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# 다중반응표면최적화: 현황평가 및 추후 연구방향
# Multiresponse Optimization: A Literature Review and Research Opportunities

정인준, 김광재
포항공과대학교 기계산업공학부

ABSTRACT

A common problem encountered in product or process design is the selection of optimal parameter levels which involve simultaneous consideration of multiresponse variables. A multiresponse problem is solved through three major stages: data collection, model building, and optimization. To date, various methods have been proposed for the optimization stage, including the desirability function approach and loss function approach. In this paper, we first propose a framework classifying the existing studies and then propose some promising directions for future research.

## 1. INTRODUCTION

Response surface methodology consists of a group of techniques used in empirical study of the relationship between a response and a number of input variables. Consequently, the experimenter attempts to find the optimal setting for the input variables that maximizes (or minimizes) the response (Box and Draper (1987), Khuri and Cornell (1996), Myers and Montgomery (2002)).

Most of the work in response surface methodology has focused on the case where there is only one response of interest. However, a common problem in product or process design is to determine the optimal parameter levels when there are multiple responses which should be considered simultaneously. Such a problem is called a multiresponse problem (Khuri (1996)).

The multiresponse problem consists of three stages: data collection (by the experimental design), model building, and optimization. In the optimization stage, two questions must be addressed: “what-to-optimize” (optimization goal) and “how-to-optimize-it” (solution technique). This paper focuses on the “what-to-optimize” aspect, assuming that the data have been collected and the response models have been fitted reasonably well. A multiresponse optimization problem is formally defined as:

$$\begin{aligned} &\text{Optimize} \quad [\hat{y}_1(\mathbf{x}), \hat{y}_2(\mathbf{x}), \Lambda\ , \hat{y}_k(\mathbf{x})] \\ &s.t. \quad \mathbf{x} \in \mathbf{\Omega}, \end{aligned} \qquad (1)$$

where $\hat{y}_i(\mathbf{x})$ denotes the estimated $i$th response ($i$ = 1, ⋯, $k$), $\mathbf{x}$ is an input variable vector, and $\mathbf{\Omega}$ is the experimental region.

To date, various methods have been proposed for multiresponse optimization, including the desirability function approach and loss function approach. This paper reviews and classifies the existing work and then proposes some promising directions for future research.

Section 2 reviews the existing approaches in multiresponse optimization. In Section 3, a new framework of classification is introduced and the existing work is classified based on the framework. Finally, conclusions and future research directions are made in Section 4.

## 2. EXISTING APPROACHES IN MULTIRESPONSE OPTIMIZATION

The existing studies in multiresponse optimization can be categorized into six major approaches: graphical, priority-based, desirability function, loss function, process capability, and probability-based approach. The last five approaches can be grouped into an analytical approach. They take a common strategy that reduces the multidimensional problem in (1) into a one-dimensional one and then solves it. Each approach is reviewed below.

### 2.1. Graphical approach

The graphical approach superimposes the response contour plots and determines an optimal solution by a visual inspection (Lind *et al.* (1960)). It had been widely used before analytical methods were developed (Hill and Hunter (1966)). This approach has a shortcoming that its usefulness is severely limited by the number of input variables and/or response. Notwithstanding, it has been utilized until recently due to its simplicity and intuitiveness (Gupta *et al.* (2001), Hamed and Sakr (2001), Theppaya and Prasertsan (2004), Huang *et al.* (2004), Huang *et al.* (2005)).

### 2.2. Priority-based approach

The priority-based approach selects the most important response among a number of responses and uses it as the objective function. The other responses are employed as constraints:

$$\begin{aligned} \text{Optimize} \quad & \hat{y}_p(\mathbf{x}) \\ \textit{s.t.} \quad & \hat{y}_s(\mathbf{x}) \in \mathbf{R}_s, \quad s = 1, \mathrm{K}, k \ (s \neq p), \\ & \mathbf{x} \in \mathbf{\Omega}, \end{aligned} \quad (2)$$

where $\hat{y}_p(\mathbf{x})$ and $\hat{y}_s(\mathbf{x})$ denote the estimated primary and secondary response, respectively, and $\mathrm{R}_s$ is a set of requirements for $\hat{y}_s(\mathbf{x})$.

