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# 얇은 원통형 쉘에 발생한 손상 규명

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# A DAMAGE IDENTIFICATION METHOD FOR THIN CYLINDRICAL SHELLS

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**ABSTRACT**

In this paper, a structural damage identification method (SDIM) is developed to identify the line crack-like directional damages generated within a cylindrical shell. First, the equations of motion for a damaged cylindrical shell are derived. Based on a theory of continuum damage mechanics, a small material volume containing a directional damage is represented by the effective orthotropic elastic stiffness, which is dependent of the size and the orientation of the damage with respect to the global coordinates. The present SDIM is then derived from the frequency response function (FRF) directly solved from the dynamic equations of the damaged cylindrical shell. In contrast with most existing SDIMs which require the modal parameters measured in both intact and damaged states, the present SDIM requires only the FRF-data measured in damaged state. By virtue of utilizing FRF-data, one may choose as many sets of excitation frequency and FRF measurement point as needed to acquire a sufficient number of equations for damage identification analysis. The numerically simulated damage identification tests are conducted to study the feasibility of the present SDIM.

**Key Words** : Vibration response(진동응답), Structural damage identification method(구조손상 검출방법), Frequency response function, FRF(주파수 응답)

## I. INTRODUCTION

The oil or gas tanks, compressor shells, boilers and airplane fuselages are the typical examples of the application of the cylindrical shell structure. Because such cylindrical shell structures should be free from disastrous structural failures due to structural damages, it is very important to detect all significant structural damages in the very early stage of damage progression. In general, the structural damages change the vibration characteristics of a structure. Therefore, in turn, the damage-induced changes in vibration characteristics can be used to detect and identify the structural damages. In most existing vibration-based structural damage identification methods (SDIMs), the modal parameters such as natural frequencies, modal damping and mode shapes and the frequency response function (FRF)-data have been widely used .

The SDIMs for cylindrical shells have been introduced by some researchers [1-2]. Srinivasan and Kot [5] proposed to use a damage index method for locating damage in circular cylindrical shells, which is basically based on the damage-induced change in the modal strain energy of a structure. They derived an expression for the damage index that requires only the radial component of shell vibrations. Ip and Tse [1] presented a feasibility study on locating damage in circular cylindrical fiber-reinforced composite shells based on natural frequencies (frequency sensitivities) and mode shape information at specific locations. Very recently Kim *et al.* [2] developed an FRF-data based SDIM to identify the locations and magnitudes of multiple local damages within a cylindrical shell. They assumed that, as did in most existing SDIMs, the local damages were isotropic and they did not take into account the directivities (orientations) of local damages. This paper is the extension of Kim *et al.* [2] to add the capability to identify the directivities of local damages, *i.e.*, directional local damages.

The failure of most structural members involves general

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degradation of elastic properties due to the localized nucleation and growth of damages (*i.e.*, voids, cavities, or cracks of the size of crystal grains) and their ultimate coalescences into the larger size of material fracture. This implies that the directivities of local damages may control the direction of crack propagation within a structure member. Because the damage directivity plays a very important role to determine the failure pattern and the remaining life of a structure member, it will be very important to identify the directivities of local damages as well, in addition to identifying their locations and severities (or magnitudes). However, to the authors' best knowledge, the main concerns of most existing SDIMs have been limited to the identification of damage locations and severities only. Thus it is mandatory to develop an FRF-based SDIM by which the directivities of local damages can be identified simultaneously: this motivates this work.

From a physical standpoint, the surface of a material fracture can be considered as the continual propagation and coalescence of film-like small cracks. Thus, it may be pertinent to consider a local damage, which is film-like and uniform through the thickness of a thin-walled structure, as the equivalent line through-crack (simply, line crack). Based on the continuum damage mechanics, Lee *et al*. [3] showed that a SMV (small material volume) containing a line crack behaves effectively orthotropic, while a SMV containing a circular crack behaves effectively isotropic. They represented the material behavior of the SMV containing a line crack in terms of the effective orthotropic elastic stiffness, which is the function of the isotropic elastic stiffness, the crack size and the crack directivity. Thus, the damage-induced change in local elastic stiffness from initially isotropic to effectively orthotropic may indicate the existence of directional damages, rather than isotropic damages. This concept of continuum damage representation may provide a tool required to identify the directivities of local damages.

First the dynamic equations of a damaged thin cylindrical shell are derived by using the continuum damage representation of a line crack-like local damage. The damage identification algorithm is then formulated from the frequency response function directly solved from the dynamic equations of damaged thin cylindrical shell. The SDIM proposed in this paper can be used to identify the directivities of multiple local damages as well, in addition to identifying their locations and severities.

