Analysis of the effects of the baseline length accuracy in integer ambiguity resolution for GPS attitude determination system

Geon-Woo Lee*, Seok-Bo Son**, Chansik Park***, Sang-Jeong Lee****

* Department of Electrical Engineering, Chungnam National University, Daejeon, Korea
(Tel : +82-42-825-3991; E-mail: liiiv@cslab.cnu.ac.kr)
**Navicom Co., Ltd, Daejeon, Korea
(Tel : +82-42-868-4533; E-mail: sbson@navicom.co.kr)
***School of Electrical and Computer Engineering, Chungbuk National University, Cheongju, Korea
(Tel : +82-43-261-3259; E-mail: chansp@cbucc.chungbuk.ac.kr)
****Division of Electrical and Computer Engineering , Chungnam National University, Daejeon, Korea
(Tel : +82-42-821-6582; E-mail: eesjl@cslab.cnu.ac.kr)

Abstract: In the GPS attitude determination system, the baseline length constraints can be used efficiently to reduce the search space. It is possible by adopting the assumptions that the baseline length doesn't change and the true baseline length is precisely known. But in real situation, the baseline length might be changed by many reasons and it is impossible to measure the true baseline length because there exists measurement error and antenna phase centre movement. In order to analyze the effect of the baseline length accuracy, one needs to know the relation between the accuracy of the baseline length and success rates of the integer ambiguity resolution. In this paper, the effect of the baseline length accuracy to the integer ambiguity resolution in the attitude determination system is analyzed by empirical method. The results show that the margins in the baseline length accuracy is less than a few cm which implies that one should take great cares when applying the attitude determination system to the flexible structures.

Keywords: GPS, Attitude, Ambiguity resolution, Baseline length accuracy, Success rates.

1. Introduction

When the attitude of vehicle is determined using carrier phase measurements from multiple GPS (Global Positioning System) antennas, the known baseline length can be used for enhancing the success rate in the integer ambiguity resolution. In this approach, the baseline length is not changed and precisely known during the integer ambiguity resolution. However, the baseline length might be changed because there is not only measurement error and antenna phase centre movement but also deformations of vehicles as it moves. And it violates the assumption and causes the decrease of integer ambiguity resolution probability.

Success rate represents the decision probability of true integer ambiguity and is used to validate whether the decided integer ambiguity is true or not [4]. Success rate depends on many factors such as number of satellite, satellite geometry, receiver noises, integer ambiguity resolution method and so on. However in GPS attitude determination system, the baseline length is utilized in integer ambiguity resolution, success rate should be modified to include the baseline length. Success rate in the original positioning application is computed from the probability density function considering the covariance of real number of integer ambiguity. Consequently in such a case, because the baseline length covariance is not considered, it is hard to expect the correct probability which indicates the obtained ambiguity is true. Unfortunately until now, there is no available result for computing the success rate considering the covariance of baseline length in attitude determination.

In this paper, the effect of the baseline length and its accuracy to success rate of integer ambiguity resolution in GPS attitude determination system is analyzed experimentally with a brief review of integer ambiguity resolution procedures. These results can be used as a guideline when GPS attitude determination system is applied to the flexible structures such as airplanes, buildings and bridges.

2. Ambiguity resolution and baseline constraints in GPS attitude determination

Precise relative positioning is possible by using carrier phase observables of GPS. To use carrier phase for positioning, however, initial integer ambiguity included in the carrier phase measurement must be resolved. But, it is very difficult to find the analytic solution because of integer constraints. Thus, search method is used for determining the integer ambiguity.

2.1 Ambiguity resolution in GPS attitude determination system

In general, integer ambiguity resolution procedure can be divided into estimation and validation. In the estimation procedure, real ambiguities and its covariance are decided using code and carrier phase measurements. And then integer ambiguities are searched by the selected estimator such as LAMBDA (Least Squares Ambiguity Data Adjustment) [4, 5] or ARCE (Ambiguity Resolution with Constraint Equation) [2]. In the validation procedure, it is decided that whether the determined integer ambiguities are true or not.

The success probability and speed of fixing true integer ambiguity is depends on many factors such as number of satellite, satellite geometry, baseline length, receiver noise and the integer estimator [1, 2, 3]. Usually the more measurements, the better results are expected. Therefore, the known baseline length information in GPS attitude determination system is extensively utilized to reduce the search space in integer ambiguity resolution.