Assuming there are only two responses of interest, Myers and Carter (1973) proposed an optimization formulation that maximizes (or minimizes) the primary response with an equality constraint on the other response. Biles (1975) extended this idea by allowing not only more than two responses, but also inequality constraints on the secondary responses. Del Castillo (1996) proposed an optimization formulation that treats the confidence regions for the stationary points of responses as constraints. More specifically, it first finds a stationary point of each response and then computes the confidence regions for the stationary points of responses. These confidence regions are used as constraints in (2).

The priority-based approach has the advantage of utilizing the existing methods in optimization. However, it does not fulfill the philosophy of the multiresponse problem to simultaneously consider the multiple responses (Kim *et al.* (2002)).

### 2.3. Desirability function approach

The desirability function approach transforms an estimated response (e.g., the *i*th estimated response $\hat{y}_i$) into a scale-free value, called a desirability (denoted as $d_i$ for $\hat{y}_i$). It is a value between 0 and 1, and increases as the corresponding response value becomes more desirable. The overall desirability *D*, another value between 0 and 1, is defined by combining the individual desirability values (i.e., $d_i$'s). Then, the optimal setting is determined by optimizing *D*.

Harrington (1965) first proposed a simple form of a desirability function. Derringer and Suich (1980) extended Harrington's approach by suggesting a more systematic transformation scheme from $\hat{y}_i$ to $d_i$. As an example, in the case of a larger-the-better-type response, the desirability function is given as:

$$d_i = \begin{cases} 0, & \hat{y}_i(\mathbf{x}) \le Y_i^{\min}, \\ \left[ \dfrac{\hat{y}_i(\mathbf{x}) - Y_i^{\min}}{Y_i^{\max} - Y_i^{\min}} \right]^t, & Y_i^{\min} \le \hat{y}_i(\mathbf{x}) \le Y_i^{\max}, \\ 1, & \hat{y}_i(\mathbf{x}) \ge Y_i^{\max}, \end{cases} \quad (3)$$

where $Y_i^{\min}$ is the minimum acceptable value of $\hat{y}_i$, $Y_i^{\max}$ is the value of $\hat{y}_i$ after which the degree of satisfaction does not increase, and $t$ is a parameter determining the desirability function shape. The desirability function proposed by Derringer and Suich contains non-differentiable points as shown in (3). Del Castillo *et al.* (1996) proposed modified desirability functions that are everywhere differentiable so that an efficient gradient-based optimization method, which requires a differentiability assumption, can be used.

The overall desirability can be obtained by aggregating the individual desirability functions using the geometric mean:

$$D = (d_1 \times d_2 \times \Lambda \times d_k)^{1/k}. \quad (4)$$

Later, different forms of aggregation have been proposed. For example, Derringer (1994) proposed the use of a weighted geometric mean. Kim and Lin (2000) suggested maximizing the lowest $d_i$, which is equivalent to maximizing the overall degree of satisfaction of all the responses.

The past studies in the desirability function approach focused mainly on the location effects of responses. However, as the Taguchi's robust design concept prevails, the recent studies attempt to consider the dispersion effects as well as the location effects (Tong *et al.* (2001), Ribardo and Allen (2003), Wu (2005), Kwon *et al.* (2005), Kim and Lin (2005)).

The major advantages of the desirability function approach are that it can incorporate a decision maker (DM)'s preference very flexibly and is easy to use in practice. However, the acquisition of the DM's preference may be quite difficult because he/she should provide the preference information assumptively on the multiple conflicting responses. To overcome this limitation, Jeong and Kim (2003, 2005) proposed an interactive optimization method to incorporate the DM's preference effectively and efficiently in the desirability function approach. Another disadvantage of this approach is that it typically ignores the correlation structure among responses. Recently, Wu (2005) considered the correlation structure by modeling the correlation coefficients among responses.