## II. EQUATUINS OF MOTION FOR DAMAGED CYLINDRICAL SHELLS

Consider an elastic, thin cylindrical shell. The shell has the radius $R$, the length $L$, and the thickness $h$ as shown in Fig. 1. The intact shell material is isotropic and has Young's modulus $E$ and Poisson's ratio $\nu$. The $x$-axis is directed along the symmetry axis of the median shell surface, the $y$-axis in the circumferential direction, and the $z$-direction along the interior normal of the meridian surface. Define the displacements in the longitudinal, circumferential and radial directions by $u(x,\theta, t)$, $v(x,\theta, t)$ and $w(x,\theta, t)$, respectively, and also define the external loads in each direction by $p_x(x,\theta, t)$, $p_y(x,\theta, t)$ and $p_z(x,\theta, t)$, respectively.

The Effective elastic stiffness $\overline{\boldsymbol{Q}}^{\boldsymbol{D}}$ with respect to the global coordinates $(x,\theta)$ for the small material volume containing a line crack-like damage can be expressed as follows [3] :

$$\overline{\boldsymbol{Q}}^{\boldsymbol{D}} = \mathbf{Q} - \Delta\overline{\mathbf{Q}} \tag{1}$$

where

$$\Delta\overline{\mathbf{Q}} = \boldsymbol{T}(\phi)^{\mathrm{T}}\left[\boldsymbol{Q}_{ij} e_{ij}\right]\boldsymbol{T}(\phi) D \tag{2}$$

In Eq. (2), $\boldsymbol{T}(\phi)$ is the coordinates transformation matrix , $\boldsymbol{Q}$ is the reduced elastic stiffness of intact isotropic solid, $e_{ij}$ is the effective material directivity parameters, $\phi$ is the damage orientation angle and $D$ is the damage magnitude

The equations of motion for a thin cylindrical shell subject to a small amplitude vibration are given by [2]

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} + p_x(x,\theta,t) = \rho h\ddot{u} \\
&\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_{\theta}}{\partial \theta} + p_\theta(x,\theta,t) = \rho h\ddot{v} \\
&\frac{\partial^2 M_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial x\partial \theta} + \frac{1}{R^2}\frac{\partial^2 M_{\theta}}{\partial \theta^2} \\
&+\frac{1}{R}N_\theta + p_z(x,\theta,t) = \rho h\ddot{w}
\end{aligned} \tag{3}$$

where the force resultants ($N_x$, $N_\theta$, $N_{x\theta}$) and the moment resultants ($M_x$, $M_\theta$, $M_{x\theta}$) are defined by

$$\begin{aligned}
\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \end{Bmatrix} &= \begin{bmatrix} \overline{K}_{11}^D & \overline{K}_{12}^D & \overline{K}_{16}^D \\ & \overline{K}_{22}^D & \overline{K}_{26}^D \\ symm & & \overline{K}_{66}^D \end{bmatrix} \begin{Bmatrix} \varepsilon_{x0} \\ \varepsilon_{\theta 0} \\ \varepsilon_{x\theta} \end{Bmatrix} \\
\begin{Bmatrix} M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix} &= -\frac{h^2}{12} \begin{bmatrix} \overline{K}_{11}^D & \overline{K}_{12}^D & \overline{K}_{16}^D \\ & \overline{K}_{22}^D & \overline{K}_{26}^D \\ symm & & \overline{K}_{66}^D \end{bmatrix} \begin{Bmatrix} \chi_x \\ \chi_\theta \\ 2\chi_{x\theta} \end{Bmatrix}
\end{aligned} \tag{4}$$

where $\varepsilon_{x0}$, $\varepsilon_{\theta 0}$ and $\varepsilon_{x\theta}$ are the membrane strains, and $\chi_x$, $\chi_\theta$ and $\chi_{x\theta}$ are the changes in curvatures. In Eq. (4), $\overline{\boldsymbol{K}}^D = [\overline{K}_{ij}^D]$ $(i, j = 1, 2, 6)$ is the membrane stiffness for the damaged cylindrical shell defined by

$$\overline{\boldsymbol{K}}^{D}(x,\theta)=\boldsymbol{K}-\varDelta\overline{\boldsymbol{K}}(x,\theta) \tag{5}$$

where $\boldsymbol{K}$ is the intact membrane stiffnesses (outside of SMV) defined by

$$\begin{aligned} &K_{11}=K_{22}=K,\quad K_{12}=K_{21}=\nu K \\ &K_{16}=K_{61}=K_{26}=K_{62}=0,\quad K_{66}=\left(\frac{1-\nu}{2}\right)K \end{aligned} \tag{6}$$

and $\varDelta\overline{\boldsymbol{K}}$ is the perturbed membrane stiffness that represents the *effective* degradation of the membrane stiffness due to the presence of the damage of magnitude *D*.

