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A PARAMETER CHANGE TEST IN RCA(1) MODEL

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Abstract

In this paper, we consider the problem of testing for parameter change in time series models based on a cusum of squares. Although the test procedure is well-established for the mean and variance in time series models, a general parameter case was not discussed in literatures. Therefore, here we develop the cusum of squares type test for parameter change in a more general framework. As an example, we consider the change of the parameters in an RCA(1) model. Simulation results are reported for illustration.

Keywords : Testing for parameter change, Cusum of squares test, RCA model, functional CLT, martingale difference.

1. Introduction

Since the seminal paper of Page (1955), the problem of testing for a parameter change has been an important issue among statisticians. Originally, the problem began with iid sample and it moved naturally into the time series context since economic time series often exhibits prominent evidences for structural change in its underlying model.

If the random observations are iid and follow a parametric model, one may consider utilizing a likelihood ratio method as in Csorgo and Horvath (1997). However, the method is no longer applicable either if the iid assumption is violated or if the underlying distribution is completely unknown. In such a case, a nonparametric approach should be considered as an alternative. From this viewpoint, here we pay attention to the cusum method for the test of parameter change.

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The cusum method is easy to handle and useful for detecting the locations of change points as seen in Inclan and Tiao (1994). However, the usage has been restricted to the change of mean, variance and distribution function (cf. Bai, 1994). An ease to use the method lies to he fact that the sample mean, variance and distribution function are all expressed as the sum of iid random variables, and the convergence result of the sequential test statistic is easily obtained.

The method for a general parameter case was not yet clarified, but an extension is straightforward. It can be shown in section 3. Before moving on to the method, we will briefly see the RCA(1) model.

2. RCA(1) model

The RCA(random coefficient autoregressive) models are widely used in many areas such as biology, engineering, finance and economics, and have been studied to investigate the effects of random perturbations of a dynamical system (cf. Tong, 1990). Many important properties of RCA models are reported in Nicholls and Quinn (1982) and Feigin and Tweedie (1985), some of which will be used in appropriate places.

Let $x_t, t = 0, 1, 2 \cdots$ be the time series of the RCA(1) model:

$x_t = (\phi + b_t)x_{t-1} + \epsilon_t$

where $\begin{pmatrix} b_t \\ \epsilon_t \end{pmatrix} \sim i.i.d.\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} w^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\right)$

(1)

Nicholls and Quinn (1982) showed that a sufficient condition for the strict stationarity and ergodicity of x_t is $\phi^2 + w^2 < 1$. Here, we assume that $E\epsilon_t^{2k} < \infty$ and $E(\phi + b_t)^{2k} < 1$ for some k that will be specified in the proof, which

immediately yields $Ex_t^{2k} < \infty$ (cf. Feigin and Tweedie, 1985).

Now, we consider the problem of testing for a change of the parameter vector $\boldsymbol{\theta} = (\phi, w^2, \sigma^2)$ based on (conditional) LSE $\hat{\boldsymbol{\theta}}$. Suppose that $x_1, \cdots, x_n$ are a sample from (1) and assume $x_0 = 0$. We intend to test the following hypotheses:

H_0 : $\boldsymbol{\theta} = (\phi, w^2, \sigma^2)$ is constant over $x_1, \cdots, x_n$
H_1 : not H_0

3. CUSUM test

Here, we can see the main theorem of this paper and it can be proved by using the property of martingale difference and functional CLT.

Theorem 1. Define the test statistic T_n by

$$T_n = \max_{J \le k \le n} \frac{k^2}{n} (\widehat{\boldsymbol{\theta}}_k - \widehat{\boldsymbol{\theta}}_n)' \Gamma^{-1} (\widehat{\boldsymbol{\theta}}_k - \widehat{\boldsymbol{\theta}}_n)$$

Suppose that $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ has the main decomposition form. If conditions

$$\sum_{t=1}^{[ns]} \boldsymbol{\xi}_{n,t} \xrightarrow{w} W_J(s)$$

and

$$\max_{1 \le k \le n} \frac{\sqrt{k}}{\sqrt{n}} |\Delta_{j,k}| = o_{P(1)}, j = 1, \cdots, J \quad \text{hold}$$

then,

under H_0, $Tn \xrightarrow{d} sup_{0 \le s \le 1} \sum_{j=1}^{J} (W_j^o(s))^2$.

We reject H_0 if T_n is large.

Theorem 1 shows that the change point test in time series models can be accomplished based on any estimators provided they satisfy regularity conditions. We can say that the cusum test is widely applicable in a broad class of time series models since it constitutes the most natural nonparametric test, and some well-known estimators, such as the method of moment estimator and the Gaussian MLE, could be employed to perform a test.

By direct application of theorem 1, we can get the cusum test for RCA(1) model.

Theorem 2. Suppose that Suppose that $E\varepsilon_1^{16} < \infty$ and $E(\phi + b_1)^{16} < 1$. Define the test statistic

$$T_n = \max_{3 \le k \le n} \frac{k^2}{n} (\widehat{\theta}_k - \widehat{\theta}_n)' \Gamma^{-1} (\widehat{\theta}_k - \widehat{\theta}_n)$$

where $\widehat{\Gamma}$ is a consistent estimator of Γ. Then under H_0,

$$Tn \xrightarrow{d} sup_{0 \le s \le 1} \sum_{j=1}^{3} (W_j^o(s))^2.$$

We reject H_0 if T_n is large.

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