Fig. 1 shows the integer ambiguity resolution procedure in GPS attitude determination system used in this paper. It is based on ARCE. In the first stage, it computes the code measurements (pseudoranges and delta pseudoranges) and it also computes the carrier phase measurements from Doppler information. The measurements from multiple receivers with multiple channels are prepared in this phase in order to apply single or double difference operations. The cycle slip and other blunders are detected and repaired in this phase [3].

In the second stage, the floating solutions and its covariance matrix are determined using the code and carrier phase measurement and the baseline length. Every integer candidates within the search space are searched and the most probable one is chosen as the possible solution. The measures to choose the most probable one are different in each method [2, 5]. The weighted square sum of the difference between the floating solution and the integer candidates are used as the measure in the ARCE.

In the validation phase, the most probable and the second most probable ones are compared. If the ratio of these two integers is sufficiently large, the most probable one is chosen as the fixed integer. Using the fixed integer, the fixed positions are computed and the fixed attitudes are computed using the fixed positions.

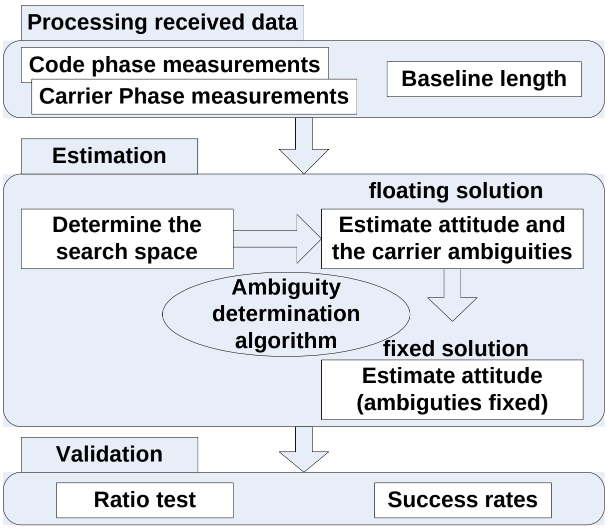

Fig. 1 Integer Ambiguity Resolution Procedure in GPS attitude determination system

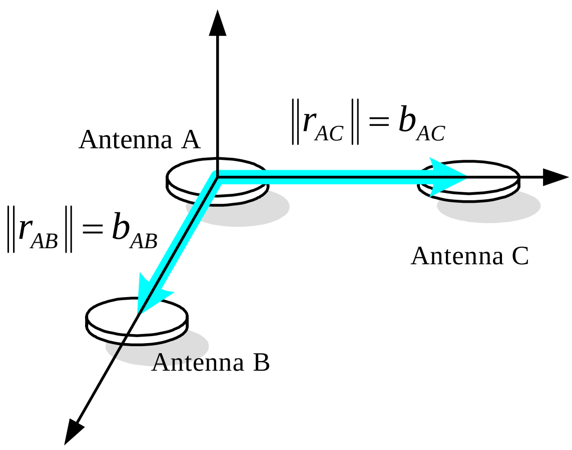

Fig. 2 Configuration of Baseline vectors

2.2 Baseline length constraint

If the known baseline length is used for fixing integer ambiguity, the search space will be reduced from 3-dimension into 2-dimension. That is, the distribution of integer ambiguity candidates will be changed from 3-dimensional elliptical spaces into 2-dimensional planes. The reduction of search space greatly reduces the computational loads and memory requirements. Thus it improves the speed of integer ambiguity resolution and therefore, it is possible to implement the real-time attitude determination system.

$$r_{AB}^T \bullet r_{AB} = b_{AB}^2 \ , \quad r_{AC}^T \bullet r_{AC} = b_{AC}^2 \tag{1}$$

$$r_{AB}^T \bullet r_{AC} = b_{AB} b_{AC} \cos\Theta \tag{2}$$

In ARCE, the search space is 3 dimensional spaces which are constructed by 3 independent integers. The search space are reduced to 2 dimensional space by using equation (1) which implies the lengths of each baseline vectors are b_{AB}, b_{AC}, respectively. Furthermore, by applying equation (2) the search space are further reduced to 1 dimensional space. Equation (2) implied that Θ, the angle between two baselines is known constant.

3. Baseline length accuracy in GPS attitude determination system

To reduce the search space, the constraints represented with the baseline length should be transformed into the other form of constraint represented with the integer ambiguity.