$$\varDelta\overline{\boldsymbol{K}}=h\varDelta\overline{\boldsymbol{Q}}=h\boldsymbol{T}(\phi)^{\mathrm{T}}\left[\boldsymbol{Q}_{ij}e_{ij}\right]\boldsymbol{T}(\phi)D \tag{7}$$

By substituting Eq. (5) into Eq. (4) and substituting the results into Eq. (3), one may obtain the equations of motion for the damaged cylindrical shell in the form as

$$\left[\overline{\boldsymbol{L}}\right]\{\boldsymbol{u}(x,\theta,t)\}+\{\boldsymbol{f}(x,\theta,t)\}=\rho h\{\ddot{\boldsymbol{u}}(x,\theta,t)\} \tag{8}$$

In Eq (5), $[\overline{\boldsymbol{L}}]=[\boldsymbol{L}]+[\boldsymbol{L}_D]$ is the matrix of differential operators for the damaged cylindrical shells, $[\boldsymbol{L}]$ is the matrix of differential operators for the intact cylindrical shells [2] and $[\boldsymbol{L}_D]$ is the perturbed matrix of differential operators defined by

$$[\boldsymbol{L}_D]=-\begin{bmatrix} L_{D11} & L_{D12} & L_{D13} \\ L_{D21} & L_{D22} & L_{D23} \\ L_{D31} & L_{D32} & L_{D33} \end{bmatrix} \tag{9}$$

where

$$\begin{aligned}
L_{D11}&=\frac{\partial}{\partial x}\left(\Delta\overline{K}_{11}\frac{\partial}{\partial x}\right)+\frac{1}{R}\frac{\partial}{\partial x}\left(\Delta\overline{K}_{16}\frac{\partial}{\partial\theta}\right)\\
&\quad+\frac{1}{R}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{16}\frac{\partial}{\partial x}\right)+\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{26}\frac{\partial}{\partial\theta}\right)\\
L_{D12}&=\frac{\partial}{\partial x}\left(\Delta\overline{K}_{16}\frac{\partial}{\partial x}\right)+\frac{1}{R}\frac{\partial}{\partial x}\left(\Delta\overline{K}_{12}\frac{\partial}{\partial\theta}\right)\\
&\quad+\frac{1}{R}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{66}\frac{\partial}{\partial x}\right)+\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{26}\frac{\partial}{\partial\theta}\right)\\
L_{D13}&=-\frac{1}{R}\left(\frac{\partial\left(\Delta\overline{K}_{12}\right)}{\partial x}+\Delta\overline{K}_{12}\frac{\partial}{\partial x}\right)\\
&\quad-\frac{1}{R^2}\left(\frac{\partial\left(\Delta\overline{K}_{26}\right)}{\partial\theta}+\Delta\overline{K}_{26}\frac{\partial}{\partial\theta}\right)\\
L_{D21}&=\frac{\partial}{\partial x}\left(\Delta\overline{K}_{16}\frac{\partial}{\partial x}\right)+\frac{1}{R}\frac{\partial}{\partial x}\left(\Delta\overline{K}_{66}\frac{\partial}{\partial\theta}\right)\\
&\quad+\frac{1}{R}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{12}\frac{\partial}{\partial x}\right)+\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{26}\frac{\partial}{\partial\theta}\right)\\
L_{D22}&=\frac{\partial}{\partial x}\left(\Delta\overline{K}_{66}\frac{\partial}{\partial x}\right)+\frac{1}{R}\frac{\partial}{\partial x}\left(\Delta\overline{K}_{26}\frac{\partial}{\partial\theta}\right)\\
&\quad+\frac{1}{R}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{26}\frac{\partial}{\partial x}\right)+\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\Delta\overline{K}_{22}\frac{\partial}{\partial\theta}\right)\\
L_{D23}&=-\frac{1}{R}\left(\frac{\partial\left(\Delta\overline{K}_{26}\right)}{\partial x}+\Delta\overline{K}_{26}\frac{\partial}{\partial x}\right)\\
&\quad-\frac{1}{R^2}\left(\frac{\partial\left(\Delta\overline{K}_{22}\right)}{\partial\theta}+\Delta\overline{K}_{22}\frac{\partial}{\partial\theta}\right)\\
L_{D31}&=\frac{1}{R}\left(\Delta\overline{K}_{12}\frac{\partial}{\partial x}\right)+\frac{1}{R^2}\left(\Delta\overline{K}_{26}\frac{\partial}{\partial\theta}\right)\\
L_{D32}&=\frac{1}{R}\left(\Delta\overline{K}_{26}\frac{\partial}{\partial x}\right)+\frac{1}{R^2}\left(\Delta\overline{K}_{22}\frac{\partial}{\partial\theta}\right)\\
L_{D33}&=-\frac{h^2}{12}\left[\frac{\partial^2}{\partial x^2}\left(\Delta\overline{K}_{11}\frac{\partial^2}{\partial x^2}\right)+\frac{1}{R^2}\frac{\partial^2}{\partial x^2}\left(\Delta\overline{K}_{12}\frac{\partial^2}{\partial\theta^2}\right)\right.\\
&\quad+\frac{2}{R}\frac{\partial^2}{\partial x^2}\left(\Delta\overline{K}_{16}\frac{\partial^2}{\partial x\partial\theta}\right)+\frac{2}{R}\frac{\partial^2}{\partial x\partial\theta}\left(\Delta\overline{K}_{16}\frac{\partial^2}{\partial x^2}\right)\\
&\quad+\frac{2}{R^3}\frac{\partial^2}{\partial x\partial\theta}\left(\Delta\overline{K}_{26}\frac{\partial^2}{\partial\theta^2}\right)+\frac{4}{R^4}\frac{\partial^2}{\partial x\partial\theta}\left(\Delta\overline{K}_{66}\frac{\partial^2}{\partial x\partial\theta}\right)\\
&\quad+\frac{1}{R^2}\frac{\partial^2}{\partial\theta^2}\left(\Delta\overline{K}_{12}\frac{\partial^2}{\partial x^2}\right)+\frac{1}{R^4}\frac{\partial^2}{\partial\theta^2}\left(\Delta\overline{K}_{22}\frac{\partial^2}{\partial\theta^2}\right)\\
&\quad\left.+\frac{2}{R^3}\frac{\partial^2}{\partial\theta^2}\left(\Delta\overline{K}_{26}\frac{\partial^2}{\partial x\partial\theta}\right)\right]-\frac{\Delta\overline{K}_{22}}{R^2}
\end{aligned} \tag{10}$$

