A Mathematical Approach to Allocate the Contributions by Applying UPFCs to Transmission System Usage

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Abstract: Competitive electricity markets necessitate equitable methods for allocating transmission usage in order to set transmission usage charges and congestion charges in an unbiased and an open-accessed basis. So in competitive markets it is usually necessary to trace the contribution of each participant to line usage, congestion charges and transmission losses, and then to calculate charges based on these contributions. A UPFC offers flexible power system control, and has the powerful advantage of providing, simultaneously and independently, real-time control of voltage, impedance and phase angle, which are the basic power system parameters on which sys-tem performance depends. Therefore, UPFC can be used efficiently and flexibly to optimize line utilization and increase system capability and to enhance transmission stability and dampen system oscillations. In this paper, a mathematical approach to allocate the contributions of system users and UPFCs to transmission system usage is presented. The paper uses a dc-based load flow modeling of UPFC-inserted transmission lines in which the injection model of the UPFC is used. The relationships presented in the paper showed modified distribution factors that modeled impact of utilizing UPFCs on line flows and system usage. The derived relationships show how bus voltage angles are attributed to each of changes in generation, injections of UPFC, and changes in admittance matrix caused by inserting UPFCs in lines. The relationships derived are applied to two test systems. The results illustrate how transmission usage would be affected when UPFC is utilized. The relationships derived can be adopted for the purpose of allocating usage and payments to users of transmission network and owners of UPFCs used in the network. The relationships can be modified or extended for other control devices.

Keywords: Distribution factors; FACTS; UPFC; Restructured power systems; Transmission pricing

1. INTRODUCTION

Transmission network is the most important component in competitive electricity markets and serves as the key mechanism for generators to compete in the supply to reach large users and distribution companies. However, with launching of restructuring in power industry [1], power system operations have demanded more power flow control needs and transmission business has undergone an increased level of attention as transmission system has been realized as the sender of transparent price signals to market participants.

In competitive electricity markets [1], energy prices and transmission pricing are highly affected by transmission congestion and other system constraints, where a congested transmission is accompanied by higher costs due to resorting to out-of-merit order as expensive generating units are dispatched to alleviate congestion [2]. Therefore, an increased attention has been paid to new devices that provide more flexibility to operate the transmission system and guarantee lower-cost mechanisms by which transmission constraints can be mitigated. The factors that justify the escalating demand for powerful control devices in restructured (deregulated) power system environments are: (a) need for more transfer capability to withstand a wider range of possible generation patterns for existing power systems without resorting to costly expansion, (b) need to reduce the constantly increasing level of risk and uncertainty associated with transmission operations and investment, (c) need to bring clean competition to electricity markets, and (d) increasing loads imposed on electric power networks motivate electric utilities to constantly look for new devices that help interconnected systems acquire adequate power transfer capabilities for the existing power transmission lines without relying on expensive or time-consuming alternatives such as up-rating existing network or building new facilities [3,4].

Recently, FACTS devices have received more attention in transmission system operations as they can be utilized to alter power system parameters in order to control power flows and stabilize system, in addition to the fact that FACTS devices have the capability of increasing transmission capabilities to the required levels [5]. The UPFC—which has been recognized as one of the best-featured FACTS devices—offers a unique combination of fast shunt and series compensations and provides a flexible power system control, therefore, it can be utilized in the power system to control line active and reactive power, achieve maximum power transfer capability, stabilize system, reduce total generation cost associated with out-of-merit order, significantly improve power system reliability, and help the system operate with more security [3,4, 6-9]. The UPFC offers a mechanism that may replace or help traditional transmission constraint mitigation methods. It can redirect or force power to flow along certain paths, therefore, it is efficient in mitigating undesired loop or parallel flows. In some cases, putting UPFC in service prevents generators to run in out-of-merit order, prevents load shedding/curtailment that would be required to maintain system security, reduces the operational and investment costs of a power system, and improves system security.

To control, operate, evaluate and price usage of power systems, it is usually necessary to know whether or not and to what extent each market participant contributes to a transmission usage [10-14]. The demand on allocation processes is of more importance in an open-accessed restructured power system in order to allocate transmission charges to different power system users using reasonable and fair allocation and pricing rules that are based on the actual usage of the transmission system. In competitive markets it is usually necessary to trace the contribution of each participant to line usage, congestion charges and transmission losses, and then to calculate charges based on these contributions.

Based on a previously derived dc-based injection model of UPFC-embedded lines by this author [3,4], the present paper derives relationships to model impact of UPFC on line flows and transmission pricing. In this paper we define modified admittances and Jacobean matrix that consider impact of UPFC. By knowing the locations and parameters of UPFC in the system and using the original admittances and Jacobean

matrix of a power system before inserting a UPFC, we use the matrix inversion lemma to derive the modified Jacobean matrix and its inverse. The relationships presented in the paper show modified distribution factors that model impact of utilizing UPFC on line flows and system usage. The relationships derived show how bus voltage angles and line flows are attributed to each of changes in generation, injections of UPFC, and changes in admittance matrix entries caused by inserting UPFC in lines. The relationships are then applied to a 4-bus, where the results illustrate how transmission usage is affected when a UPFC is utilized and show how contributions of a generator to line power flows change after adding a UPFC. The derivations presented in the paper can be used to price usage of UPFC and to assess transmission pricing, where the factors presented can be used to allocate charges of the transmission networks to market participants.