### 2.4. Loss function approach

The loss function approach aims to find the optimal parameter setting by minimizing the expected loss function. Pignatiello (1993) first proposed the use of a squared error loss function in multiresponse optimization:

$$L(\mathbf{y}(\mathbf{x})) = (\mathbf{y}(\mathbf{x}) - \boldsymbol{\theta})'\mathbf{C}(\mathbf{y}(\mathbf{x}) - \boldsymbol{\theta}), \quad (5)$$

where $\mathbf{y}(\mathbf{x})$ is a vector of response variables, $\boldsymbol{\theta}$ is the target vector of responses, and $\mathbf{C}$ is the cost matrix representing the relative importance of each response. Then, the expected loss, which is to be minimized, can be derived as:

$$E[L(\mathbf{x})] = (E(\mathbf{y}(\mathbf{x})) - \boldsymbol{\theta})'\mathbf{C}(E(\mathbf{y}(\mathbf{x})) - \boldsymbol{\theta}) + trace[\mathbf{C}\boldsymbol{\Sigma}_{\mathbf{y}}(\mathbf{x})], \tag{6}$$

where $\boldsymbol{\Sigma}_{\mathbf{y}}(\mathbf{x})$ is the variance-covariance matrix of the responses. Tsui (1999) extended the Pignatiello's model, which was developed only for a nominal-the-best-type response, to the cases for larger-the-better and smaller-the-better-type responses.

Vining (1998) proposed a modification to the Pignatiello's model by employing a vector of the estimated responses $\hat{\mathbf{y}}(\mathbf{x})$ in loss function, instead of $\mathbf{y}(\mathbf{x})$. Consequently, the expected loss can be expressed as:

$$E[L(\mathbf{x})] = (E[\hat{\mathbf{y}}(\mathbf{x})] - \boldsymbol{\theta})'\mathbf{C}(E[\hat{\mathbf{y}}(\mathbf{x})] - \boldsymbol{\theta}) + trace[\mathbf{C}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}}(\mathbf{x})], \tag{7}$$

where the $\boldsymbol{\Sigma}_{\hat{\mathbf{y}}}(\mathbf{x})$ is the variance-covariance matrix of the predicted responses. The Vining's approach includes Khuri and Conlon (1981)'s method using the generalized distance concept as a special case.

Ko *et al.* (2005) proposed an improvement over the Pignatiello's and Vining's models. They employ $\hat{\mathbf{y}}_{new}(\mathbf{x})$ in the loss function, as opposed to $\mathbf{y}(\mathbf{x})$ in the Pignatiello's or $\hat{\mathbf{y}}(\mathbf{x})$ in Vining's model. Then, the expected loss is expressed as:

$$E[L] = (E[\hat{\mathbf{y}}(\mathbf{x})] - \boldsymbol{\theta})'\mathbf{C}(E[\hat{\mathbf{y}}(\mathbf{x})] - \boldsymbol{\theta}) + trace[\mathbf{C}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}}(\mathbf{x})] + trace[\mathbf{C}\boldsymbol{\Sigma}_{\mathbf{y}}(\mathbf{x})]. \tag{8}$$

The expected loss in Equation (8) includes both the variance of the responses and the variance of the predicted responses. Thus, the Ko *et al.*'s model is a more comprehensive model and includes both the Pignatiello's and Vining's models as special cases.

The loss function approach originated from the Taguchi's robust design concept and, thus, naturally considers the dispersion effects of responses (i.e., $\boldsymbol{\Sigma}_{\mathbf{y}}(\mathbf{x})$ in (6) and (8)). It also considers the correlation structure among responses. Wu and Chyu (2004) considered both correlation structure and dispersion effects although they used a different model with (5)-(8). Elsayed and Chen (1993), Ribeiro and Elsayed (1995), and Lamghabbar *et al.* (2004) did not consider the correlation structure. Ames *et al.* (1997) did not consider both. The loss function approach is statistically sound, but requires several statistical assumptions as a compensation for it.