## III. FORCED VIBRATION RESPONSES OF A DAMAGED CYLINDRICAL SHELL

Assume that a harmonic load $p_z(x,\theta,t)$ is applied at a point $(x_F,\ \theta_F)$ of a cylindrical shell, only in the direction normal to the surface of the cylindrical shell, the forced vibration responses of a damaged cylindrical shell can be assumed in the form

$$\begin{aligned}\{\boldsymbol{u}(x_M,\theta_M,t)\}=&\left(\sum_{I=1}^{M}\{\boldsymbol{U}_I\}\frac{W_I(x_F,\theta_F)}{\Omega_I^2-\omega^2}\right.\\ &\left.+\sum_{I=1}^{M}\sum_{J=1}^{M}\lambda_{IJ}\{\boldsymbol{U}_I\}\frac{W_J(x_F,\theta_F)}{(\Omega_I^2-\omega^2)(\Omega_J^2-\omega^2)}\right)F_o e^{i\omega t}\end{aligned} \tag{11}$$

where $\{\boldsymbol{U}_I\}=\{U_I\ \ V_I\ \ W_I\}^{\mathrm{T}}$ $(I = 1, 2, 3, ..., M)$ are the normal modes of the intact cylindrical shell, $(x_M,\ \theta_M)$ represents the measurement point of the forced vibration responses. The matrix $\boldsymbol{\lambda}=[\lambda_{IJ}]$ is the damage influence matrix (DIM) which reflects the influence of damage defined by

$$\lambda_{IJ} = \int_A \{\boldsymbol{U}_I\}^T [\mathbf{L}_D] \{\boldsymbol{U}_J\} dx d\theta \tag{12}$$