2. UPFC MODELING

The UPFC comprises shunt and series control elements and offers a unique combination of fast shunt and series compensations [3,4,6-9]. The UPFC injects a voltage in series with the transmission line through a series transformer. The active power involved in the series injection is taken from the transmission line through a shunt transformer, and the UPFC generates or absorbs the needed reactive power locally by the switching operation of its two converters. A UPFC offers flexible power system control, and has the powerful advantage of providing, simultaneously and independently, real-time control of voltage, impedance and phase angle, which are the basic power system parameters on which system performance depends. Therefore, UPFC can be used efficiently and flexibly to optimize line utilization and increase system capability and to enhance transmission stability and dampen system oscillations.

Fig. 1 represents a lossless UPFC-embedded transmission line that connects node i to node j. This representation is the combined network that includes both the UPFC and the _-model of the transmission line [3]. Each of the two converters is represented by a voltage source in series with a reactance. The transmission line is represented by the lossless two-port network. The complex voltages $U_{ij} = U_{ij}\angle\delta_{ij}$ and $E_{ij} = E_{ij}\angle\beta_{ij}$ are respectively the controllable voltages inserted from the series branch and the shunt branch of the UPFC in the line connecting bus i to bus j, and $V_i = V_i\angle\theta_i$ and $V_j = V_j\angle\theta_j$ are the complex voltages at nodes i and j, respectively. The reactance x_{se}/x_{sh} refers to the reactance associated with the series/shunt voltage source of the UPFC, and x_l/x_c refers to the line series/shunt reactance [3].
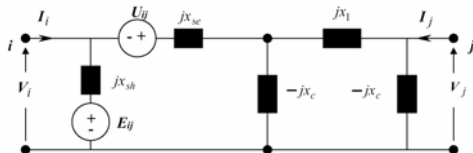

Fig. 1 Representation of a lossless UPFC-embedded line.

Let the admittances y_1 and y refer, respectively, to the series and shunt admittances of the two-port network of the π-equivalent circuit of a transmission line before adding the UPFC, and the admittances y_{se} and y_{sh} refer, respectively, to

the series and shunt admittances associated with the series and shunt voltage sources of the UPFC. These admittances can be expressed as:

$$y_1 = \frac{1}{jx_1} = -j\frac{1}{x_1} = jb_1 \tag{1}$$

$$y = \frac{1}{-jx_c} = j\frac{1}{x_c} = jb_c \tag{2}$$

$$y_{se} = \frac{1}{jx_{se}} = -j\frac{1}{x_{se}} = jb_{se} \tag{3}$$

$$y_{sh} = \frac{1}{jx_{sh}} = -j\frac{1}{x_{sh}} = jb_{sh} \tag{4}$$

Before inserting the UPFC in the line connecting buses i and j, the admittance matrix of the line is:

$$Y^o = \begin{bmatrix} y_{ii}^o & y_{ij}^o \\ y_{ji}^o & y_{jj}^o \end{bmatrix} = \begin{bmatrix} y_1 + y & -y_1 \\ -y_1 & y_1 + y \end{bmatrix} \tag{5}$$

After inserting the UPFC in the line connecting buses i and j, the admittance matrix becomes:

$$Y = \begin{bmatrix} y_{ii} & y_{ij} \\ y_{ji} & y_{jj} \end{bmatrix} = \begin{bmatrix} y_1 + y + \Delta y_{ii} & -y_1 + \Delta y_{ij} \\ -y_1 + \Delta y_{ji} & y_1 + y + \Delta y_{jj} \end{bmatrix} \tag{6}$$

$$= \begin{bmatrix} y_{ii}^o + \Delta y_{ii} & y_{ij}^o + \Delta y_{ij} \\ y_{ji}^o + \Delta y_{ji} & y_{jj}^o + \Delta y_{jj} \end{bmatrix}$$

The last equation can be expressed as:

$$Y = Y^o + \Delta Y \tag{7}$$

Where

$$\Delta Y = \begin{bmatrix} \Delta y_{ii} & \Delta y_{ij} \\ \Delta y_{ji} & \Delta y_{jj} \end{bmatrix} = \begin{bmatrix} \Delta y_{io} - \Delta y_{ij} & \Delta y_{ij} \\ \Delta y_{ji} & \Delta y_{jo} - \Delta y_{ij} \end{bmatrix} \tag{8}$$

Using Kirchof's laws and with some tedious mathematical manipulations we can express I_i and I_j in Fig. 1 as follows:

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_{ii} & y_{ij} \\ y_{ji} & y_{jj} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} y_1 + y & -y_1 \\ -y_1 & y_1 + y \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} + \begin{bmatrix} \Delta y_{ii} & \Delta y_{ij} \\ \Delta y_{ji} & \Delta y_{jj} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

Or

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_1 + y + \Delta y_{ii} & -y_1 + \Delta y_{ij} \\ -y_1 + \Delta y_{ji} & y_1 + y + \Delta y_{jj} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{10}$$

The entries of the admittance matrices in (9) and (10), in terms of reactances and susceptances are given by:

$$y_{ii} = j\left[\frac{x_1 - x_c}{x_1 x_c + x_{se}x_c - x_{se}x_1} - \frac{1}{x_{sh}}\right] \tag{11}$$

$$= j\left[\frac{b_{se}b_1 + b_{se}b_c}{b_{se} + b_1 + b_c} + b_{sh}\right]$$

$$y_{ij} = y_{ji} = j\frac{x_c}{x_1 x_c + x_{se}x_c - x_{se}x_1} = -j\frac{b_{se}b_1}{b_{se} + b_1 + b_c} \tag{12}$$

$$y_{jj} = j\frac{-x_c^2 + x_1 x_c + 2x_c x_{se} - x_1 x_{se}}{x_c(x_1 x_c + x_c x_{se} - x_1 x_{se})} \tag{13}$$

$$= j\frac{b_{se}b_1 + b_{se}b_c + 2b_1 b_c + b_c^2}{b_{se} + b_1 + b_c}$$

$$\Delta y_{ij} = \Delta y_{ji} = j\left[\frac{x_c}{x_1 x_c + x_c x_{se} - x_1 x_{se}} - \frac{1}{x_1}\right] \tag{14}$$

$$= j\frac{b_1^2 + b_1 b_{se}}{b_{se} + b_1 + b_c}$$

$$\Delta y_{ii} = \Delta y_{io} - \Delta y_{ij} = j\left[\frac{x_{se}(x_1 - x_c)^2}{x_c(x_1 x_c + x_c x_{se} - x_1 x_{se})} - \frac{1}{x_{sh}}\right] \tag{15}$$

$$= j\frac{b_{sh}(b_{se} + b_1 + b_c) - b_c(b_1 + b_c) - b_1^2 - b_1 b_c}{b_{se} + b_1 + b_c}$$

$$\Delta y_{jj} = \Delta y_{jo} - \Delta y_{ij} = j\frac{x_{se}x_c}{x_1(x_1 x_c + x_c x_{se} - x_1 x_{se})} \tag{16}$$

$$= -j\frac{b_1^2}{b_{se} + b_1 + b_c}$$

$$\Delta y_{io} = -j\left[\frac{1}{x_{\text{sh}}} + \frac{x_{\text{se}}(x_c - x_1)}{x_c(x_1 x_c + x_c x_{\text{se}} - x_1 x_{\text{se}})}\right] \quad (17)$$

$$= j\left[b_{\text{sh}} - \frac{b_c(b_1 + b_c)}{b_{\text{se}} + b_1 + b_c}\right]$$

$$\Delta y_{jo} = j\frac{x_{\text{se}}}{x_1 x_c + x_c x_{\text{se}} - x_1 x_{\text{se}}} = j\frac{b_1 b_c}{b_{\text{se}} + b_1 + b_c} \quad (18)$$

Effect of inserting UPFC between buses i and j, shown by (7)–(9), can be seen as adding admittances in parallel with the original admittances of the π-equivalent circuit of the transmission line (line without the UPFC). The effect of inserting UPFC on original admittances is illustrated by Fig. 2.
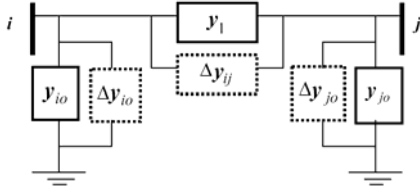

Fig. 2 Effect of inserting UPFC on original line admittances.

Let P^0, B^0 and θ^0 refer, respectively, to vector of net power injections, susceptance matrix and vector of bus voltage angles of the base case (case 0, i.e., before inserting UPFCs), and P, B and θ refer, respectively, to vector of net power injections, susceptance matrix and vector of bus voltage angles of the modified case (after inserting UPFCs). As the shunt reactances and susceptances do not contribute to real power flows we can express real power flow equations of the power system before inserting UPFCs by the following dc load flow equation:

$$B^0\theta^0 = P^0 \quad (19)$$

where the ijth and iith entries of $B0$ matrix are given by:

$$b_{ij}^0 = b_{ji}^0 = \text{imag}\{-y_1\} = 1/x_1; \quad i, j = 1, 2, \dots, \text{NB};$$
$$i \neq j \quad (20)$$
$$b_{ii}^0 = \sum_{j=1, j\neq i}^{\text{NB}} b_{ij}^0; \quad i = 1, 2, \dots, \text{NB}$$

