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A Simple Beam Model for Thin-Walled Composite Blades with Closed, Two-Cell Sections

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KEY WORDS: Mixed method, Composite blades, Two-cell airfoil section, Closed-form solution

ABSTRACT

A simple beam model based on a mixed method is proposed for the analysis of thin-walled composite blades with a two-cell airfoil section. A semi-complementary energy functional is used to obtain the beam force-displacement relations. The theory accounts for the effects of elastic couplings, shell wall thickness, warping, and warping restraint. All the kinematic relations as well as the cross-section stiffnesses are evaluated in a closed-form through the current beam formulation. The theory has been applied to two-cell composite blades with extension-torsion couplings and fairly good correlation has been observed in comparison with a detailed analysis and other literature.

1. Introduction

In general, the composite rotor blades are built-up structures made of different materials for the skin and spar and are normally of closed single- or multi-celled cross-sections and are thin-walled except near the root where they become thick-walled. In the analysis of composite blades, there is a need to properly model the local behavior of the shell wall as a reaction to the global deformation of the blade.

During last couple of decades, a few selective research activities have been devoted to model and analyze thin-walled composite beams and blades with multi-cell sections. Mansfield [1], Chandra and Chopra [2], Volovoi and Hodges [3], and Jung and Park [4] are the representative ones. Most approaches found in the literature have been formulated through either a displacement [2] or a force method [1]. The former is based on suitable approximations to the displacement field of the shell wall of the section. The assumed displacement field is used to compute the strain energy, and the beam stiffness relations are obtained by introducing relevant energy principles. This method is quite straightforward and easy to apply but there is no systematic method to determine the distribution of warpings. In the force method, also called the flexibility formulation, the assumed direct stress field in the shell wall is used to obtain the distribution of the shear stress and the related warpings from the equilibrium equations of the shell wall. The flexibility method provides a systematic method to determine the warping functions.

Recently, the mixed method that combines both the displacement and force methods by using a variational-asymptotic framework [3] or the Reissner's functional [4] has been developed and proved accurate enough for the analysis of elastically-coupled, thin-walled composite blades. However, in spite of many advantages inherent in

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the mixed approach, the formulation process is rather complex and generally not easy to follow with.

In the present work, a simple and concise approach based on a mixed method is proposed toward modeling and analyzing the thin-walled composite blades with two-cell sections. A closed-form expression is obtained for the cross-section stiffness coefficients as well as the distribution of shear across the section. The theory is validated by comparison of the static response of two-celled composite blades with experimental results found in the literature and also with those of a detailed finite element analysis using the MSC/NASTRAN.

2. Formulation

Fig. 1 shows the geometry and coordinate system of a composite blade with two-cell section. Two different systems of coordinate axes are used: an orthogonal Cartesian coordinate system (x, y, z) for the blade and a curvilinear coordinate system (x, s, n) for the shell wall of the section. The global deformations of the beam are (U, V, W) along the x, y and z axes, and ϕ is the elastic twist about the x-axis. The local shell deformations are (u, v_t, v_n) along the x, s and n directions, respectively. Allowing the transverse shear deformations, the local deformations at an arbitrary point on the shell wall can be expressed as

$$\begin{aligned} u &= u^0 + n\psi_x \\ v_t &= v_t^0 + n\psi_s \\ v_n &= v_n^0 \end{aligned} \tag{1}$$

where the superscript 0 denotes the variable defined at the mid-plane of the shell wall and ψ_x, ψ_s represent rotations about the s- and x- axes, respectively. The shell mid-plane displacements can be obtained in terms of the beam displacements and rotations as:

$$\begin{aligned} v_t^0 &= Vy_{,s} + Wz_{,s} + r\phi \\ v_n^0 &= Vz_{,s} - Wy_{,s} - q\phi \end{aligned} \tag{2}$$

where r and q are the coordinates of an arbitrary point on the shell wall in the (n, s) coordinate system. Assuming small strains, the strain-displacement relation of the shell wall can be obtained as:

$$\begin{aligned} \varepsilon_{xx} &= U_{,x} + z\beta_{y,x} + y\beta_{z,x} - \overline{\omega}\phi_{,xx} \\ \gamma_{xs} &= \gamma_{xy}y_{,s} + \gamma_{xz}z_{,s} = u_{,s}^0 + V_{,x}y_{,s} + W_{,x}z_{,s} + r\phi_{,x} \end{aligned} \tag{3}$$

where γ_{xy} and γ_{xz} represent the transverse shear strains of the blade in the horizontal and vertical directions, respectively, and $\overline{\omega}$ is the sectorial area of the section. In Eq. (3), the cross-section rotations of the blade, β_y and β_z, are defined as:

$$\begin{aligned} \beta_y &= \gamma_{xz} - W_{,x} \\ \beta_z &= \gamma_{xy} - V_{,x} \end{aligned} \tag{4}$$