Substituting the normal modes of a cylindrical shell simply supported at both ends into Eq. (12) than we can obtain the damage influence matrix as follows

$$\boldsymbol{\lambda} = [\lambda_{IJ}] = \int_A \mathbf{M}_{IJ}(x,\theta)\boldsymbol{\Lambda}(x,\theta)\, dx d\theta \tag{13}$$

where $\boldsymbol{M}_{IJ}\,(x,\theta)$ is the one by six matrix defined by

$$\mathbf{M}_{IJ}(x,\theta) = \left[ M_{IJ}^{(1)} \;\; M_{IJ}^{(2)} \;\; M_{IJ}^{(3)} \;\; M_{IJ}^{(4)} \;\; M_{IJ}^{(5)} \;\; M_{IJ}^{(6)} \right] \tag{14}$$

where

$$\begin{aligned}
M_{IJ}^{(1)} &= \left( P_{1I}P_{1J} + \frac{h^2}{12} \right) \left( \frac{m\pi}{L} \right)^2 \left( \frac{r\pi}{L} \right)^2 W_I W_J \\
M_{IJ}^{(2)} &= \left[ \left( P_{1I}P_{2J}\frac{1}{R} + \frac{h^2}{12R^2} \right) \left( \frac{m\pi}{L} \right)^2 s^2 \right. \\
&\quad + \left( P_{2I}P_{1J}\frac{1}{R} + \frac{h^2}{12R^2} \right) n^2 \left( \frac{r\pi}{L} \right)^2 \\
&\quad \left. + P_{1I}\frac{1}{R}\left( \frac{m\pi}{L} \right)^2 + P_{1J}\frac{1}{R}\left( \frac{r\pi}{L} \right)^2 \right] W_I W_J \\
M_{IJ}^{(3)} &= -\left( P_{1I}P_{1J}\frac{1}{R} + P_{1I}P_{2J} + \frac{h^2}{6R} \right) \left( \frac{m\pi}{L} \right)^2 W_I \frac{\partial^2 W_J}{\partial x \partial \theta} \\
&\quad - \left( P_{1I}P_{1J}\frac{1}{R} + P_{2I}P_{1J} + \frac{h^2}{6R} \right) \left( \frac{r\pi}{L} \right)^2 \frac{\partial^2 W_I}{\partial x \partial \theta} W_J \\
M_{IJ}^{(4)} &= \left[ \left( P_{2I}P_{2J}\frac{1}{R^2} + \frac{h^2}{12R^4} \right) n^2 s^2 + P_{2I}\frac{1}{R^2} n^2 \right. \\
&\quad \left. + P_{2J}\frac{1}{R^2}s^2 + \frac{1}{R^2} \right] W_I W_J \\
M_{IJ}^{(5)} &= -\left[ \left( P_{2I}P_{1J}\frac{1}{R^2} + P_{2I}P_{2J}\frac{1}{R} + \frac{h^2}{6R^3} \right) n^2 \right. \\
&\quad \left. + P_{1J}\frac{1}{R^2} + P_{2J}\frac{1}{R} \right] W_I \frac{\partial^2 W_J}{\partial x \partial \theta} \\
&\quad - \left[ \left( P_{1I}P_{2J}\frac{1}{R^2} + P_{2I}P_{2J}\frac{1}{R} + \frac{h^2}{6R^3} \right) s^2 \right. \\
&\quad \left. + P_{1I}\frac{1}{R^2} + P_{2I}\frac{1}{R} \right] \frac{\partial^2 W_I}{\partial x \partial \theta} W_J \\
M_{IJ}^{(6)} &= \left( P_{1I}P_{1J}\frac{1}{R^2} + P_{1I}P_{2J}\frac{1}{R} + P_{2I}P_{1J}\frac{1}{R} \right. \\
&\quad \left. + P_{2I}P_{2J} + \frac{h^2}{3R^2} \right) \frac{\partial^2 W_I}{\partial x \partial \theta} \frac{\partial^2 W_J}{\partial x \partial \theta}
\end{aligned} \tag{15}$$

and In Eq. (13), $\Lambda(x,\theta)$ is the six by one vector defined by

$$\boldsymbol{\Lambda}(x,\theta) = \left\{ \Delta\overline{K}_{11} \;\; \Delta\overline{K}_{12} \;\; \Delta\overline{K}_{16} \;\; \Delta\overline{K}_{22} \;\; \Delta\overline{K}_{26} \;\; \Delta\overline{K}_{66} \right\}^{\mathrm{T}} \tag{16}$$