where NB refers to the number of buses. After inserting UPFCs, real power flow equations are given by the following modified dc load flow equation:

$$B\theta = P \quad (21)$$

The matrix B can be seen as a modification to the base case matrix B^0 as follows:

$$B = B^0 + \Delta B \quad (22)$$

where the ijth and iith entries of matrices B and ΔB are given by:

$$\Delta b_{ij} = \Delta b_{ji} = \text{imag}\{-\Delta y_{ij}\} = \frac{x_c}{x_1 x_c + x_c x_{\text{se}} - x_1 x_{\text{se}}} - \frac{1}{x_1}$$

$$= \frac{b_1^2 + b_1 b_{\text{se}}}{b_{\text{se}} + b_1 + b_c} \quad (23\text{-}a)$$

$$\Delta b_{ii} = \sum_{j=1, j\neq i}^{\text{NB}} \Delta b_{ij} \quad (23\text{-}b)$$

$$b_{ij} = b_{ji} = \text{imag}\{y_{ij}\} = \frac{x_c}{x_1 x_c + x_{\text{se}} x_c - x_{\text{se}} x_1}$$

$$= -\frac{b_{\text{se}} b_1}{b_{\text{se}} + b_1 + b_c} \quad (24)$$

$$b_{ii} = \sum_{j=1, j\neq i}^{\text{NB}} b_{ij} \quad (25)$$

Let $P_{i,ij}^{sh}$ and $P_{i,ij}^{se}$ refer, respectively, to the real power drawn at bus i due to shunt and series sources of the UPFC in the line

i-j. The UPFC-embedded line shown in Fig. 1 can be modeled by the equivalent real power injection _-model shown in Fig. 3, which fits the traditional load flow equations. For the equivalent model shown in Fig. 3, the susceptances b_{io} and b_{jo}, the real powers drawn (negative injections) $P_{i,ij}^{sh}$ and $P_{i,ij}^{se}$ at bus i and $P_{i,ij}^{se}$ at bus j, and the real power flows, under the assumption of flat profile of voltages, are given by [3]:
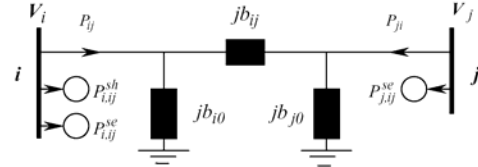

Fig. 3 The π-model of the UPFC-embedded line.

$$b_{io} = \text{imag}(y_{io}^o + \Delta y_{io}^o) = -\frac{1}{x_{\text{sh}}} + \frac{x_1}{x_1 x_c + x_{\text{se}} x_c - x_{\text{se}} x_1} \quad (26)$$

$$b_{jo} = \text{imag}(y_{jo}^o + \Delta y_{jo}^o) = \frac{1}{x_c} + \frac{x_{\text{se}}}{x_1 x_c + x_{\text{se}} x_c - x_{\text{se}} x_1} \quad (27)$$

$$P_{i,ij}^{\text{se}} = \left\{\frac{x_1 - x_c}{x_1 x_c + x_{\text{se}} x_c - x_{\text{se}} x_1}\right\} U_{ij}\sin(\theta_i - \delta_{ij}) \quad (28)$$

$$P_{i,ij}^{\text{sh}} = \frac{1}{x_{\text{sh}}} E_{ij}\sin(\theta_i - \beta_{ij}) \quad (29)$$

$$P_{j,ij}^{\text{se}} = \left\{\frac{x_c}{x_1 x_c + x_{\text{se}} x_c - x_{\text{se}} x_1}\right\} U_{ij}\sin(\theta_j - \delta_{ij}) \quad (30)$$

$$P_{ij} = -P_{ji} = -b_{ij}(\theta_i - \theta_j) \quad (31)$$

If $\Re_i/\Re_j$ refers to the set of buses that are connected to bus i/j, the load flow balance equations at buses i and j can be expressed as follows:

$$P_i^G - P_i^D = P_i^{\text{se}} + P_i^{\text{sh}} - \sum_{j\in\Re_i} b_{ij}(\theta_i - \theta_j); \quad i = 1, 2, \dots, \text{NB} \quad (32)$$

$$P_j^G - P_j^D = P_j^{\text{se}} - \sum_{i\in\Re_j} b_{ji}(\theta_j - \theta_i); \quad j = 1, 2, \dots, \text{NB} \quad (33)$$

where

$$P_i^{\text{se}} = \sum_j P_{i,ij}^{\text{se}}, \quad P_i^{\text{sh}} = \sum_j P_{i,ij}^{\text{sh}}, \quad P_j^{\text{se}} = \sum_i P_{j,ij}^{\text{se}} \quad (34)$$