3.1 Search space determination using baseline length constraint

The linearized double difference carrier phase measurement for 3 independent integer ambiguity is given in (3) where l_I is the difference between measured carrier phase and the computed value, H_I is measurement matrix consist of the difference of line of sight vector, r_I is the baseline vector, λ is the wavelength (19.2cm for L1), N_I is the integer ambiguity and w_I is the receiver noise presumed as white Gaussian. Applying weighted least squares; the estimate of the baseline vector is obtained as in equation (4).

$$l_I = H_I r_I + \lambda N_I + w_I \tag{3}$$

$$\hat{r}_I = H_I^{-1}(l_I - \lambda N_I) \tag{4}$$

The baseline length constraint can be expressed as equation (5). This constraint can be transformed into equation (6), using Cholesky decomposition, $(H_I H_I^T)^{-1} = (LL^T)^{-1}$ and low triangle matrix L^{-1} is given in equation (7).

$$b^2 = \hat{r}_I^T \hat{r}_I = (l_I - \lambda N_I)^T H_I^{-T} H_I^{-1} (l_I - \lambda N_I) \tag{5}$$

$$b^2 = [L^{-1}(l_I - \lambda N_I)]^T [L^{-1}(l_I - \lambda N_I)] \tag{6}$$

$$L^{-1} = \begin{bmatrix} \kappa_{11} & 0 & 0 \\ \kappa_{21} & \kappa_{22} & 0 \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{bmatrix} \tag{7}$$

$$L^{-1}(l_I - \lambda N_I) \equiv \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} \kappa_{11}(l_1 - \lambda n_1) \\ \kappa_{21}(l_1 - \lambda n_1) + \kappa_{22}(l_2 - \lambda n_2) \\ \kappa_{31}(l_1 - \lambda n_1) + \kappa_{32}(l_1 - \lambda n_2) + \kappa_{33}(l_1 - \lambda n_3) \end{bmatrix} \quad (8)$$

Using equation (8), the constraint in equation (6) becomes equation (9).

$$\hat{r}_I^T \hat{r}_I = b^2 = k_1^2 + k_2^2 + k_3^2 \quad (9)$$

From equation (9), baseline constraints and receiver noise covariance, following search spaces are determined. All integer candidates in the interval of equation (10) are searched. For an integer candidate n_1, the next search interval in equation (11) is computed. Now, for an integer candidate (n_1, n_2), using equation (12), two candidates of n_3 is computed.

$$-\frac{b}{\lambda\kappa_{11}} + \frac{l_1}{\lambda} \le n_1 \le \frac{b}{\lambda\kappa_{11}} + \frac{l_1}{\lambda} \quad (10)$$

$$-\frac{\sqrt{b^2 - k_1^2}}{\lambda\kappa_{11}} + \xi_1 \le n_2 \le \frac{\sqrt{b^2 - k_1^2}}{\lambda\kappa_{22}} + \xi_1 \quad (11)$$

$$n_3 = \frac{\pm\sqrt{b^2 - k_1^2 - k_2^2}}{\lambda\kappa_{33}} + \xi_2 \quad (12)$$

$$\xi_1 = \frac{\kappa_{21}(l_1 - \lambda n_1) + \kappa_{22} l_2}{\lambda\kappa_{22}} \quad (13)$$

$$\xi_2 = \frac{\kappa_{31}(l_1 - \lambda n_1) + \kappa_{32}(l_2 - \lambda n_2) + \kappa_{33} l_3}{\lambda\kappa_{33}} \quad (14)$$

Using above procedure, 3 dimensional searches reduces to 2 dimensional searches and therefore, the implementation of GPS attitude determination system with off-the-shelf microprocessor is possible. However, it assumes that the baseline length is precisely known and constant. If the baseline length has an error, then the search space in equation (10) and (11) no longer holds and it should be modified. The relation between the known baseline length accuracy and the performance of the integer ambiguity resolution is not known quantitatively. It can be qualitatively explained as in next section.

3.2 The effects of the baseline length accuracy

For the short baseline, it could be assumed that the satellite signals are parallel to each receiver. In the integer ambiguity resolution using double difference measurement, the integer ambiguity to be determined is not the number of cycles between the satellite and receiver, but the number of cycles between the each receiver with a direction to the satellite as in Fig. 3 [3].