From Eq.(7), Eq. (13) gives the damage influence matrix $\boldsymbol{\lambda}$ as

$$\boldsymbol{\lambda} = \sum_{l=1}^{N} (\boldsymbol{\alpha}^l + \boldsymbol{\beta}^l \cos 2\phi_l + \boldsymbol{\gamma}^l \sin 2\phi_l) D_l \tag{17}$$

where, $\boldsymbol{\alpha}^l = [\alpha_{IJ}^l], \boldsymbol{\beta}^l = [\beta_{IJ}^l]$ and $\boldsymbol{\gamma}^l = [\gamma_{IJ}^l]$ are computed from

$$\begin{aligned}
\alpha_{IJ}^l &= K \int_{\theta_{Dl}-\bar{\theta}_l}^{\theta_{Dl}+\bar{\theta}_l} \int_{x_{Dl}-\bar{x}_l}^{x_{Dl}+\bar{x}_l} \left[ \left( \frac{1+\nu^2}{1-\nu^2} \right) M_{IJ}^{(1)} + \left( \frac{2\nu}{1-\nu^2} \right) M_{IJ}^{(2)} \right. \\
&\quad \left. + \left( \frac{1+\nu^2}{1-\nu^2} \right) M_{IJ}^{(4)} + \left( \frac{1-\nu}{2(1+\nu)} \right) M_{IJ}^{(6)} \right] dx d\theta \\
\beta_{IJ}^l &= K \int_{\theta_{Dl}-\bar{\theta}_l}^{\theta_{Dl}+\bar{\theta}_l} \int_{x_{Dl}-\bar{x}_l}^{x_{Dl}+\bar{x}_l} \left( -M_{IJ}^{(1)} + M_{IJ}^{(4)} \right) dx d\theta \\
\gamma_{IJ}^l &= -\frac{1}{2} K \int_{\theta_{Dl}-\bar{\theta}_l}^{\theta_{Dl}+\bar{\theta}_l} \int_{x_{Dl}-\bar{x}_l}^{x_{Dl}+\bar{x}_l} \left( M_{IJ}^{(3)} + M_{IJ}^{(5)} \right) dx d\theta
\end{aligned} \tag{18}$$

## IV. DAMAGE IDENTIFICATION THEORY

In general, it is easier to measure the radial displacement $w(x,\theta,t)$ rather than to measure the longitudinal displacement $u(x,\theta,t)$ or circumferential displacement $v(x,\theta,t)$, the inertance FRF of $w(x,\theta,t)$ measured from a damaged cylindrical shell will be considered as the experimentally measured data required to identify the damages within a cylindrical shell. The inertance FRF is defined as the ratio of the acceleration to the applied force as

$$\mathcal{A}(\omega; x_M, \theta_M) = \frac{\ddot{w}(x_M, \theta_M, t)}{p_z(x_F, \theta_F, t)} = -\omega^2 W(x_M, \theta_M) \tag{19}$$

where $\ddot{w}(x_M, \theta_M, t)$ is the radial acceleration measured at a point ($x_M$, $\theta_M$) and $p_z(x_F, \theta_F, t)$ is the point force applied at a point ($x_F$, $\theta_F$) normal to the surface of a cylindrical shell. Applying the external load $p_z(x_F, \theta_F, t)$ and the radial displacement $w(x_M, \theta_M, t)$ computed from Eq. (11) into Eq. (19) yields

$$\mathcal{A}^D(\omega; x_M, \theta_M) = \mathcal{A}(\omega; x_M, \theta_M) + \Delta\mathcal{A}(\omega; x_M, \theta_M) \tag{20}$$

where $\mathcal{A}$ is the inertance FRF measured from the intact cylindrical shell and $\Delta\mathcal{A}$ is the perturbed inertance FRF due to the presence of damage. They are given by

$$\mathcal{A}(\omega; x_M, \theta_M) = -\omega^2 \boldsymbol{\Psi}_M^{\mathrm{T}} \, diag[\Omega^2 - \omega^2] \boldsymbol{\Psi}_F \tag{21}$$

$$\Delta\mathcal{A}(\omega; x_M, \theta_M) = -\omega^2 \boldsymbol{\Psi}_M^{\mathrm{T}} \boldsymbol{\lambda} \boldsymbol{\Psi}_F \tag{22}$$

where

$$\boldsymbol{\Psi}_M = \begin{Bmatrix} \vdots \\ \frac{W_I(x_M,\theta_M)}{\Omega_I^2-\omega^2} \\ \vdots \end{Bmatrix}, \ \boldsymbol{\Psi}_F = \begin{Bmatrix} \vdots \\ \frac{W_I(x_F,\theta_F)}{\Omega_I^2-\omega^2} \\ \vdots \end{Bmatrix} \tag{23}$$