The active power needed by the series converter is provided from the ac power system by the shunt converter through the dc link (power exchanged between converters). The active power delivered to the shunt converter ($P_{ij}^{ex,sh}$) must satisfy the active power needed by the series converter ($P_{ij}^{ex,se}$) [3,4,6,9,10], therefore, this constraint should be enforced in power flow calculations i.e:

$$P_{ij}^{\text{ex,sh}} - P_{ij}^{\text{ex,se}} = 0 \quad (35)$$

The real powers developed by (exchanged between) the series and shunt voltage sources ($P_{ij}^{ex,sh}$ and $P_{ij}^{ex,se}$) are calculated as [3]:

$$P_{ij}^{\text{ex,sh}} = \frac{E_{ij}}{x_{\text{sh}}}\sin(\theta_i - \beta_{ij}) \quad (36)$$

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$$P_{ij}^{\text{ex,se}} = -\frac{x_c - x_1}{x_1 x_{se} - x_{se}x_c - x_1 x_c} U_{ij} \sin(\delta_{ij} - \theta_i)$$
$$+ \frac{x_c}{x_1 x_{se} - x_{se}x_c - x_1 x_c} U_{ij} \sin(\delta_{ij} - \theta_j) \tag{37}$$

3. DERIVATIONS OF DISTRIBUTIONS FACTORS CONSIDERING IMPACT OF UPFC

In the following subsections let the superscript (o) refers to system variables, matrices and vectors when the UPFC is not included in the system (base case values) and let the absence of the superscript in these system variables, matrices and vectors refers to the situation when the UPFC is included.

3.1. Review of dc distribution factors

Distribution factors are factors calculated based on linear (dc) load flow model. They include *generation shift distribution factors* (GSDFs or *A* factors), and *generalized generation distribution factors* (GGDFs or *D* factors), *generalized load distribution factors* (GLDFs or *C* factors). It has been a common practice to use dc distribution factors in security and contingency analyses, where they have been used to approximately determine the impact of generation and load values on transmission line flows. In the recent years, these factors are suggested as a mechanism to allocate transmission usage payments as these factors can be efficiently utilized to evaluate usage of transmission capacity. To recover the total fixed transmission costs, the distribution factors can be used by transmission providers to allocate transmission payments to different users. By using these factors, the allocation can be attributed to transaction-related net power injections, to generators, or to loads. The dc distribution factors are [10-12,15]:

3.2. Distribution factors considering impact of UPFC

The base case dc load flow equation, without the UPFCs, is given by the sparse linear equation mentioned in (19). However, when UPFCs are utilized in the system, the equation that represents the system can be expressed as follows:

$$B\theta = P = P^o + \Delta P \tag{38}$$

where

$$\Delta P = \Delta P^G + \Delta P^C \tag{39}$$

where the NB×1 vectors P^0, ΔP, ΔP^G and ΔP^C, respectively, represent vector of the net power injections of the base case (when UPFCs are not included in the system), change in net power injection, change in net power injection due to change in generation, and change in net power injection due to injections of UPFCs. The *i*th element for each of these vectors is given by:

$$\Delta P_i^C = -(P_i^{se} + P_i^{sh}) = -\sum_{j \in \mathfrak{R}_i}(P_{i,ij}^{se} + P_{i,ij}^{sh}); \tag{40}$$
$$i = 1, 2, \ldots, \text{NB}$$

$$\Delta P_i^G = P_{G_i} - P_{G_i}^o; \qquad i = 1, 2, \ldots, \text{NB} \tag{41}$$

$$P_i^o = P_{G_i}^o - P_{D_i}^o; \qquad i = 1, 2, \ldots, \text{NB} \tag{42}$$

After inserting UPFCs in the system, the correction to the base case dc load flow Jacobean matrix is given by:

$$B = B^o + \Delta B \tag{43}$$

where ΔB is the susceptance matrix that reflects impact of inserting the UPFCs. When a UPFC is inserted in the line *k* connecting buses *r* and *s*, near bus *r*, we can express ΔB as:

$$\Delta B = U_k R_k^{\mathsf{T}} \tag{44}$$

Vector U_k has zero entries except two entries; it has 1 in the

*r*th position and .1 in the *s*th position, with *r* and *s* representing buses at the terminals of the line in which the UPFC is inserted, and vector R_k also has zero entries except two non-zero entries in the *r*th and *s*th positions. The non-zero elements of U_k and R_k are given by:

$$U_k(r) = 1, \qquad U_k(s) = -1 \tag{45}$$
$$R_k(r) = -R_k(s) = \text{imag}\{\Delta y_{ij}\}$$
$$= \frac{X_c}{X_1 X_c - X_1 X_{se} + X_{se} X_c} - \frac{1}{X_1} \tag{46}$$

Therefore, B in (43) can be expresses as:

$$B = B^o + U_k R_k^{\mathsf{T}} \tag{47}$$

For the general case, when the system has *q* of UPFCs, we can express ΔB and B matrices as:

$$\Delta B = UR^{\mathsf{T}} \tag{48}$$
$$B = B^o + UR^{\mathsf{T}} \tag{49}$$

The product UR^T in (48) and (49) can be reformulated as follows:

$$UR^{\mathsf{T}} = [U_1 U_2 \ldots U_q][R_1 R_2 \ldots R_q]^{\mathsf{T}} \tag{50}$$

