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시간에 따른 탄성지지 열탄성 접촉에 대한 열접촉저항의 영향
Effects of Thermal Contact Resistance on Transient Thermoelastic Contacts for an Elastic Foundation

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Key Words : Transient thermoelastic contacts, Elastic foundation, Thermal contact resistance, Unilateral contact condition

ABSTRACT

The paper presents a numerical solution to the problem of a hot rigid indenter sliding over a thermoelastic Winkler foundation with a thermal contact resistance at constant speed. It is shown analytically that no steady-state solution can exist for sufficiently high temperature or sufficiently small normal load or speed, regardless of the thermal contact resistance. However, the steady state solution may exist in the same situation if the thermal contact resistance is considered. This means that the effect of the large values of temperature difference and small value of force or velocity which occur at no steady state can be lessened due to the thermal contact resistance.

When there is no steady-state, the predicted transient behavior involves regions of transient stationary contact interspersed with regions of separation regardless of the thermal contact resistance. Initially, the system typically exhibits a small number of relatively large contact and separation regions, but after the initial transient, the trailing edge of the contact area is only established and the leading edge loses contact, reducing the total extent of contact considerably. As time progresses, larger and larger number of small contact areas are established, until eventually the accuracy of the algorithm is limited by the discretization used.

1. Introduction

When two bodies slide against each other, frictional heating at the interface causes thermoelastic deformation which modifies the contact pressure distribution. Hills and Barber[1] gave an analytical solution for sliding Hertzian contact, using a thermoelastic Green's function to reduce the problem to the solution of an integral equation with a Bessel function kernel. A remarkable feature of their results was that no steady-state solution could be found in certain ranges of the applied load and sliding speed without violation of the unilateral contact constraints. Similar results were demonstrated by Yevtushenko and Ukhanska[2] for a problem with interfacial thermal contact resistance, which was not a function of pressure. Jang[3] showed that similar problems arise in the simpler case in which the contacting bodies are replaced by elastic foundation. He developed a numerical algorithm for the transient problem in this case and showed that the contact area tends to break down into a number of smaller regions as sliding progresses. Even more surprising is the fact that this process appears to continue without limit, leading to larger and larger numbers of smaller contact areas. Existence theorems can be proved for the corresponding transient problem, so we must conclude that in these parameter ranges the system must undergo periodic or random transient variations in contact conditions.

In this study, Jang's analysis is extended to the sliding without friction of a hot rigid perfectly conducting indenter over a linear thermoelastic Winkler foundation with a thermal contact resistance, which is not a function of pressure. We will investigate the effects of the thermal contact resistance on the transient thermoelastic contact problems.

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2. Statement of the Problem

Consider the problem illustrated in Fig. 1, where an indenter at temperature T_o is pressed into the foundation with a force F and moves to the right at constant speed V. The mechanical behavior of the foundation is defined by the statement that the local contact pressure p is proportional to the local indentation u --- i.e. $u(x,t)=cp(x,t)$, where c is the elastic foundation compliance. We also assume that lateral thermal conduction in the foundation can be neglected so that it behaves likes a set of parallel one-dimensional rods oriented normal to the interface and each rod acts independently of its neighbors.

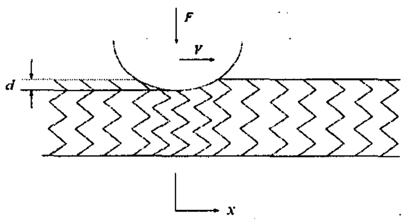

Fig. 2.1 Geometry configuration of transient thermal contact

If the indenter contacts with a surface at time $t=t_0$ with a thermal resistance $1/hA$ where A is the contact area, the temperature for $y<0,\ t>t_0$ is given by Schneider[4]. The corresponding thermal displacement on the surface can be shown to be

$$\delta(x,t)=T_0\left(\frac{2\alpha\sqrt{x(t-t_0)}}{\sqrt{\pi}}+\frac{\alpha k}{h}e^{h^2x(t-t_0)/k^2}erfc\left(\sqrt{\frac{h^2x(t-t_0)}{k^2}}\right)-\frac{\alpha k}{h}\right) \quad (1)$$

where x, k and α are the thermal diffusivity, the thermal conductivity, and the coefficient of the thermal expansion, respectively. If contact at x ends at $t=t_1$, the thermal displacement will remain constant at the value $\delta(x,t_1)$ for $t>t_1$.

Using these results, the gap function can be defined as follows,

$$g(x,t)=g_0(x,t)-d(t)-\delta(x,t)+u(x,t), \quad (2)$$

where

$$g_0(x,t)=(x-Vt)^2/2R \quad (3)$$

is the gap between the indenter and an undeformed foundation and d is an unknown rigid body displacement which will generally vary with time.