Substituting Eq. (17) into Eq. (22) gives

$$\sum_{l=1}^{N}\Big[a^l(\omega;x_M,\theta_M)+b^l(\omega;x_M,\theta_M)\cos 2\phi_l + c^l(\omega;x_M,\theta_M)\sin 2\phi_l\Big]D_l = \Delta\mathcal{A}(\omega;x_M,\theta_M) \tag{24}$$

where

$$\begin{aligned} a^l(\omega;x_M,\theta_M) &= -\omega^2\ \boldsymbol{\Psi}_M^{\mathrm{T}}(\omega;x_M,\theta_M)\ \boldsymbol{\alpha}^l\ \boldsymbol{\Psi}_F(\omega) \\ b^l(\omega;x_M,\theta_M) &= -\omega^2\ \boldsymbol{\Psi}_M^{\mathrm{T}}(\omega;x_M,\theta_M)\ \boldsymbol{\beta}^l\ \boldsymbol{\Psi}_F(\omega) \\ c^l(\omega;x_M,\theta_M) &= -\omega^2\ \boldsymbol{\Psi}_M^{\mathrm{T}}(\omega;x_M,\theta_M)\ \boldsymbol{\gamma}^l\ \boldsymbol{\Psi}_F(\omega) \end{aligned} \tag{25}$$

Equation (23) provides the relationship between the damage information (*i.e.*, damage magnitudes $D_l$ and damage orientations $\phi_l$) and the damage-induced change in frequency response function (*i.e.*, $\Delta\mathcal{A}$). Thus, once $\Delta\mathcal{A}$ is experimentally measured from the damaged shell, Eq. (25) can be used to identify the unknown damage information.

For a chosen set of excitation frequency ($\omega$) and measurement point ($x_M$, $\theta_M$), Eq. (23) provides an algebraic equation for unknown effective damage magnitudes $D_l$ and damage orientations $\phi_l$. Thus, by properly choosing as many different sets of ($\omega$;$x_M$,$\theta_M$) as required, $2N$ for instance, a set of simultaneous algebraic equations may be obtained in the form as

$$\boldsymbol{X}(\boldsymbol{\Phi})\mathbf{D} = \Delta\mathcal{A} \tag{26}$$

where

$$\begin{aligned} \mathbf{D} &= \{D_1 \quad D_2 \quad \cdots \quad D_N\}^{\mathrm{T}} \\ \boldsymbol{\Phi} &= \{\phi_1 \quad \phi_2 \quad \cdots \quad \phi_N\}^{\mathrm{T}} \\ \Delta\mathcal{A} &= \{\Delta\mathcal{A}_1 \quad \Delta\mathcal{A}_2 \quad \cdots \quad \Delta\mathcal{A}_{2N}\}^{\mathrm{T}} \end{aligned} \tag{27}$$

and

$$\begin{aligned} \boldsymbol{X}(\boldsymbol{\Phi}) &= \boldsymbol{A} + \boldsymbol{B}\,diag[\cos 2\phi] + \boldsymbol{C}\,diag[\sin 2\phi] \\ \boldsymbol{A} &= [a_{kl}] = a^l(\omega;x_M,\theta_M)_k \quad (k=1,\ 2,\ ...,\ 2N) \\ \boldsymbol{B} &= [b_{kl}] = b^l(\omega;x_M,\theta_M)_k \quad (l=1,\ 2,\ ...,\ N) \\ \boldsymbol{C} &= [c_{kl}] = c^l(\omega;x_M,\theta_M)_k \end{aligned} \tag{28}$$

Equation (26) represents the structural damage identification algorithm developed in this paper for locating many line crack-like directional damages and also for identifying their severities (*i.e.*, effective damage magnitudes) and orientations with respect to the reference coordinates.

Equation (26) is non-linear equation with respect to unknown damage magnitude $D$ and damage orientation angle $\phi$. In this paper, we can find the damage magnitude $D$ and damage orientation angle $\phi$ in used Newton-Raphson method.

## V. NUMERICAL ILLUSTRATIONS AND DISCUSSIONS

As an illustrative example, consider a cylindrical shell which is simply-supported at both ends. The cylindrical shell has the radius $R$ = 0.125$m$, length $L$ = 0.3$m$, thickness $h$ = 0.003$m$, Young's Modulus $E$ = 206$GPa$, Poisson's ratio $\nu$ = 0.33, and the mass density $\rho$ = 7850$kg/m^3$.