Assuming the hoop stress flow N_{ss} is negligibly small, the constitutive relations for the shell wall of the section is written as

$$\begin{Bmatrix} N_{xx} \\ N_{xs} \end{Bmatrix} = \begin{bmatrix} A'_{11} & A'_{16} \\ A'_{16} & A'_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xs} \end{Bmatrix} \tag{5}$$

with

$$A'_{11} = A_{11} - \frac{A_{12}^2}{A_{22}}, \quad A'_{16} = A_{16} - \frac{A_{12}A_{26}}{A_{22}}, \quad A'_{66} = A_{66} - \frac{A_{26}^2}{A_{22}}$$

where A_{ij} are the laminate stiffness coefficients for the extension in the classical lamination theory. It is convenient to write Eq. (5) in a semi-inverted form as:

$$\begin{Bmatrix} N_{xx} \\ \gamma_{xs} \end{Bmatrix} = \begin{bmatrix} A_{n\varepsilon} & A_{n\gamma} \\ -A_{n\gamma} & A_{\gamma\gamma} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ N_{xs} \end{Bmatrix} \tag{6}$$

where

$$A_{n\varepsilon} = A'_{11} - \frac{A'^2_{16}}{A'_{66}}, \quad A_{n\gamma} = \frac{A'_{16}}{A'_{66}}, \quad A_{\gamma\gamma} = \frac{1}{A'_{66}}$$

In order to assess the semi-inverted constitutive relations into the beam formulation, a modified form of Reissner's semi-complimentary energy functional Φ_R is introduced:

$$\Phi_R = \frac{1}{2}[N_{xx}\varepsilon_{xx} - \gamma_{xs}N_{xs}] \tag{7}$$

The stiffness matrix relating beam forces with beam displacements is obtained by using the variational statement of the Reissner functional which is given by

$$\delta \int_0^l \oint \left(\Phi_R + \gamma_{xs}N_{xs}\right) ds\, dx = 0 \tag{8}$$

where l is the length of the blade. The parenthesis of Eq. (8) represents the strain energy density of the blade. The unknown shear flow N_{xs} can be determined from the continuity condition of the shell wall which is given as:

$$\oint u_{,s}^0 ds = 0 \tag{9}$$

By using Eqs. (3) and (6), the shear strain is given by the relation,

$$\gamma_{xs} = -A_{n\gamma}\varepsilon_{xx} + A_{\gamma\gamma}N_{xs} = u_{,s}^0 + V_{,x}y_{,s} + W_{,x}z_{,s} + r\phi_{,x} \tag{10}$$

Integrating Eq. (10) from 0 to s and invoking the continuity condition for each wall of the section defined as in Eq. (9), yields the following set of equations:

$$\begin{aligned}(\alpha_1+\alpha_3)n_I-\alpha_3 n_{II} &= 2A_I\phi_{,x}+\int_{C_1+C_3}A_{n\gamma}\varepsilon_{xx}ds\\ -\alpha_3 n_I+(\alpha_2+\alpha_3)n_{II} &= 2A_{II}\phi_{,x}+\int_{C_2+C_3}A_{n\gamma}\varepsilon_{xx}ds\end{aligned} \tag{11}$$

where n_I and n_{II} are unknown shear flows for each cell of the section, A_I and A_{II} are the enclosed areas of each cell, and C_i (i = 1, 2, 3) are the contour lengths of the cell segment (see Fig. 2). The shear flow components corresponding to each of the three curves C_1, C_2, and C_3 are n_I, n_{II}, and n_I-n_{II}, respectively. The α_i (i = 1, 2, 3) appeared in Eq. (11) are defined as

$$\alpha_1=\int_{C_1}A_{\gamma\gamma}ds;\quad \alpha_2=\int_{C_2}A_{\gamma\gamma}ds;\quad \alpha_3=\int_{C_3}A_{\gamma\gamma}ds \tag{12}$$

