An Evaluation of Effective Atomic Number in Compton Scattering System Using Low-Energy Gamma Rays

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1. Introduction

The attenuation coefficient, the electron density and the effective atomic number are basic quantities in determining the penetration of gamma rays in matter. There are many applications in the medical radiation dosimetry, medical imaging and inspection systems using gamma rays where it is required to determine the effective atomic number, $Z_{\text{eff}}$, of target materials. However, it is not easy to find the effective atomic number in a transmission gamma radiation system. Scattered gamma rays provide useful additional information as they have interacted with the materials of interest. The effective atomic number is determined in several ways depending on the type of photon scattering processes considered in the scattering system.

In this study, a new approach using the ratio of the total attenuation coefficient to the incoherent attenuation coefficient is suggested for evaluating the $Z_{\text{eff}}$ number of compound materials in a Compton scattering system with a Co-57 gamma source. In order to achieve acceptable accuracy of the approach, the ratio is processed by using the least squares method with many reference compounds.

2. Method and Results

1. Theory

For the low energy range of gamma rays where the pair production effect does not exist, the mass attenuation coefficient $\mu_t$ of an individual element can be given as a sum of three terms: photoelectric absorption $\mu_{\text{pe}}$, Rayleigh scattering $\mu_R$ and Compton scattering $\mu_C$

$$\mu_t = \frac{1}{\rho} \left( \mu_{\text{pe}} + \mu_R + \mu_C \right) = \frac{N_A}{A} \left( Z \sigma_C + \sigma_t + \sigma_R \right) \quad (1)$$

where $\rho$, $A$ and $Z$ are the density, mass number and atomic number, respectively. $N_A$ is the Avogadro's number. The Compton scattering cross section $\sigma_C$ is given by the Klein-Nishina formula. The photoelectric effect and Rayleigh scattering cross section, $\sigma_t$ and $\sigma_R$, can be determined by the following empirical formula

$$\sigma_C = a Z^m, \sigma_t = a \frac{Z^m}{E^k}, \sigma_R = b \frac{Z^n}{E^l} \quad (2)$$

where the constants $a$, $b$, $m$, $k$, $n$ and $l$ are determined to give the best fit to actual attenuation coefficients.

For a chemical compound a single number cannot present the atomic number uniquely across the entire energy region. Thus, an effective atomic number is used for characterizing the compound and the partial process of gamma rays in the material. The number is defined as

$$Z_{\text{eff}} = \left( \sum w_i Z_i^q \right)^{1/q} \quad (3)$$

where $q$ is 1, $m$ and $n$ for the effective atomic number $Z_C$ in Compton scattering, for $Z_{\text{pe}}$ in photoelectric effect and for $Z_R$ in Rayleigh scattering, respectively. $w_i$ is the weight fraction of the $i$th element in the compound. Hence the total mass attenuation coefficient can be expressed as

$$\frac{\mu_t}{\mu_C} = \frac{N_A}{A} \left( Z_C \sigma_{\text{KN}} + a \frac{Z^m}{E^k} + b \frac{Z^n}{E^l} \right) \quad (4)$$

Although all the effective numbers $Z_C$, $Z_{\text{pe}}$ and $Z_R$ are related each other, for an unknown compound it is not easy to determine each of them by solving Eq.(4) because they can be seen as independent unknowns.

2. Calculation of Effective Atomic Number for a Compton Scattering System

In practice it is actually good enough to know just one effective atomic number among the effective atomic numbers defined in Eq.(3). To obtain an effective atomic number of compounds in a scattering system in which the total and Compton scattering attenuation coefficients can be obtained with measurements, a new approach is suggested. Making a ratio $\mu_t/\mu_C$, one obtains

$$\frac{\mu_t}{\mu_C} = \frac{1}{\sigma_{\text{KN}}} \left( \sigma_{\text{KN}} + a \frac{Z^m}{E^k} + b \frac{Z^n}{E^l} \right) \quad (5)$$

In Eq.(5), $\sigma_{\text{KN}}$ is independent of the atomic number. The term of $Z^{m-1}$, which represents for Rayleigh scattering, contributes a valid value to the total attenuation coefficient only when the gamma rays are scattered in a small angle or with high energy. Hence it can be assumed that the effective atomic number $Z_{\text{eff}}$ of the compound is defined by

$$Z_{\text{eff}} = \left( \sum w_i Z_i^{m-1} \right)^{1/m-1} \quad (6)$$

where the value of $m$ can be determined by applying Eq.(2) for the attenuation coefficients of the elements.

