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구간치 퍼지수의 상관계수

Correlation coefficients of interval-valued fuzzy numbers

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요약

본 논문에서는 리만 적분 대신에 쇼케이 적분을 활용하여 구간치 퍼지수의 상관계수를 정의한다. 또한 이들에 관한 성질등을 조사한다.

Abstract

In this paper, we define correlation coeeficients of interval-valued fuzzy numbers by utilizing Choquet integrals. Also we discuss some properties of them.

Keywords : fuzzy measures, Choquet integrals, interval-valued fuzzy numbers, correlation coefficients.

1. Introduction.

In this paper, we consider interval-valued fuzzy numbers which were introduced at first by Gorzalczany([1]) and Turksen([2]). Utilizing Riemann integral, G. Wang and X. Li([3]) defined the correlation coefficient of interval-valued fuzzy numbers and studied some of its important properties. They had been applied to the fields of approximate inference, signal transmission and control etc.

The main purpose of this paper is to define correlation coefficients of interval-valued fuzzy numbers by utilizing Choquet integral instead of Riemann integral and to discuss some properties of them.

2. Interval-numbers and interval-valued fuzzy numbers.

In this section, we introduce some results of interval numbers and interval-valued fuzzy numbers about operations and orders. Throughout this paper, let I be a closed unit interval, i.e., $I=[0,1]$. Let $[I]=\{\bar{a}=[a^-,a^+] \mid a^- \leq a^+, a^-, a^+ \in I\}$ especially for arbitrary $a \in I$, putting $a=[a,a]$, then $a \in [I]$ is obvious.

For any $\bar{a}=[a^-,a^+], \bar{b}=[b^-,b^+] \in [I]$, we define

$\bar{a}\wedge\bar{b}=[a^-\wedge b^-, a^+\wedge b^+]$,

$\bar{a}\vee\bar{b}=[a^-\vee b^-, a^+\vee b^+]$,

$\bar{a}'=[1-a^+, 1-a^-]$,

$\bar{a} \le \bar{b}$ if and only if $a^- \le b^-$, $a^+ \le b^+$,

$\bar{a} < \bar{b}$ if and only if $\bar{a} \le \bar{b}$ and $\bar{a} \ne \bar{b}$,

$\bar{a} \subset \bar{b}$ if and only if $b^- \le a^- \le a^+ \le b^+$.

Definition 2.1 Let X be an ordinary set, a mapping $A : X \to [I]$ is called an interval-valued fuzzy number on X if

(1) A is normal, i.e., there is at least an $x_0 \in X$ such that $A(x_0) = [1,1]$;

(2) A is convex,

i.e., $A(\lambda x + (1-\lambda)y) \ge A(x) \wedge A(y)$ for each $x, y \in X$ and $\lambda \in [0,1]$;

(3) A^- and A^+ are upper semi-continuous;

(4) The closure of $A_{(0,0)}$ is compact.

Let $IFN(X)$ denote the class of all interval-valued fuzzy numbers on X. For each $A \in IFN(X)$, let

$$A(x) = [A^-(x), A^+(x)],\ A^-(x) \le A^+(x),$$

where $x \in X$ and $A^- : X \to I$ ($A^+ : X \to I$) is called a lower fuzzy number (an upper fuzzy number) of A, simply write $A = [A^-, A^+]$. Especially, A is called a degenerate ordinary set if $A^-(x) = A^+(x) = 1$ or $A^-(x) = A^+(x) = 0$ for all $x \in X$.

For any $A, B \in IFN(X)$ and $x \in X$, we define

$(A \wedge B)(x) = A(x) \wedge B(x)$,

$(A \vee B)(x) = A(x) \vee B(x)$,

$$A'(x) = (A(x))' = [1 - A^+(x), 1 - A^-(x)] = [A^{+\prime}(x), A^{-\prime}(x)],$$

$A = B$ if and only if $A^-(x) = B^-(x)$, $A^+(x) = B^+(x)$ for all $x \in X$.

Definition 2.2 Let $A \in IFN(X)$ and $[\lambda_1, \lambda_2] \in [I]$.

We define

$A_{[\lambda_1,\lambda_2]} = \{x \in X \mid A^-(x) \ge \lambda_1,\ A^+(x) \ge \lambda_2\}$,

$A_{(\lambda_1,\lambda_2)} = \{x \in X \mid A^-(x) > \lambda_1,\ A^+(x) > \lambda_2\}$,

$A_{[\lambda_1,\lambda_2)} = \{x \in X \mid A^-(x) \ge \lambda_1,\ A^+(x) > \lambda_2\}$,

$A_{(\lambda_1,\lambda_2]} = \{x \in X \mid A^-(x) > \lambda_1,\ A^+(x) \ge \lambda_2\}$.

Then the above ordinary set are called $[\lambda_1, \lambda_2]$-cut set of A, (λ_1, λ_2)-strong cut set of A, $[\lambda_1, \lambda_2)$- right strong cut set of A and $(\lambda_1, \lambda_2]$-left strong cut set of A.

Theorem 2.3 ([3]) $A \in IFN(X)$ if and only if A^- and A^+ are ordinary fuzzy numbers.