With the error in the baseline length, the baseline length constraint is represented as the shadow region not as line. Thus, if the baseline length is changed C into B or D, then the integer ambiguity might be changed N to N1 or N2, respectively as in Fig. 3. The effect of baseline length error is more complex than expressed in Fig. 3 because it is affected by many factors like the number of satellites, geometry of satellites, receiver noise and baseline length and so on. The wrongly determined integer ambiguity gives unpredictable large attitude error especially for the short baseline.

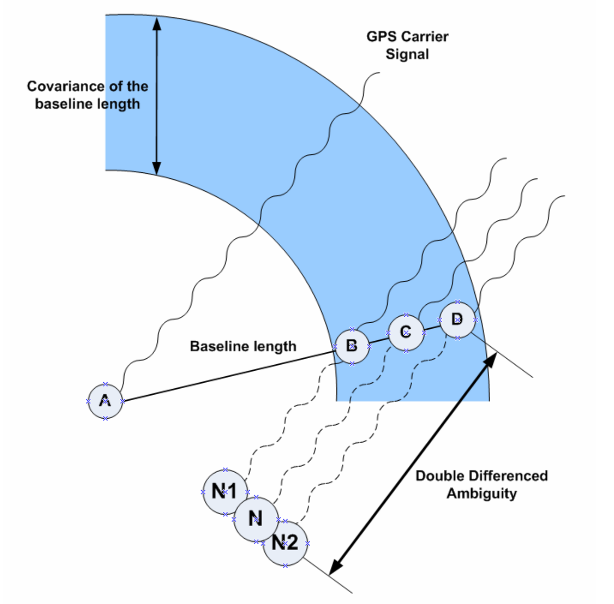

Fig. 3 Effects of the baseline length accuracy

4. Experimental results

The data acquired from the flight test using Bandi airplane is used for analyzing the effect of the baseline length accuracy to the success rate of integer ambiguity resolution. The dedicated GPS receiver with three antennas is installed on the airplane as shown in Fig. 2. The length of each baseline is 70cm. The code and carrier phase measurements from 3 antennas are stored for 1200 seconds flight. This paper just shows about the result of 1st baseline.

Figure 4 shows the satellites constellation during the experiments. Seven satellites are observed during experiment with interruption. The PDOP (Position Dilution of Precision) during the experiment is given in Fig. 5. The average of PDOP is about 2.36.

In order to see the effect of baseline length accuracy to the success rate, intentional error is added to the baseline length. The success rates for 70cm ± 3cm baseline length are computed with 1mm interval. The variation of the experimental success rates of integer ambiguity resolution according to the baseline length error is shown in Fig. 6. The success rate is rapidly reduced when the larger than 1cm errors are occurred. In addition, it can be seen that the success rate has the maximum at 70cm + 5mm. Because the graph is symmetric around 70cm, the assumption of 70cm true baseline length seems correct. It might be suspected that non white Gaussian error properties can cause this result. Multipath is the most suspected one.

The theoretic success rates which is computed by integrating the probability mass do not consider the effect of baseline length and gives smaller values than the experimental values. It implies that the theoretic success rate is not a valid measure in the attitude determination application. Figure 7 shows it. The average of theoretic success rates are about 56.1%. It is below than the experimental success rates. After all, if the theoretic success rates are considered validating the experimental results, it could cause many problems in the attitude determination system. Figure 8 also represents the number of common satellites for the 1st baseline. Two pictures are closely related with each other.

The result clearly shows that the changes in the baseline length affects the search space and reduces the success rate. It requires great care when applying the integer ambiguity resolution method to the applications where larger than 1cm error might be possible. In this case, the better result might be expected without considering the erroneous baseline length information. In the applications such as long winged airplane, bridge and building monitoring, sometimes 1cm error in baseline length is expected even if not common.