First assume that the cylindrical shell has a line crack and investigate its effects on the natural frequencies of the cylindrical shell. The line crack is 0.015$m$ long and it is centered at ($x_D$, $\theta_D$) = (0.135$m$, 0.9$\pi$). To compute the effective elastic stiffnesses $Q_{ij}^D$ for the SMV containing the line crack, the dimensions of the SMV are chosen as $2\overline{x} = 0.3m$ and $2R\overline{\theta} = 0.025\pi\, m$ so that the effective damage magnitude becomes $D = 0.3$.

Next, the numerically simulated damage identification tests are conducted to validate the present SDIM. Two example problems are considered: (a) the shell with a line crack-like damage (one-damage problem), and (b) the shell with three line crack-like damages (three-damage problem). The details of the line crack-like damages considered for two example problems are given in Table 1. The cylindrical shells are divided into 100 equal-sized finite segments, and the damage identification analyses are conducted to determine the effective damage magnitudes and orientations within all finite segments. A point harmonic force is applied at the point ($x_F$ = 0.15$m$, $\theta_F$ = $\pi$), and Eq. (20) is used to simulate the inertance FRFs at each center of finite segments.

Figures (a) and (b) show the damage identification results for the one-damage problem and the three-damage problem, respectively. The results shown in Figs. (a) and (b) are those sufficiently converged after 6 iterations for the one-damage problem and 25 iterations for the three-damage problem, respectively. One can clearly see from Figs. (a) and (b) that the present SDIM certainly has the capability to identify the directivities of multiple local damages, in addition to the capability to identify their locations and severities.

## VI. CONCLUSIONS

In the present paper, the equations of motion for the thin uniform cylindrical shells with multiple line crack-like directional damages are derived. A structural damage identification algorithm is then derived by using the frequency response functions (FRF) solved from the equations of motion. To provide the capability to identify the directivities of local damages as well, the local damages are considered as the line crack-like directional damages and they are represented by the effective orthotropic elastic stiffness based on a theory of continuum damage mechanics. As the result, the present SDIM can be used to identify the locations, severities, and the orientations of multiple local damages, all together at a time. The numerically simulated damage identification tests are conducted to evaluate the performance of the present FRF-based SDIM. The effects of any possible noises in the measured FRF-data on the damage identification results are also numerically tested. Experimental verifications will be provided in the next coming paper.

## ACKNOWLEDGEMENT

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**Table 1. Damage information pre-specified for the damage identification tests**

| Example problems | Effective damage magnitude | Damage orientation (degrees) | Damage location ($x_D$, $\theta_D$) |
|---|---|---|---|
| One-damage (a) | $D = 0.3$ | $\phi = 30°$ | (0.135$m$, 0.9$\pi$) |
| Three-damage (b) | $D_1 = 0.3$ | $\phi_1 = 0°$ | (0.075$m$, 0.5$\pi$) |
| | $D_2 = 0.4$ | $\phi_2 = 30°$ | (0.135$m$, 1.3$\pi$) |
| | $D_3 = 0.2$ | $\phi_3 = 45°$ | (0.225$m$, 0.9$\pi$) |

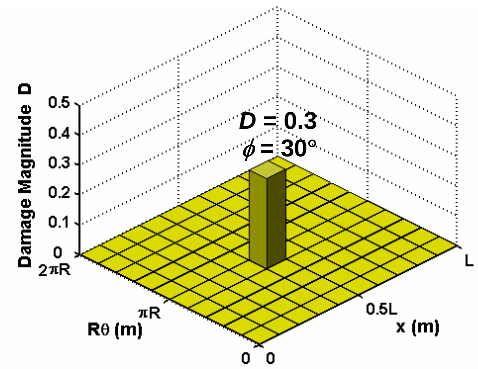


**Fig. (a) Damage identification results for the one-damage problem**

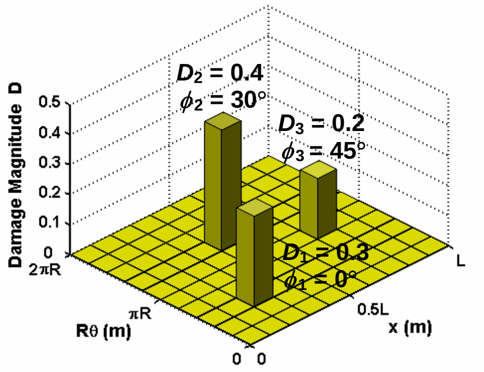


**Fig. (b) Damage identification results for the three-damage problem**

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