If I is the identity matrix of the appropriate size, then the inverse of the modified Jacobean matrix B in (49), using the matrix inversion lemma [24], can be expressed as:

$$B^{-1} = [B^o + UR^{\mathsf{T}}]^{-1} = B^{o^{-1}} + M = X^o + M \tag{51}$$

where

$$M = -[X^o U(I + R^{\mathsf{T}} X^o U)^{-1} R^{\mathsf{T}} X^o] \tag{52}$$

Using the inverse given by (51) and (52), we can express θ in (38) as:

$$\theta = B^{-1}P = B^{-1}(P^o + \Delta P^G + \Delta P^C)$$
$$= (X^o + M)(P^o + \Delta P^G + \Delta P^C)$$
$$= X^o P^o + X^o(\Delta P^G + \Delta P^C) + M(P^o + \Delta P^G + \Delta P^C)$$
$$= \theta^o + \Delta\theta \tag{53}$$

where

$$\theta^o = X^o P^o \tag{54}$$
$$\Delta\theta = X^o(\Delta P^G + \Delta P^C) + M(P^o + \Delta P^G + \Delta P^C) \tag{55}$$

Eq. (53) can also be expressed as:

$$\theta = X^o P^o + (X^o + M)\Delta P^G + MP^o + (X^o + M)\Delta P^C \tag{56}$$
$$= X^o P^o + B^{-1}\Delta P^G + MP^o + B^{-1}\Delta P^C$$
$$= \theta^o + \Delta\theta^G + \Delta\theta^M + \Delta\theta^C$$

Where

$$\Delta\theta^G = (X^o + M)\Delta P^G \tag{57}$$
$$\Delta\theta^C = (X^o + M)\Delta P^C \tag{58}$$
$$\Delta\theta^M = MP^o \tag{59}$$

Note that we decomposed $\Delta\theta$ (the vector of angle changes) into the following three components: $\Delta\theta^G$, $\Delta\theta^C$ and $\Delta\theta^M$. The vector $\Delta\theta^G$ represents the change in bus voltage angles due to the change in generation vector, while the vector $\Delta\theta^C$ represents the change in bus voltage angles due to the net injections of UPFCs and $\Delta\theta^M$ represents the change in bus voltage angles due to the modification of the admittance matrix caused by UPFCs. Let F_{ij}^o and ΔF_{ij} refer, respectively, to the initial real power flow and the change in real power flow in the line *i-j*. Line flows, with UPFCs are utilized, can be expressed as:

$$F_{ij} = H\theta = (H^o + \Delta H)(\theta^o + \Delta\theta) \tag{60}$$
$$= (H^o\theta^o + H\Delta\theta + \Delta H\theta^o)$$

The last equation can be expressed as:

$$F_{ij} = F_{ij}^o + \Delta F_{ij}; \qquad \Delta F_{ij} = H\Delta\theta + \Delta H\theta^o \tag{61}$$

where H is the *NL×NB* line flow matrix whose *k*th row has two nonzero entries in the *i*th position and in the *j*th position and zero entries elsewhere, with *i* and *j* representing buses at the terminals of the *k*th line (from bus *i* to bus *j*). The entries

of this matrix are given by:

$$H_{ki} = -H_{ik} = B_{ij} = B_{ij}^o + \Delta B_{ij} \qquad (62)$$

The changes in line flows given by (61), based on (56)–(59), can be expressed as:

$$\Delta F_{ij} = \Delta H \theta^o + H \Delta \theta = \Delta H \theta^o + H(\Delta \theta^G + \Delta \theta^C + \Delta \theta^M) \quad (63)$$

Or

$$\Delta F_{ij} = \Delta H B^{o^{-1}} P^o + H B^{-1} \Delta P^G + H M P^o + H B^{-1} \Delta P^C \qquad (64)$$

$$= (\Delta H B^{o^{-1}} + H M) P^o + H B^{-1} \Delta P^G + H B^{-1} \Delta P^C$$

To simplify next expressions, let $X^o = B^{o^{-1}}$ and $X = B^{-1}$, then the last equation can be written as:

$$\Delta F_{ij} = (\Delta H X^o + H M) P^o + H X \Delta P^G + H X \Delta P^C \qquad (65)$$

$$= \Delta F_{ij}^B + \Delta F_{ij}^G + \Delta F_{ij}^C$$

The vectors ΔF_{ij}^G, ΔF_{ij}^C and ΔF_{ij}^B in (64) represent, respectively, vector of changes in line flows due to vector of changes in generation, vector of changes in line flows due to net injections of UPFCs, and vector of changes in line flows due to modification of the admittance matrix caused by inserting UPFCs. The vectors of changes in line flows, as given by (64), can be expressed as:

$$\Delta F_{ij}^B = (\Delta H X^o + H M) P^o \qquad (66)$$

$$\Delta F_{ij}^G = H X \Delta P^G \qquad (67)$$

$$\Delta F_{ij}^C = H X \Delta P^C \qquad (68)$$

As we have shown in the modeling of UPFC, inserting UPFCs in the system imposes changes on admittance values. Therefore, A, D and C factors change consequently. If x_{mn} refers to the reactance of the line m-n, then the A factors of the modified case can be expressed as follows:

$$A_{mn,i} = \frac{X_{mi} - X_{ni}}{x_{mn}} = (X_{mi} - X_{ni}) B_{mn} \qquad (69)$$

$$= ([X_{mi}^o + \Delta X_{mi}] - [X_{ni}^o + \Delta X_{ni}])[B_{mn}^o + \Delta B_{mn}]$$

$$= (X_{mi}^o - X_{ni}^o) B_{mn}^o + (X_{mi}^o - X_{ni}^o) \Delta B_{mn}$$

$$+ (\Delta X_{mi} - \Delta X_{ni}) B_{mn} = A_{mn,i}^o + \Delta A_{mn,i}$$

where $\Delta A_{mn,i}$ is given by:

$$\Delta A_{mn,i} = (X_{mi}^o - X_{ni}^o) \Delta B_{mn} + (\Delta X_{mi} - \Delta X_{ni}) B_{mn} \qquad (70)$$

For changes in injections at all system buses, the change in power flow of the line m-n is given by:

$$\Delta F_{mn} = \sum_{b \neq r} A_{mn,i} \Delta P_i \qquad (71)$$

Where

$$\sum_{i \neq r} \Delta P_i + \Delta P_r = 0 \qquad (72)$$

The D factors of the modified case are defined as:

$$F_{mn} = \sum_{i=1}^{N} D_{mn,i} G_i \qquad (73)$$

Where

$$D_{mn,i} = D_{mn,r} + A_{mn,i} \qquad (74)$$

$$D_{mn,r} = \frac{F_{mn} - \sum_{i=1, i \neq r}^{N} A_{mn,i} G_i}{\sum_{i=1}^{N} G_i} \qquad (75)$$

The positive injections due to UPFCs are treated as generations, while negative injections are treated as loads. Therefore, it is important to emphasize that G_i in (72) and (74) refers to total generation at bus i plus the positive injection due to UPFC at bus i, if any. The C factors for the modified case are defined as:

$$F_{mn} = \sum_{j=1}^{N} C_{mn,j} L_j \qquad (76)$$

Where

$$C_{mn,j} = C_{mn,r} - A_{mn,j} \qquad (77)$$

$$C_{mn,r} = \frac{F_{mn}^p + \sum_{j=1, j \neq r}^{N} A_{mn,j} L_j}{\sum_{j=1}^{N} L_j} \qquad (78)$$

Again, note that L_j in (75) and (77) refers to total load at bus j plus the negative injection due to UPFCs at bus j, if any.

4. A FOUR-BUS TEST SYSTEM

Fig. 4 represents a four-bus test system with three generators and two loads. System and UPFC data are given in [3]. In this paper, in order to verify and illustrate the presented derivations we show the results of the optimal power flow (OPF) for this system for two cases, where we use the total generation cost as the objective function of the OPF [3]. The two cases are (1) results of OPF without UPFC (case 0), and (2) results of OPF with UPFC (case 1). The results of this test system, for the two cases, are shown in Tables 1–4. System results are given in p.u. for generator outputs and UPFC injections and in radians for voltage angles.

Table 1 gives outputs of generators for both cases and changes in generations and injections of UPFC. It also shows angles for both cases and their changes in addition to decomposition of angle changes into the three components given in (56)–(59). Note that the change in any variable in Table 1 is calculated as the value of the variable for case 1 minus the value of the variable for case 0.

Table 2 shows line flows of the two cases and gives changes in flows because of adding the UPFC. It also shows decomposition of line flow changes into the three components given in (65)–(67). Again, the change in a line flow in Table 2 is calculated as the value of the line flow for case 1 minus the value of the line flow for case 0.

The GSDFs (A factors) and the GGDFs (D factors) for both cases are given respectively in Tables 3 and 4. These factors can be used to allocate contribution of generators and UPFC to line usage and consequently can be utilized to calculate system usage charges or credits. Transmission usage of both cases using D factors is depicted in Tables 5 and 6. The contribution of generator at bus i to the line flow in the line m-n is calculated as $F_{mn,Gi}^0 = D_{mn,i}^0 G_i^0$ for the base case (case 0) and as $F_{mn,Gi} = D_{mn,i} G_i$ for the modified case (case 1). Note that in case 1 the UPFC is modeled as a generation (positive injection) of 0.5134 p.u. at bus 3.