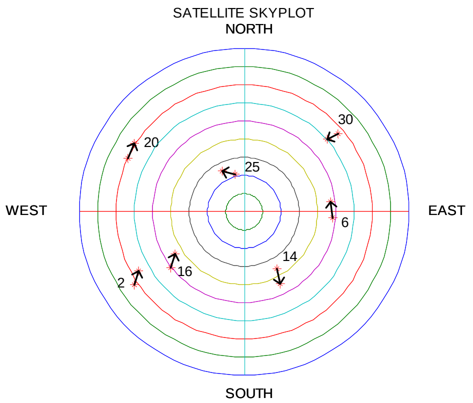

Fig. 4 Satellite constellation during the experiment

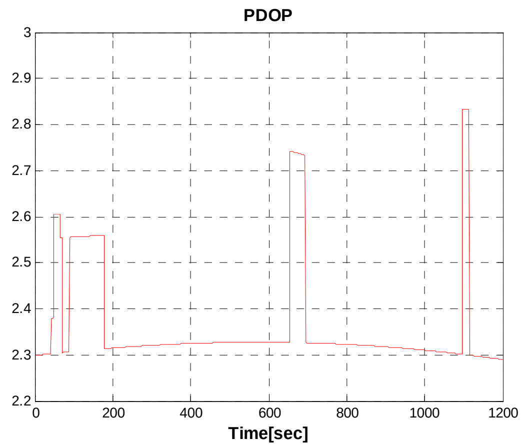

Fig. 5 PDOP during the experiment

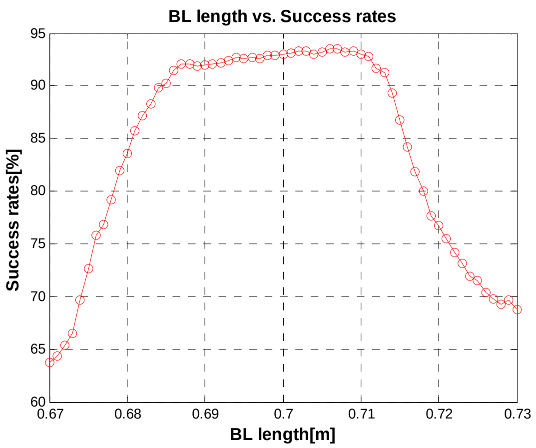

Fig. 6 Effect of the baseline length accuracy in success rates

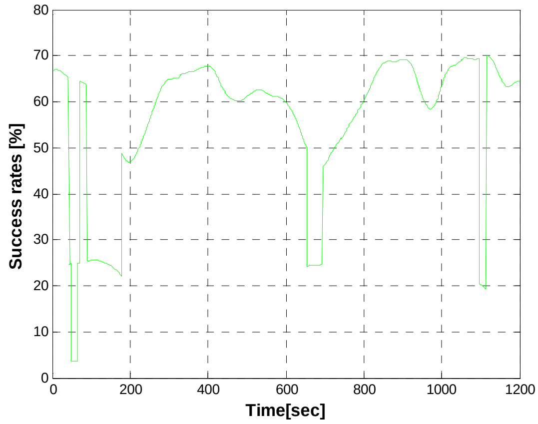

Fig. 7 Theoretic success rates

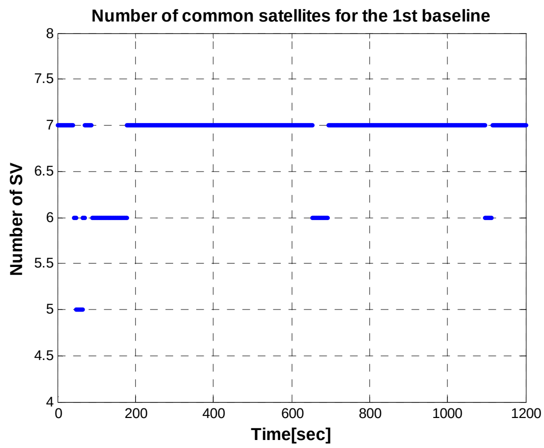

Fig. 8 Number of Satellites

5. Conclusions

In the GPS attitude determination system, the baseline length constraints can be used efficiently to reduce the search space. It is possible to reduce 3 dimensional search to 2 dimensional search by adopting the assumptions that the baseline length doesn’t change and the true baseline length is precisely known. But in real situation, the baseline length might be changed by many reasons and it is impossible to measure the true baseline length because there is the measurement error and antenna phase centre movement. In order to analyze the effect of the baseline length accuracy, one needs to know the relation between the accuracy of the baseline length and success rates of the integer ambiguity resolution.

In this paper, the effect of the baseline length accuracy to the integer ambiguity resolution in the attitude determination system is analyzed by empirical method. The results from real flight test show that the margins in the baseline length accuracy are less than a few cm. It implies that one should take great cares when applying the attitude determination system to the flexible structures such as long winged airplane, high dynamic vehicles, bridge and building monitoring. And one should also take great cares when using the theoretic success rates for validating the experimental attitude results.

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