# Intelligent Digital Redesign Based on Periodic Control

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ABSTRACT

This paper presents a new linear-matrix-inequality-based intelligent digital redesign (LMI-based IDR) technique to match the states of the analog and the digital T-S fuzzy control systems at the intersampling instants as well as the sampling ones. The main features of the proposed technique are: 1) the fuzzy-model-based periodic control is employed, and the control input is changed n times during one sampling period; 2) The proposed IDR technique is based on the approximately discretized version of the T-S fuzzy system, but its discretization error vanishes as n approaches the infinity. 3) some sufficient conditions involved in the state matching and the stability of the closed-loop discrete-time system can be formulated in the LMIs format.

**Key words** : intelligent digital redesign (LMI-based IDR), fuzzy-model-based control, Digital control, T-S fuzzy system.

## 1. Introduction

Intelligent Digital redesign (IDR) has gained tremendously increasing attention as yet another efficient design tool of sampled-data fuzzy control [1]-[6]. The term IDR involves converting a well-designed analog fuzzy-model-based control into an equivalent digital one maintaining the property of the original analog control system in the sense of state-matching, by which the benefits of both the analog control and the advanced digital technology can be achieved.

This paper studies a periodic control for T-S fuzzy systems by using the LMI-based IDR method. The main features of the proposed method are four-fold. First, the fuzzy-model-based periodic control is employed, and the control input is changed n times during one sampling period. Second, the proposed periodic control scheme can improve the state-matching performance in the long sampling limit. Finally, some sufficient conditions involved in the state matching and the stability of the closed-loop discrete-time system can be formulated in the LMIs format.

## 2. Preliminaries

Consider a nonlinear system described by

$$x_c(t) = f(x_c(t), u_c(t)) \qquad (1)$$

where $x(t) \in R^n$ is the state vector, $u_c(t) \in R^m$ is analog control input, and the subscript “c” means the analog control.

To facilitate the control design, we will develop a simplified model, which can represent the local linear input-output relations of the nonlinear system. This type of models is referred as T-S fuzzy models. The fuzzy dynamical model corresponding to (1) is described by the following IF-THEN rules [1]-[6].:

$R_k$ : IF $z_1(t)$ is about $\Gamma_{k1}$ and L and $z_p(t)$ is about $\Gamma_{kp}$,

THEN $x_c(t) = A_k x_c(t) + B_k u_c(t)$ (2)

where $R_k, k \in I_q = \{1,2,K, q\}$, is the $k$ th fuzzy rule, $z_r(t), r \in I_p = \{1,2,K, p\}$, is the $r$ th premise variable, and $\Gamma_{kr}, (k,r) \in I_q \times I_p$, is the fuzzy set. Then, given a pair $(x_c(t),u_c(t))$, using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of (2) has the form

$$\dot{x}_c(t)=\sum_{k=1}^{q}\theta_k(z(t))(A_k x_c(t)+B_k u_c(t)) \quad (3)$$

where $\theta_k(z(t))=\frac{w_k(z(t))}{\sum_{k=1}^{q}w_k(z(t))}$,

$w_k(z(t))=\sum_{r=1}^{p}\Gamma_{kr}(z_r(t))$, and $\Gamma_{kr}(z_r(t))$ is the grade of membership of $z_r(t)$ in $\Gamma_{kr}$. The possibly time-varying parameter vector $\theta\in R^q$ belongs to a convex polytope $\Theta$, where

$$\Theta:=\left\{\sum_{k=1}^{q}\theta_k=1,\quad 0\le\theta_k\le 1\right\}$$

It is clear that as $\theta$ varies inside $\Theta$, $\sum_{k=1}^{q}\theta_k(z(t))A_k$ and $\sum_{k=1}^{q}\theta_k(z(t))B_k$ range over a matrix polytope

$$\left[\sum_{k=1}^{q}\theta_k(z(t))A_k,\sum_{k=1}^{q}\theta_k(z(t))B_k\right]\in\mathbf{Co}\{(A_k,B_k),k\in I_q\}$$

where **Co** denotes the convex hull. In this note, the stabilization of the polytopic model (3) is equivalent to the simultaneous stabilization of its vertices $(A_k,B_k),k\in I_q$.