## 2.5. Process capability approach

The process capability approach derives a process capability index using the estimated mean and standard deviation of a response. The overall capability index is obtained by combining the individual process capability indices. Then, the optimal setting is determined by maximizing the overall capability index.

Barton and Tsui (1991) proposed a performance centering as a process capability index:

$$PC_i = \min\left\{\frac{\hat{\mu}_i(\mathbf{x}) - Y_i^{\min}}{\hat{\sigma}_i(\mathbf{x})}, \frac{Y_i^{\max} - \hat{\mu}_i(\mathbf{x})}{\hat{\sigma}_i(\mathbf{x})}\right\}, \tag{9}$$

where $PC_i$, $\hat{\mu}_i(\mathbf{x})$, and $\hat{\sigma}_i(\mathbf{x})$ are the performance centering measure, the estimated mean, and the estimated standard deviation of the $i$th response variable, respectively. Then, they suggested maximizing the minimum of $PC_i$'s. Plante (1999) extended the Barton and Tsui's approach by developing several multicriteria models based on the performance centering. Plante (2001) proposed the use of two typical process capability indices, *Cpk* and *Cpm*:

$$Cpk_i = \min\left\{\frac{\hat{\mu}_i(\mathbf{x}) - Y_i^{\min}}{3\hat{\sigma}_i(\mathbf{x})}, \frac{Y_i^{\max} - \hat{\mu}_i(\mathbf{x})}{3\hat{\sigma}_i(\mathbf{x})}\right\}, \tag{10}$$

$$Cpm_i = \frac{Y_i^{\max} - Y_i^{\min}}{6\sqrt{\hat{\sigma}_i(\mathbf{x}) + (\hat{\mu}_i(\mathbf{x}) - \theta_i)^2}}, \tag{11}$$

where $\theta_i$ is the target of the $i$th response variable. As shown in (9) and (10), $PC_i$ and $Cpk_i$ are fundamentally the same index. Then, he suggested maximizing the (weighted) geometric mean of $Cpk_i$'s (or $Cpm_i$'s). Ch'ng (2005) proposed to maximize the weighted sum of $Cpm_i$'s. Köksalan and Plante (2003) proposed an interactive optimization method to incorporate the DM's preference in the process capability approach.

The process capability approach has the advantages that its indices, *Cpk* and *Cpm*, are familiar to quality practitioners and it considers the dispersion effects of responses because the indices involve the variance term (i.e., $\hat{\sigma}_i(\mathbf{x})$ in (9)-(11)). However, it does not consider the correlation structure of responses.

## 2.6. Probability-based approach

The probability-based approach assumes a multivariate probability distribution of a multivariate response $\mathbf{Y}$. It first models the distributional parameters in terms of input variables and then finds the optimal setting which maximizes the probability that all responses simultaneously meet their specifications.

Chiao and Hamada (2001) assumed the multivariate normal distribution with mean $\boldsymbol{\mu} = (\mu_1, \mu_2, \cdots, \mu_k)'$ and variance-covariance matrix $\boldsymbol{\Sigma}$ (the diagonal elements of which are the variances $\sigma_1^2, \sigma_2^2, \cdots, \sigma_k^2$ and the off-diagonal elements of which are the covariances $\rho_{ij}\sigma_i\sigma_j$ ($1 \le i < j \le k$), where $\rho_{ij}$ is the correlation between the $i$th and $j$th responses). The joint probability density function, $f(\mathbf{Y};\ \boldsymbol{\mu}, \boldsymbol{\Sigma})$, is given as:

$$f(\mathbf{Y}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{(k/2)}|\boldsymbol{\Sigma}|^{1/2}} e^{-(1/2)(\mathbf{Y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{Y}-\boldsymbol{\mu})'}. \quad (12)$$

The distributional parameters are modeled as a function of $\mathbf{x}$: $\hat{\mu}_i(\mathbf{x})$, $\hat{\sigma}_i(\mathbf{x})$, and $\hat{\rho}_{ij}(\mathbf{x})$. Then, they suggested maximizing the proportion of conformance, $P(\mathbf{Y} \in \mathbf{S} | \mathbf{x})$, where $\mathbf{S}$ is a set of specifications for the responses. Peterson (2004) and Miró-Quesada *et al.* (2004) estimated the distributional parameters in the multivariate $t$ distribution using a Bayesian approach.