The boundary condition for contact and separation regions can be written

$$separation\ p(x,t)=0\ ;\ g(x,t)>0$$

$$contact\ p(x,t)>0\ ;\ g(x,t)=0 \quad (4)$$

and equilibrium requires that

$$F=\int_C p(x,t)dx, \quad (5)$$

where C is the contact region.

3. Dimensionless Formulation

The number of independent parameters can be reduced by utilizing the self-similarity of the punch profile. There are two length scales in the problem --- the radius R and a force-related quantity $L=\sqrt{3\alpha fcFR}$ We define the dimensionless coordinates $\hat{x}=x/L$, $\hat{t}=Vt/L$ and other dimensionless quantities through $\hat{\delta}=R\delta/L^2$, $\hat{g}=Rg/L^2$, $\hat{d}=Rd/L^2$, $\hat{p}=cRp/L^2$, $\hat{H}=k/h\sqrt{V/(kL)}$. Introducing these definitions into Eqs. (2,3,4,6) yields

$$\hat{\delta}(\hat{x},\hat{t})=\sqrt{\frac{3\pi\lambda}{8}}\left(2\sqrt{\frac{\hat{t}-\hat{t}_0(\hat{x})}{\pi}}-\hat{H}+\hat{H}e^{\left(\frac{\hat{t}-\hat{t}_0(\hat{x})}{\hat{H}^2}\right)}erfc\left(\frac{\sqrt{\hat{t}-\hat{t}_0(\hat{x})}}{\hat{H}}\right)\right)$$

$$;\ \hat{t}_0(\hat{x})<\hat{t}<\hat{t}_1(\hat{x}) \quad (6)$$

$$\hat{g}(\hat{x},\hat{t})-\hat{p}(\hat{x},\hat{t})=\frac{(\hat{x}-\hat{t})^2}{2}-\hat{d}\hat{t}-\hat{\delta}(\hat{x},\hat{t}) \quad (7)$$

and

$$\int_{\hat{c}} \hat{p}(\hat{x}, \hat{t})\, d\hat{x} = 1, \tag{8}$$

where $\lambda \equiv 8a^2 T_0^2 xR/(3\pi cFV)$.

Notice that with this formulation, the dimensionless parameters governing the evolution of the process are λ which can be seen as a ratio between thermoelastic and elastic effects and $\hat{H}$

4. Steady-State Solution

Since the contacting body moves at a constant speed, it is natural to expect the long-time behavior to become invariant in a frame of reference moving with the body. In particular, the indentation $\hat{d}$ would then be independent of $\hat{t}$ Denoting the value of this constant by d_0, we can then find the leading edge $\hat{a}(\hat{t})$ of the contact area by enforcing $\hat{g}=0$, $\hat{p}=0$ in Eq. (7), with the result $\frac{(\hat{a}-\hat{t})^2}{2} = d_0$, since the thermal expansion must be zero before contact starts. It follows that $\hat{a}(\hat{t}) = \sqrt{2d_0} + \hat{t}$, or alternatively that $\hat{t_0}(\hat{x}) = \hat{x} - \sqrt{2d_0}$.

The expansion in the contact area can now be calculated from Eq. (6) and the contact pressure from (7). The trailing edge of the contact area $\hat{b}(\hat{x})$ is defined by the condition that the contact pressure goes to zero. One solution of the resulting equation is clearly $\hat{a}(\hat{t})$ and the other is the one real root which comes from the above analysis with $\hat{p}(\hat{x}, \hat{t})=0$. Once $\hat{a}, \hat{b}$ have been determined, the corresponding value of λ can be obtained.

For the special case where $d_0=0$, the corresponding pressure distribution is

$$\hat{p}(\hat{x}, \hat{t}) = -\frac{(\hat{x}-\hat{t})^2}{2} + \sqrt{\frac{3\pi\lambda}{8}}\left(2\sqrt{\frac{t-x}{\pi}} - \hat{H} + \hat{H}e^{\left(\frac{\hat{t}-\hat{x}}{\hat{H}^2}\right)} erfc\left(\frac{\sqrt{t-x}}{\hat{H}}\right)\right) \tag{9}$$

and Eq. (8) then yields

$$1 = \int_{\hat{b}}^{a} \hat{p}(\hat{x}, \hat{t})\, d\hat{x} \tag{10}$$

$$= \frac{-\xi}{12}\left(2\xi^2 + \sqrt{6\lambda}(3\hat{H}\sqrt{\pi} - 4\sqrt{\xi}\right) + \frac{\hat{H}}{2}\sqrt{\frac{3\pi\lambda}{2}}\left(\frac{2\hat{H}}{\sqrt{\pi}}\sqrt{\xi} + e^{(\xi)/\hat{H}^2} \times \hat{H}^2 erfc\left(\frac{\sqrt{\xi}}{\hat{H}}\right) - \hat{H}^2\right), \tag{11}$$

where $\xi = \hat{t} - \hat{x}$.