In this paper, a well-constructed analog fuzzy-model-based control law, which will be employed in redesigning the digital controller. The controller is described by the following IF-THEN rules:

$R_k$ : IF $z_1(t)$ is about $\Gamma_{k1}$ and L and $z_p(t)$ is about $\Gamma_{kp}$,

THEN $u_c(t)=\bar{K}_k x_c(t)$, (4)

and its defuzzified output is

$$u_c(t)=\sum_{k=1}^{q}\theta_k(z(t))\bar{K}_k x_c(t) \quad (5)$$

Therefore, main purpose of this paper is to find the digital equivalent of the following analogously controlled system:

$$\dot{x}_c(t)=\sum_{k=1}^{q}\sum_{l=1}^{q}\theta_k(z(t))\theta_l(z(t))(A_k+B_k\bar{K}_l)x_c(t) \quad (6)$$

# 3. Main Results

## 3.1. Discretization of T-S fuzzy Systems

In the following, let $h_0$ and $h$ be the sampling time and the control update time, respectively. For convenience, we take $h=\frac{h_0}{N}$ for a positive integer $N$, where $N$ is an input multiplicity [7]. Then, $t=ih_0+jh$ for $i\in Z_0$ and $j\in Z_{[0,N-1]}$, where the indexes $i$ and $j$ indicate sampling and control update instants, respectively.

By interfacing an ideal sampler and a zero-order holder between the plant and a controller, the digital fuzzy control system is represented by

$$\dot{x}_d(t)=\sum_{k=1}^{q}\theta_k(z(t))(A_k x_d(t)+B_k u_{dk}(t)). \quad (7)$$

where $u_d(t)=u_d(ih_0+jh)$ for $t\in[ih_0+jh,ih_0+jh+h)$, $i\in Z_0$, $j\in Z_{[0,N-1]}$ is the periodic control input vector, the control input is changed $N$ times during one sampling time $h_0$, and the subscript "d" means the analog control. The periodic control input takes the following form:

$$u_{dk}(ih_0+jh)=\sum_{l=1}^{q}\theta_l(z(ih_0+jh))K_{kl}x_d(ih_0+jh) \quad (8)$$

where $x_d(ih_0+jh)$ is not required to obtain $u_d(ih_0+jh)$ because it will be predicted from $x_d(ih_0)$ after each control update.

**Lemma 1.** *[5]* If $\theta_k(z(t))=\theta_k(z(ih_0+jh))$ for $t\in[ih_0+jh,ih_0+jh+h)$, $i\in Z_0$, $j\in Z_{[0,N-1]}$, and $e^{\sum_{k=1}^{q}\theta_k(z(ih_0+jh))A_k h}=$, $\sum_{k=1}^{q}\theta_k(z(ih_0+jh))e^{A_k h}$ then the discretized system of (7) with sampling time $h$ is as follows:

$$x_d(ih_0+jh+h)=\sum_{k=1}^{q}\theta_k(z(ih_0+jh))\times(G_k x_d(ih_0+jh)+H_k u_{dk}(ih_0+jh)) \quad (9)$$

where $G_k = e^{A_k h}$ and $H_{kl} = (G_k - I)A_k^{-1}B_l$.

In order to predict $x_d(ih_0 + jh)$ in (8), we will develop a general form of solutions to (9) controlled by (8) for $x_d(ih_0 + jh)$ with the arbitrary initial state $x_d(ih_0)$.

**Corollary 1.** The solution to (9) closed by (8) for $x_d(ih_0 + jh)$ with the arbitrary initial state $x_d(ih_0)$ is given by

$$x_d(ih_0 + jh) = \prod_{v=1}^{j} \Big(\sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh)) \times\theta_l(z(ih_0 + jh - vh))(G_k + H_k K_{kl})\Big)x_d(ih_0) \quad (10)$$

for $i \in Z_0$ and $j \in Z_{[1,N-1]}$.

*Proof.* The closed-loop system (9) with (8) is described by

$$x_d(ih_0 + jh + h) = \sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + jh)) \times\theta_l(z(ih_0 + jh))(G_k + H_k K_{kl})x_d(ih_0 + jh) \quad (11)$$

Replacing $j$ in (11) to $j-1$ leads

$$x_d(ih_0 + jh) = \sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + jh - h)) \times\theta_l(z(ih_0 + jh - h))(G_k + H_k K_{kl})x_d(ih_0 + jh - h)$$

We compute

$$x_d(ih_0 + h) = \sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0))\theta_l(z(ih_0)) \times (G_k + H_k K_{kl}) x_d(ih_0)$$

$$\begin{aligned} x_d(ih_0 + 2h) &= \sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + h)) \times\theta_l(z(ih_0 + h))(G_k + H_k K_{kl}) \times x_d(ih_0 + h) \\ &= \sum_{k_0=1}^{q}\sum_{l_0=1}^{q}\sum_{k_1=1}^{q}\sum_{l_1=1}^{q} \theta_{k_0}(z(ih_0 + h)) \times\theta_{l_0}(z(ih_0 + h))\theta_{k_1}(z(ih_0)) \\ &\quad \times\theta_{l_1}(z(ih_0))(G_{k_0} + H_{k_0} K_{k_0 l_0}) \times (G_{k_1} + H_{k_1} K_{k_1 l_1}) x_d(ih_0) \end{aligned}$$

for $(k_0, j_0, k_1, j_1) \in \underbrace{I_q \times I_q \times I_q}_{4}$. Proceeding forward, we can readily obtain (10) for $j > 0$. □