The major advantages of the probability-based approach are that it naturally considers the correlation structure by assuming a multivariate probability distribution. However, it requires several statistical assumptions and barely allows the DM's involvement.

### 2.7. Other approaches

First, there are a few studies that are not directly included in but closely related to the desirability function approach. Lai and Chang (2004) and Kumar and Goel (2002) proposed a fuzzy modeling approach that is almost the same with the desirability function approach. Peterson (2000) proposed a combined approach of the desirability function and probability-based approach.

Second, several variants of the loss function approach have appeared in the literature. Riberiro *et al.* (2000), Teeravaraprug and Cho (2002), and Savage and Seshadri (2003) appended the cost-of-loss concept to the loss function approach. Tong and Su (1997), Su and Tong (1997), Antony (2000), Yang and Chou (2005), Wang and Tong (2005), and Liao (2005) considered the quality losses in each experimental run as responses. Chen (1997), Lu and Antony (2002), Maghsoodloo and Chang (2001), Wu (2002), and Tong *et al.* (2004a, 2004b) considered the signal-to-noise rations in each experimental run as responses. Romano *et al.* (2004) proposed a loss function method integrating both robust parameter design and tolerance design. The Logothetis and Haigh (1988), Artiles-Leon (1996), Tong *et al.* (1997), and Jayaram and Ibrahim (1997)'s works are also related to the loss function approach.

As others, Reddy *et al.* (1997) and Xu *et al.* (2004) proposed a goal programming approach and Kumar *et al.* (2000) proposed the use of a utility concept in the Taguchi method.

## 3. CLASSIFICATION OF EXISTING WORK

The existing work in multiresponse optimization, reviewed in the previous section, is classified in this section. The classification is performed in two aspects: statistical properties and the DM's preference. Through the classification, limitations of the existing work and thus insights on the new direction in multiresponse optimization could be identified. It should be noted that the papers in the analytical approach in Section 2 are used in the classification.

### 3.1. Classification based on statistical properties

The classification based on statistical properties is performed via three points: (i) correlation structure among responses, (ii) robustness of response, and (iii) quality of response models.

<u>Correlation structure among responses</u>

The correlation structure means the strength of relationships among responses. The first column of Table 1 shows the results of classification based on the consideration of the correlation structure. All the work in the priority-based and process capability approach does not consider the correlation structure at all. Most of work in the desirability function approach does not consider the correlation structure, but a recent paper attempts to tackle it. Wu (2005) considered the correlation structure as mentioned in Subsection 2.3.

On the other hand, half the work in the loss function approach considers the correlation structure. All the work in the probability-based approach considers the correlation structure. This is because both loss function and probability-based approach, in general, formulate the problem with vectors and matrices and, thus, naturally consider the variance-covariance matrix of responses.

<u>Robustness of response</u>

The robustness refers to the low sensitivity of the response to other factors, that is, the small dispersion effect. Two types of sensitivity have been addressed: robustness to uncontrollable (noise) factors and robustness to parameter fluctuation. The robustness to uncontrollable factors means how large the variance of a response is at specific setting of input variables. On the other hand, the robustness to parameter fluctuation means how large the variance of a response is amplified by the parameter fluctuation of input variables.

The second column of Table 1 shows the results of classification based on the consideration of the robustness of response. All the work in the priority-based approach and the majority in the desirability function and probability-based approach do not consider the robustness. But, several recent papers in the desirability function approach began to consider the robustness, as the Taguchi's robust design concept becomes more prevailing. Tong *et al* (2001), Ribardo and Allen (2003), Wu (2005), and Kim and Lin (2005) considered only the robustness to uncontrollable factors, while Kwon (2005) considered both the robustness to uncontrollable factors and the robustness to parameter fluctuation.

