) Proposed SFLC

Concluding Remarks

- Design of Simple FLC called SFLC
- Properties of SFLC: 1-dim. Rule Table
  - ✓ Decrement of No. of Control Rules
  - ✓ Decrement of No. of Tuning Parameters
  - ✓ Alleviation of Computational Complexity
  - ✓ Easy of Generation, Modification, and Tuning of Control Rules
- Stability Analysis for Proposed SFLC: Absolute Stability
- Simulation using Inverted Pendulum System

References

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[3] G.-C. Hwang and S.-C. Lin, "A Stability Approach to Fuzzy Control Design for Nonlinear Systems," *Fuzzy Sets and Systems*, vol. 48, pp. 279-287, 1992.

[4] B.-J. Choi, S.-W. Kwak and B. K. Kim, "Design of a Single-Input Fuzzy Logic Controller and Its Properties," *Fuzzy Sets and Systems*, vol. 106, no. 3, pp. 299-308, 1999.

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