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In order to obtain the $Z_{\text{eff}}$ the ratio $\mu_t/\mu_C$ is expressed in the form of a polynomial function. The coefficients of the polynomial function can be determined by using any cross section database for many compounds. Most biological tissues and compounds used in clinical dosimetry have $Z_{\text{eff}}$ numbers for all photon processes in the range from 5 to 35. $Z_{\text{eff}}$ numbers of drug and explosive materials which are the most interested objects of inspection systems are in this range. Hence for illustrating the method about 200 compounds having $Z_{\text{eff}}$ in the range from 5 to 35 are chosen, the attenuation coefficients were obtained from XCOM Database\(^5\). The ratios $\mu_t/\mu_C$ of these compounds and the fitting curves for gamma energy at 98keV and 122keV are shown in Fig. 2 as an example. The degree of the polynomial function was chosen to give satisfactory fit to the actual ratios $\mu_t/\mu_C$ of the compounds.

Figure 2 shows that the ratio $\mu_t/\mu_C$ increases as $Z_{\text{eff}}$ increases, therefore the polynomial functions of the ratio $\mu_t/\mu_C$ can be easily solved by a numerical calculation. The calculation results for some common compounds which have the $Z_{\text{eff}}$ in the interested range, from 5 to 35, at 98keV and 122keV energies are shown in Table 1 as an example. The error in the Table is the percentage error of the $Z_{\text{eff}}$ calculated from the ratio $\mu_t/\mu_C$ of the compounds by the bisect numerical method for both energies relative to the $Z_{\text{eff}}$ obtained from Eq.(6). The errors about 2.5% demonstrate that the suggested method is a valid approach to evaluate the $Z_{\text{eff}}$ of compounds by a scattering system.

### Table 1 Calculated values of $Z_{\text{eff}}$ for some common compounds

<table>
<thead>
<tr>
<th>Material</th>
<th>$Z_{\text{eff}}$</th>
<th>Calculated $Z_{\text{eff}}$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethylene</td>
<td>5.740</td>
<td>5.747, 5.725</td>
<td>0.11, 0.26</td>
</tr>
<tr>
<td>Bakelite</td>
<td>6.372</td>
<td>6.422, 6.416</td>
<td>0.78, 0.69</td>
</tr>
<tr>
<td>TNT</td>
<td>7.198</td>
<td>7.288, 7.274</td>
<td>1.24, 1.06</td>
</tr>
<tr>
<td>Teflon</td>
<td>8.498</td>
<td>8.686, 8.647</td>
<td>2.21, 1.75</td>
</tr>
<tr>
<td>PVC</td>
<td>14.530</td>
<td>14.504, 14.417</td>
<td>0.18, 0.78</td>
</tr>
<tr>
<td>Steel</td>
<td>25.807</td>
<td>26.449, 26.319</td>
<td>2.49, 1.99</td>
</tr>
<tr>
<td>Brass</td>
<td>30.415</td>
<td>30.800, 30.655</td>
<td>1.27, 0.79</td>
</tr>
</tbody>
</table>

### 3. Conclusions

A new simple evaluation of the effective atomic number $Z_{\text{eff}}$ of material in a Compton scattering system is performed by the expression of the ratio $\mu_t/\mu_C$ as a polynomial function of $Z_{\text{eff}}$. The method can be applied to the low range of gamma energies where the Compton scattering does not dominate. For the compounds which have $Z_{\text{eff}}$ from 5 to 35, the calculations have shown that the results are given with reasonable errors, less than 2.5%, good enough to be used in any further calculation of medical science or inspection purposes.

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### REFERENCES