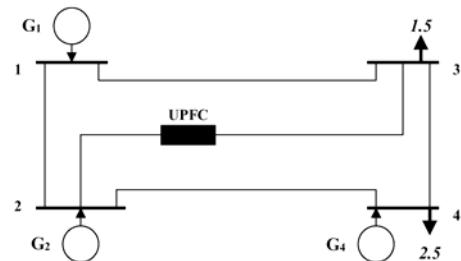
Fig. 4 Four-bus test system.

5. CONCLUSIONS

It is a beneficial interest to utilize UPFCs in power system, as they are capable of altering power system parameters in order to control power flows, improve power transfer capability, stabilize system, and reduce costs. Usage of UPFC becomes of more interest in recently restructured power systems for justifiable technical and economic reasons.

In both regulated and deregulated (restructured) power systems it is usually essential to allocate transmission usage, charges and credits among power system users. Some of the suggested approaches to calculate contributions of system users to line flows are based on dc generation and load distribution factors. The allocation process is of more interest as competitive electricity markets necessitate equitable methods for allocating transmission usage in order to set transmission usage charges and congestion charges in an unbiased and an open-accessed basis.

Table 1
Results of the test system: generations, generation changes, powers of UPFC, bus voltage angles, and changes in angles

Variable	Bus i			
	1	2	3	4
$P_{G_i}^o$	1.5000	1.7500	–	0.7500
P_{G_i}	1.0417	2.4583	–	0.5000
ΔP_i^G	−0.4583	0.7083	–	−0.2500
ΔP_i^C	0.0000	−0.5134	0.5134	0.0000
θ_i^o	0	−0.1000	−0.2000	−0.3500
θ_i	0	−0.0375	−0.1708	−0.3308
$\Delta\theta_i$	0	0.0624	0.0292	0.0191
$\Delta\theta_i^G$	0	0.0631	0.0285	0.0193
$\Delta\theta_i^G$	0	−0.0105	−0.1978	−0.3254
$\Delta\theta_i^M$	0	0.0263	−0.0263	0.0053
$\Delta\theta_i^C$	0	−0.0270	0.0270	−0.0054

Table 2
Results of the test system: flows, changes in line flows, and decomposition of changes in line flows

Line	F_{ij}^o	F_{ij}	ΔF_{ij}	ΔF_{ij}^B	ΔF_{ij}^G	ΔF_{ij}^C
1–2	0.500	0.1878	−0.3122	−0.131479	−0.315777	0.135082
1–3	1.000	0.8538	−0.1462	0.131479	−0.142573	−0.135082
2–3	1.000	0.6663	−0.3337	−0.236662	0.173291	−0.270299
2–4	1.250	1.4664	0.2164	0.105183	0.219282	−0.108065
3–4	0.500	0.5336	0.0336	−0.105187	0.030718	0.108065

Table 3
A factors of the system for both cases

Line	Case 0				Case 1			
	G_1	G_2	G_3	G_4	G_1	G_2	G_3	G_4
1–2	0.000	−0.5862	−0.4138	−0.5172	0.000	−0.6316	−0.3684	−0.5263
1–3	0.000	−0.4138	−0.5862	−0.4828	0.000	−0.3684	−0.6316	−0.4737
2–3	0.000	0.3448	−0.3448	0.0690	0.000	0.2632	−0.2632	0.0526
2–4	0.000	0.0690	−0.0690	−0.5862	0.000	0.1053	−0.1053	−0.5789
3–4	0.000	−0.0690	0.0690	−0.4138	0.000	−0.1053	0.1053	−0.4211

Table 4
D factors of the system for both cases

Line	Case 0				Case 1			
	G_1	G_2	G_3	G_4	G_1	G_2	G_3	G_4
1–2	0.4784	−0.1078	0.0647	−0.0388	0.4858	−0.1458	0.1174	−0.0405
1–3	0.5216	0.1078	−0.0647	0.0388	0.5142	0.1458	−0.1174	0.0405
2–3	0.0862	0.4310	−0.2586	0.1552	0.0284	0.2915	−0.2348	0.0810
2–4	0.3922	0.4612	0.3233	−0.1940	0.3437	0.4490	0.2384	−0.2353
3–4	0.2328	0.1638	0.3017	−0.1810	0.2102	0.1049	0.3155	−0.2108

In this paper, a mathematical approach to allocate the contributions of system users and UPFCs to transmission system usage was presented. The paper used a dc-based load flow modeling of UPFC-inserted transmission lines in which the injection model of the UPFC is used. The relationships presented in the paper showed modified distribution factors that modeled impact of utilizing UPFCs on line flows and system usage. The derived relationships show how bus voltage angles are attributed to each of changes in generation, injections of UPFC, and changes in admittance matrix caused by inserting UPFCs in lines. The relationships derived were applied to two test systems. The results illustrated how transmission usage would be affected when UPFC was utilized. The relationships derived can be adopted for the purpose of allocating usage and payments to users of transmission network and owners of UPFCs used in the network. The relationships can be modified or extended for other control devices.

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