On the other hand, all the work in the process capability approach and the majority in the loss function approach consider the robustness. This is because, as mentioned in Subsections 2.4 and 2.5, the loss function approach originated from the Taguchi's robust design concept and the process capability approach uses the indices involving the estimated standard deviation of a response.

Quality of response models

The quality of response models refers to how reliable the estimated response models are. Two approaches have been proposed in this regard: quality of description and quality of prediction. The quality of description means a measure of how well the estimated response models explain data. The $R^2$, adjusted $R^2$, or mean squared error can be employed as measures of the quality of description. On the other hand, the quality of prediction means how large the variance of a model itself is at specific setting of input variables.

The last column of Table 1 shows the results of classification based on the consideration of the quality of response models. Most of work in all the approaches does not consider the quality of response models. But, two papers each in the desirability function, loss function, and probability-based approach consider the quality of response models. Kim and Lin (2000, 2005) proposed a method to adjust the desirability function shape by incorporating the levels of the quality of description. Vining (1998) and Ko *et al.* (2005) considered the quality of prediction by employing the variance-covariance matrix of the predicted responses (i.e., $\mathbf{\Sigma}_{\hat{\mathbf{y}}}(\mathbf{x})$ in (7) and (8), respectively). Peterson (2004) and Miró-Quesada *et al.* (2004) also employed the variance-covariance matrix of the predicted responses.

Table 1. The results of classification based on statistical properties

| Approach | Existing paper | Correlation | Robustness* | Quality** |
|---|---|---|---|---|
| Priority-based approach | Myers and Carter (1973) | | | |
| | Biles (1975) | | | |
| | Del Castillo (1996) | | | |
| Desirability function approach | Derringer and Suich (1980) | | | |
| | Derringer (1994) | | | |
| | Del Castillo *et al.* (1996) | | | |
| | Kim and Lin (2000) | | | ○′ |
| | Tong *et al.* (2001) | | ○ | |
| | Jeong and Kim (2003) | | | |
| | Ribardo and Allen (2003) | | ○ | |
| | Wu (2005) | ○ | ○ | |
| | Kwon *et al.* (2005) | | ⊙ | |
| | Jeong and Kim (2005) | | | |
| | Kim and Lin (2005) | | ○ | ○′ |
| Loss function approach | Pignatiello (1993) | ○ | ○ | |
| | Elsayed and Chen (1993) | | ○ | |
| | Ribeiro and Elsayed (1995) | | ⊙ | |
| | Ames *et al.* (1997) | | | |
| | Vining (1998) | ○ | | ○ |
| | Tsui (1999) | ○ | ○ | |
| | Lamghabbar *et al.* (2004) | | ○ | |
| | Wu and Chyu (2004) | ○ | ○ | |
| | Ko *et al.* (2005) | ○ | ○ | ○ |
| Process capability approach | Barton and Tsui (1991) | | ○ | |
| | Plante (1999) | | ○ | |
| | Plante (2001) | | ○ | |
| | Köksalan and Plante (2003) | | ○ | |
| | Ch'ng (2005) | | ○ | |
| Probability-based approach | Chiao and Hamada (2001) | ○ | ○ | |

| | | | |
|---|---|---|---|
| | Peterson (2004) | ○ | ○ |
| | Miró-Quesada *et al.* (2004) | ○ | ○ |

* In the Robustness column, the symbol ⦿ represents that the corresponding paper considers both the robustness to uncontrollable factors and the robustness to parameter fluctuation, while the symbol ○ only the robustness to uncontrollable factors.

** In the Quality column, the symbol ○′ represents that the corresponding paper considers only the quality of description, while the symbol ○ only the quality of prediction.

Summary

An overall look on Table 1 indicates that the robustness of response is most considered in the existing work among the three properties, while the quality of response models least considered. The loss function and probability-based approach incorporates the properties more than the others. The ratios at which papers incorporate more than two properties at once are 56% in the loss function approach and 67% in the probability-based approach, respectively. The process capability approach considers only the robustness of response. The desirability function approach also, in general, considers only that criterion. The priority-based approach does not consider all the properties at all. Up to now, the number of papers considering all the properties is nothing but one, that is, Ko *et al.* (2005). Fortunately, however, the recent papers intend to incorporate the properties altogether. In the future, a new work in multiresponse optimization should fulfill the requirement that considers all the properties.