Only positive values of d_0 are admissible and it can be shown that the integral in (8) is a monotonically increasing function of d_0 in the range $d_0 > 0$. Thus, there is no steady-state solution of the assumed form if the value of the right hand side in Eq. (11) is greater than 1. Notice that the integral in Eq. (11) has two parameters, λ and $\hat{H}$ to identify the steady state solution. Fig. 2. shows the stability diagram in which the thermal contact resistance is included. The steady state solutions are obtained when λ is less than 1 regardless of the value of $\hat{H}$ However, even when λ is greater than 1 and $\hat{H}$ is above a certain value, the steady state solution can exist.

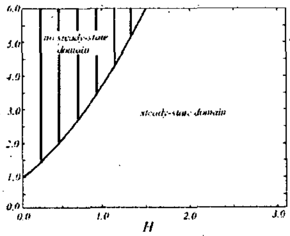

Fig. 2 : Stablity diagram for λ and $\hat{H}$

To determine how the system behaves at large values of time for $\lambda > 1$ and $\hat{H} > 0$, a numerical solution of the problem has been developed, which is described in the next section.

5. Numerical Implementation

The contact problem can be discretized in space and time by dividing the elastic foundation into discrete strips of width $\Delta\hat{x}$ and proceeding in increments of time $\Delta\hat{t}$. The numerical algorithm explained below was developed by Jang[3]. Note that in the numerical simulation, the discrete strips of width $\Delta \hat{x} = 0.001$ and the increments of time $\Delta \hat{t} = 0.001$.

6. Results

7.1 Contact area and rigid body penetration

When $\lambda > 1$, the transient behavior of the system depends upon the values of $\hat{H}$. Fig. 3 shows the extent of the contact area and the rigid body penetration $\hat{a}$ as functions of time $\hat{t}$ for $\lambda = 6.0$ and $\hat{H} = 2.0$. In the initial transient, the leading edge of the contact area remains unchanged, while the trailing edge moves, reducing the total extent of contact. It shows that a steady state with a single contact area is established after about $\hat{t} = 12$, confirming that the system settles into a steady state.

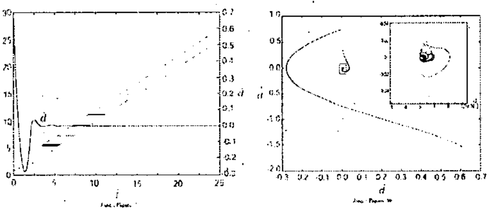

Fig. 3 : Extent of contact area and rigid body penetration $\hat{a}$ as function of time $\hat{t}$ and phase diagram for $\lambda = 6.0$ and $\hat{H} = 1.5$

When $\hat{H}$ decreases more and the system settle into no steady state, the duration of the initial transient increases and involves a succession of separated contact areas and oscillations in the value of $\hat{a}$ incessantly. Figs. 6 shows the results for $\lambda = 6.0$ and $\hat{H} = 0.5$. For Fig. 6, it shows that the system has no steady state and its state is characterized as contact with numerous small intervening region of separation after about $\hat{t} = 4$.

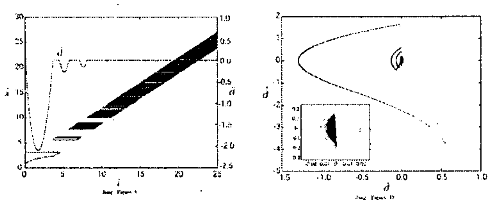

Fig. 4 : Extent of contact area and rigid body penetration $\hat{a}$ as function of time $\hat{t}$ and phase diagram for $\lambda = 6.0$ and $\hat{H} = 0.5$

For $\lambda = 6.0$ and $\hat{H} = 0.1$, larger separation zones alternate with relatively small zones of contact.

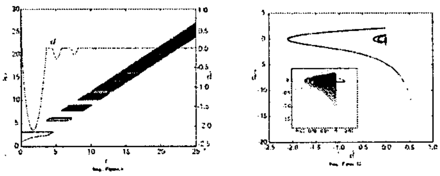

Fig. 3 : Extent of contact area and rigid body penetration $\hat{a}$ as function of time $\hat{t}$ and phase diagram for $\lambda = 6.0$ and $\hat{H} = 0.1$

7. Summary

The investigation presents a numerical solution to the problem of a hot rigid indenter sliding over a thermoelastic Winkler foundation with a thermal resistance at a constant speed. The numerical solution shows that the steady state solution, when it exists, is the final condition regardless of the initial conditions imposed. The results also show that the thermal contact resistance affects the long-term behavior of the system along with the parameter λ, a ratio between thermoelastic and elastic effects. Regardless of the thermal contact resistance, the steady state solutions are obtained when λ is less than 1. However, when λ is greater than 1, the steady state solution can exist according to the thermal contact resistance.

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