Theorem 2.5 ([3]) Let R be the set of all real numbers and $[a, b]$ an closed interval in R with $a \le b$. If we take $X = R$ or $[a, b]$ and If $A \in IFN(X)$ and $[\lambda_1, \lambda_2] \in [I]/\{[0,0]\}$, then

(1) $A_{[\lambda_1,\lambda_2]}$ is a closed interval in X;

(2) there exists a closed interval $[c, d]$ in X such that

$$A_{(0,0)} \subset [c, d].$$

3. Basic properties of Choquet integrals.

A fuzzy measure μ on a measurable space $(\Omega, \mathfrak{I})$ is an extended real-valued function $\mu : \mathfrak{I} \to [0, \infty]$ satisfying

(i) $\mu(\emptyset) = 0$,

(ii) $\mu(A) \le \mu(B)$, whenever $A, B \in \Im$, $A \subset B$.

(iii) for every increasing sequence $\{A_n\}$ of measurable sets , we have

$$\mu(\cup_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n),$$

(iv) for every decreasing sequence $\{A_n\}$ of measurable sets and $\mu(A_1) < \infty$, we have

$$\mu(\cap_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n).$$

Definition 3.1 ([5,6,7]) (1) The Choquet integral of a measurable function f with respect to a fuzzy measure μ on $E \in \Omega$ is defined by

$$(C)\int_E f d\mu = \int_0^\infty \mu(\{x \in X | f(x) > r\} \cap E) dr$$

where the integral on the right-hand side is an ordinary one.

(2) A measurable function f is called Choquet integrable if the Choquet integral of f can be defined and its value is finite.

Instead of $(C)\int_X f d\mu$, we write $(C)\int f d\mu$.

Definition 3.2 ([5,6,7]) Let f, g be measurable functions. Then we say that f is comonotonic to g, in symbol $f \sim g$ if we have

$$f(x) < f(x') \Rightarrow g(x) \le g(x') \text{ for all } x, x' \in X.$$

We say $f: X \to R^+$ is in $L_c^1(\mu)$ if and only if f is measurable and $(C)\int f\, d\mu < \infty$. We note that "$x \in X$ $\mu-a.e.$" stands for "$x \in X$ μ-almost everywhere". The property $P(x)$ holds for $x \in X$ $\mu-a.e.$ means that there is a measurable set E such that $\mu(A) = 0$ and the property $P(x)$ holds for all $x \in E^c$, where E^c is the complement of E.

Theorem 3.3([5,6,7]) Let $f, g, h : X \to R^+$ be measurable functions. Then we have

(1) $f \sim f$,

(2) $f \sim g \;\Rightarrow\; g \sim f$,

(3) $f \sim a$ for all $a \in R^+$,

(4) $f \sim g$ and $f \sim h \Rightarrow f \sim (g+h)$.

Theorem 3.4([5,6,7]) Let $f, g : X \to R^+$be nonnegative measurable functions.

(1) If $f \le g$, then $(C)\int f d\mu \le (C)\int g d\mu$.

(2) If $A \subset B$ and $A, B \in \Omega$, then

$$(C)\int_A f d\mu \le (C)\int_B g d\mu.$$

(3) If $f \sim g$ and $a, b \in R^+$, then

$$(C)\int (af + bg) d\mu = a(C)\int f d\mu + b(C)\int g d\mu.$$

4. Main results.

In this section, by utilizing Choquet integrals, we define an interval-valued correlation coefficient and discuss some properties of them.

Definition 4.1 Let $a, b \in R^+$ with $a \le b$ and $A, B \in IFN([a,b])$.

(1) We define

$$m(A,B) = [m^-(A,B), m^+(A,B)]$$

where

$$m^-(A,B) = \frac{1}{\mu([a,b])}(C)\int A^-B^- + A'B^{-\prime}) d\mu$$

and $m^{+}(A,B)=\frac{1}{\mu([a,b])}(C)\int A^{+}B^{+}+A^{+\prime}B^{+\prime})d\mu.$

(2) An interval-valued correlation coefficient with respect to A and B is defined by

$$\rho(A,B) = \left[\frac{m^{-}(A,B)}{\sqrt{m^{-}(A,A)m^{-}(B,B)}}, \frac{m^{+}(A,B)}{\sqrt{m^{+}(A,A)m^{+}(B,B)}}\right]$$

We note that A is called a degenerate ordinary set if

$$A^{-}(x)=A^{+}(x)=1 \text{ or } A^{-}(x)=A^{+}(x)=0$$

for all $x\in X$.

Theorem 4.4 If $A,B\in IFN([a,b])$ and $A^{-}\sim B^{-}$ and $A^{+}\sim B^{+}$, then we have

$$[0,0]\le\rho(A,B)\le[1,1].$$

Theorem 4.5 Let $A,B\in IFN([a,b])$. Then $\rho(A,B)=[1,1]$ if and only if $A=B$ $\mu-a.e.$ on $[a,b]$.

Theorem 4.6 Let $A,B\in IFN([a,b])$. Then $m(A,B)=[0,0]$ if and only if A,B are degenerate ordinary sets $\mu-a.e.$ on $[a,b]$.

We remark that (1) if a fuzzy measure μ is a classical measure, then these results and the results of G. Wang and X. Li([3]) are the same, because Choquet integral is equal to Riemann integral, and
(2) we consider the concepts of information energy by means of definition of the correlation coefficients, utilizing Choquet integrals instead of Riemann integrals.

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