### 3.2. Classification based on the DM's preference

In general, the analytical approach requires the DM's preference on the tradeoffs among multiple responses. The preference is represented through preference parameters. (The shape parameter $t$ in (3) is an example of the preference parameter.) It can be extracted at one of the following timings: before, during, and after solving the problem. The classification based on the DM's preference is performed via two points: (i) type of preference parameter and (ii) timing of extracting the DM's preference.

Type of preference parameter

The preference parameters in multiresponse optimization generally include the target (of a response), specifications (of a response), and relative weights (among responses). The priority-based approach incorporates the target and specifications. The desirability function approach incorporates all these parameters. In addition to those, it includes the shape as another parameter. The loss function approach incorporates the target and relative weights. The process capability approach incorporates all the parameters, but it does not include any other parameter. The probability-based approach incorporates only the specifications.

The desirability function approach has the most preference parameters, while the probability-based approach has the least parameters. This means that the desirability function approach allows the DM to have flexible options to provide his/her preference information in various ways, while the probability-based approach operates with the least options extracting the DM's preference.

Timing of extracting the DM's preference

Existing multiresponse optimization methods can be viewed as the multiobjective optimization (MOO) classification system. Generally, the MOO literature assorts various optimization methods into three categories by the timing of the DM's preference into a model: prior preference articulation, progressive preference articulation, posterior preference articulation methods (Hwang *et al.* (1979)). Prior preference articulation methods require that all the preference of the DM be extracted prior to solving the problem. Progressive preference articulation methods — often referred to as, interactive methods — require that the DM input his/her preference information into a model during the problem solving process. Posterior preference articulation methods do not need any substantial articulation of the DM's preference before or during the problem solving process, but they necessitate it when he/she selects the most satisfactory solution among non-dominated solutions.

Most of work in all the approaches is categorized into prior preference articulation methods in MOO (Park *et al.* (2000), Park and Kim (2005)). But, Jeong and Kim (2003, 2005) and Köksalan and Plante (2003) proposed an interactive method. Although not included in the major approaches, Montgomery and Bettencourt (1977), Mollaghasemi and Evans (1994), and Boyle and Shin (1996) also proposed an interactive method. As posterior preference articulation methods, Ilhan *et al.* (1992), Song *et al.* (1995), Loy *et al.* (2000), and Istadi and Amin (2005) exists.

Prior preference articulation methods have been criticized in that a considerable burden is imposed on the DM in the preference extraction process. Posterior preference articulation methods have the disadvantage that the number of non-dominated solutions generated is often too large and, thus, it is a difficult task to choose the most satisfactory solution. In the case of interactive methods, however, it is easy and effective to extract the DM's preference since he/she has only to provide the information by a local level in an interactive manner. In the future, it is highly demanded to use and develop the interactive method in multiresponse optimization.

## 4. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

The review and the classification of the existing work in multiresponse optimization have been proposed. The review and the classification aim to provide useful information about multiresponse optimization and to show the future directions, respectively. This paper has described the existing work under the category of the graphical, priority-based, desirability function, loss function, process capability, and probability-based approach. Then, it has been classified in two aspects: statistical properties and the DM's preference.

In the future, a new study in multiresponse optimization should be made to consider the three statistical properties: correlation structure of among responses, robustness of response, and quality of response models. Also, it should be developed to extract the DM's preference effectively and efficiently. An interactive method is a very useful alternative with regard to the preference extraction.

The preference extraction issue in multiresponse optimization can be well resolved through a combination of the desirability function approach and the interactive method's concept. As mentioned in Subsection 3.2, the desirability function approach has the most preference parameters such as the target, specifications, shape, and relative weights and, thus, allows the DM to have flexible options to provide his/her preference information through such parameters. Therefore, an interactive method utilizing these parameters integratively as an interaction medium would be the best alternative.

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