# On the Fuzzy Impulsive Function
# 퍼지 충격함수에 관하여

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**abstract**

In this paper we study Laplace transform of the fuzzy Dirac delta function and the examples of fuzzy impulsive differential equations.

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## I. Introduction

Many application in engineering and physics are ofter acted upon by an external force of large magnitude that acts only for a very short period of time. For example, a vibrating airplane wing could be struck by lighting, a mass on a spring could be given a sharp blow by a ball peen hammer, a ball could be sent soaring when struck violently by some kind of club. For solving this fuzzy type make use of the notion a fuzzy impulsive. To solve such a fuzzy logical problem mathematically, for $a>0,\ t_0>0$, we can define the fuzzy function

$$\tilde{\delta}_a(t-t_0)=\begin{cases}0 & 0\le t\le t_0-a\\ \dfrac{1}{\tilde{2}a} & t_0-a\le t\le t_0+a\\ 0 & t\ge t_0+a\end{cases}$$

(1.1)

where $\tilde{\cdot}$ is about $\cdot$. The function $\tilde{\delta}_a(t-t_0)$ is called a unit fuzzy impulse since it possesses the integration property

$$\int_0^\infty \tilde{\delta}(t-t_0)\,dt=\tilde{1}.$$

(1.2)

In practice it is convenient to work with another type of unit fuzzy impulse that is defined by the limit

$$\tilde{\delta}(t-t_0)=\lim_{a\to 0}\tilde{\delta}_a(t-t_0)$$

(1.3)

The expression $\tilde{\delta}(t-t_0)$ said to be the fuzzy Dirac delta function which is useful in representing

an instantaneous impulse at time $t=t_0$.

It is possible to obtain the Laplace transform of the fuzzy Dirac delta function by the formal assumption that

$$\mathcal{L}\{\tilde{\delta}(t-t_0)\}=\lim_{a\to 0}\mathcal{L}\{\tilde{\delta}_a(t-t_0)\}.$$

(1.4)

## II. Laplace transform of the fuzzy Dirac delta function

**Theorem.** For $t_0>0$ and $a>0$

(1) $\int_0^\infty \tilde{\delta}_a(t-t_0)\,dt=\tilde{1}$

(2) $\mathcal{L}\{\tilde{\delta}(t-t_0)\}=e^{-st_0}\cdot\tilde{1}$ .

**Example.** Solve the initial value problem

$$y''+y=c\tilde{\delta}(t-2\pi)$$

(2.6)

subject to $y(0)=1$ and $y'(0)=0$, where

$c \in R$ is positive constant.

It could serve as models for describing the motion of a fuzzy mass on a spring moving in a medium in which damping negligible. The fuzzy mass is release from rest 1 unit below the equilibrium position and at $t = 2\pi$ seconds the fuzzy mass is given a sharp blow.

Solution. From the Laplace transform of the differential (2.6) is

$$s^2 Y(s) - s + Y(s) = ce^{-2\pi s}\tilde{1}$$

or

$$Y(s) = \frac{s}{s^2+1} + \frac{ce^{-2\pi s}}{s^2+1}\tilde{1}$$

(2.7)

Utilizing the inverse form of the translation, we find the solution

$$y(t) = \cos t + csin(t-2\pi)u(t-2\pi) \cdot \tilde{1}$$

(2.8)

Put

$$[y(t)]^\alpha = \cos t + c\sin t(t-2\pi)u(t-2\pi)\frac{2}{3-\alpha}, \quad \cos t + c\sin(t-2\pi)u(t-2\pi)\frac{2}{\alpha+1}$$

(2.9)

for $\alpha \in [0,1]$. Let $T > 0$. Consider the following the solution set

$X^\alpha = \{[y(t)]^\alpha \mid [y(t)]^\alpha$ satisfies eq.(2.9) for $t \in [0, T]$ and $\alpha \in [0,1]\}$.

Nonempty is obvious since we can select $\alpha \in [0,1]$. Let $[y(t)]^\alpha \in X^\alpha$, then there is $\alpha \in [0,1]$ such that

$$|[y(t)]^\alpha| \le \sqrt{c^2\left|\frac{2}{\alpha+1} - \frac{2}{3-\alpha}\right|} \le \frac{4}{3}c,$$

Thus $X^\alpha$ is bounded. Let $[y]^{\alpha_k} \in X^\alpha$ for each $[y]^{\alpha_k}$, then there is $\alpha_k \in [0,1]$ such that $\alpha_k \to \alpha \in [0,1]$ and

$$\lim_{k\to\infty} [y]^{\alpha_k} = \lim_{k\to\infty} \cos t + c\sin(t-2\pi)u(t-2\pi)\frac{2}{3-\alpha_k}, \quad \cos t + c\sin(t-2\pi)u(t-2\pi)\frac{2}{\alpha_k+1}$$

$$\cos t + c\sin(t-2\pi)u(t-2\pi)\frac{2}{3-\alpha}, \quad \cos t + c\sin(t-2\pi)u(2-2\pi)\frac{2}{\alpha+1} = [y(t)]^\alpha$$

because $[\tilde{1}]^\alpha = \frac{2}{3-\alpha}, \frac{2}{\alpha+1}$ is closed. Thus $X^\alpha$ is compact. From the (2.8) the solution can be written as

$$y(t) = \begin{cases} \cos t & 0 \le t < 2\pi \\ \cos t + c \cdot \tilde{1}\sin t & t \ge 2\pi \end{cases}$$

We see from the solution $y(t)$ that the mass exhibiting simple harmonic motion it is struck at $t = 2\pi$. The influence of the fuzzy unit impulse is to increase the amplitude of vibration to $\sqrt{(c \cdot \tilde{1})^2 + 1}$ for $t > 2\pi$.

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