Substituting (10) to $x_d(ih_0 + jh)$ in (9) controlled by (8), we can obtain the following discretized version of the closed-loop digital fuzzy system with (7) and (8):

$$x_d(ih_0 + jh + h) = \prod_{v=0}^{j}\Big(\sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh)) \times\theta_l(z(ih_0 + jh - vh))(G_k + H_k K_{kl})\Big) \times x_d(ih_0) \quad (12)$$

for $i \in Z_0$ and $j \in Z_{[0,N-1]}$.

**Corollary 2.** In the the analogously controlled fuzzy system (6),

- the approximate discrete-time model can be also obtained as

$$x_c(ih_0 + jh + h) = \sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + jh)) \times\theta_l(z(ih_0 + jh))\Xi_{kl} x_c(ih_0 + jh) \quad (13)$$

where $\Xi_{kl} = e^{(A_k + B_k \bar{K}_l)h}$.

- the solution to (13) for $x_c(ih_0 + jh)$ with the arbitrary initial state $x_c(ih_0)$ is given by

$$x_c(ih_0 + jh) = \prod_{v=1}^{j}\Big(\sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh)) \times\theta_l(z(ih_0 + jh - vh))\Xi_{kl}\Big)x_c(ih_0) \quad (14)$$

for $i \in Z_0$ and $j \in Z_{[1,N-1]}$.

Therefore, from (13) and (14), we directly obtain the following discrete-time representation of (6):

$$x_c(ih_0 + jh + h) = \prod_{v=0}^{j}\Big(\sum_{k=1}^{q}\sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh)) \times\theta_l(z(ih_0 + jh - vh))\Xi_{kl}\Big)x_c(ih_0) \quad (15)$$

for $i \in Z_0$ and $j \in Z_{[1,N-1]}$.

### 3.2 Design of the periodic controller using IDR method

The IDR problem for the system (8) is the problem to design a periodic control law (9) such that i) the origin $x=0$ is a globally asymptotically stable equilibrium point of the closed-loop system

$$x_d(t)=\sum_{k=1}^{q}\sum_{l=1}^{q}\theta_k(z(t))\theta_l(z(ih_0+jh))$$
$$\times\ A_k x_d\ t\ + B_k K_{kl} x_d\ ih_0(\theta 0)h\ ,$$

(16)

and ii) by comparing (12) and (15), to realize $x_c(ih_0+h)=x_d(ih_0+h)$ under the assumption that $x_c(ih_0)=x_d(ih_0)$, $K_{kl}$ was numerically synthesized for $\|\Xi_{kl}-G_k-H_k K_{kl}\|$ to be a minimizer in the induced 2-norm sense.

**Theorem 1.** If there exist $Q=Q^T \succ 0$ and constant matrices $F_k$ such that the following generalized eigenvalue problem (GEVP) has solutions:

*Minimize*$_{Q,F_k}$ $\gamma$ *subject to*

$$\begin{bmatrix} -\gamma Q & (\bullet)^T \\ \Xi_{kl}Q-G_kQ-H_kF_{kl} & -\gamma I \end{bmatrix} \prec 0,\quad k,l\in I_q$$

$$\begin{bmatrix} -Q & (\bullet)^T \\ G_kQ+H_kF_{kk} & -Q \end{bmatrix} \prec 0,\quad k,l\in I_q$$

$$\begin{bmatrix} -Q & (\bullet)^T \\ \dfrac{G_kQ+H_kF_{kl}+G_lQ+H_lF_{lk}}{2} & -Q \end{bmatrix} \prec 0,\quad k,l\in I_q$$

then the state $x_d(ih_0+jh)$ of the discrete-time representation (12) closely matches the discrete-time representation (15), and (12) is globally asymptotically stable in the sense of Lyapunov, where $(\bullet)^T$ denotes the transposed element in symmetric positions.

### 4. Closing Remarks

This paper proposed the periodic control design using the LMI approach for the fuzzy system. Some sufficient conditions were derived for stabilization and state matching of the discretized model by the fast discretization. The proposed periodic control scheme can improve the state-matching performance in the long sampling limit.

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