By substituting the axial strain Eq. (3) into Eq. (11), the unknown shear flows are obtained as

$$\{n\}=[f]\{q_b\} \tag{13}$$

where

$$\begin{aligned}\{n\}&=\lfloor n_I\ \ n_{II}\rfloor^T\\ [f]&=\begin{bmatrix} f_{x1} & f_{y1} & f_{z1} & f_{\phi 1} & f_{\omega 1}\\ f_{x2} & f_{y2} & f_{z2} & f_{\phi 2} & f_{\omega 2}\end{bmatrix}\\ \{q_b\}&=\lfloor U_{,x}\ \ \beta_{y,x}\ \ \beta_{z,x}\ \ \phi_{,x}\ \ \phi_{,xx}\rfloor^T\end{aligned} \tag{14}$$

By inserting Eqs. (3) and (13) into Eq. (8), one can obtain the following set of beam forces-displacements relations,

$$\{F_b\}=\lfloor N\ \ M_y\ \ M_z\ \ T\ \ M_\omega\rfloor^T=[K_{bb}]\{q_b\} \tag{15}$$

where N is the axial force, M_y and M_z are the bending moments about y and z directions, respectively, T is the twisting moment and M_ω is the Vlasov bi-moment. The cross-section stiffness matrix $[K_{bb}]$ relates the cross-section force and moment resultants with beam displacements at an Euler-Bernoulli level of approximation for extension and bending and Vlasov level for torsion.

3. Results and Discussion

Numerical simulations are carried out for coupled composite blades with two-cell airfoil section. The blade is clamped at one end and warping restrained at both ends. The geometry and the material properties of the blade are given in Table 1. Blades with three different ply layups representing extension-torsion couplings are examined. Table 2 shows the details of the layup used in the study.

Fig. 3 shows the comparison results for the tip bending slope and the induced tip twist of the three different blades under a unit tip shear load. The present results are compared with the experimental test data as well as the theoretical results obtained by Chandra and Chopra [2]. It is noted that a displacement-based approach is adopted in Ref. 2 in order to describe the theory. For a comparison purpose, a detailed finite element analysis results obtained using the MSC/NASTRAN is also included in Fig. 3. It is seen that, the predictions of the present method are in good agreement with other results and show better correlation with experimental results than those with Chandra and Chopra [2] in spite of the simple membrane shell model for the present formulation. The structural responses obtained by the present method are within 4.5% of the experimental results.

Fig. 4 presents the tip twist responses of the extension-torsion coupled blades under a unit tip torque load. Good correlation between the present theory and other predictions is also obtained.

4. Conclusion

In the present work, a closed-form analysis for coupled composite blades with multiple cell sections was performed. The analysis model included the effects of elastic couplings, shell wall thickness, torsion warping and constrained warping. The beam force-displacement relations of the blade were obtained by using the Reissner's semi-complementary energy functional. The resulting (5x5) stiffness matrix idealized the blade at an Euler-Bernoulli level of approximation for extension and bending and Vlasov for torsion. The theory was correlated with experimental test data and detailed finite element results for coupled composite blades with a two-cell airfoil section. Good correlation of responses with experimental results was obtained for the cases considered in this study. The error was less than 4.5% for extension-torsion coupled composite blades.

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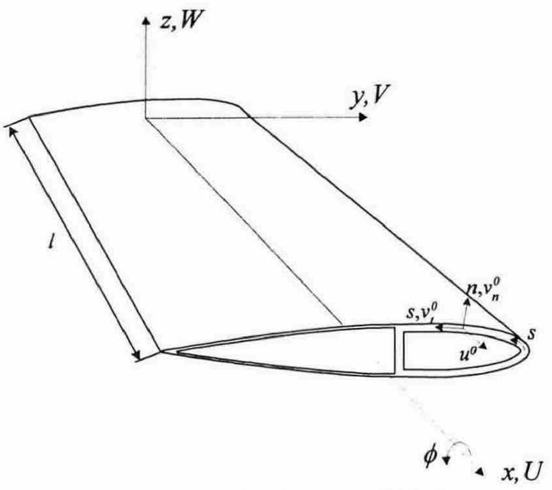

Fig. 1 Schematic of a two-cell blade.

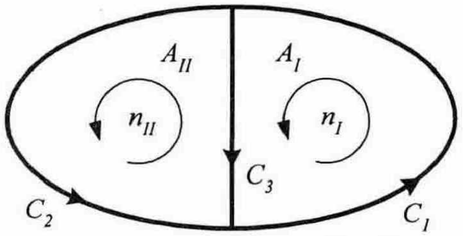

Fig. 2 Definition of a two-cell section.

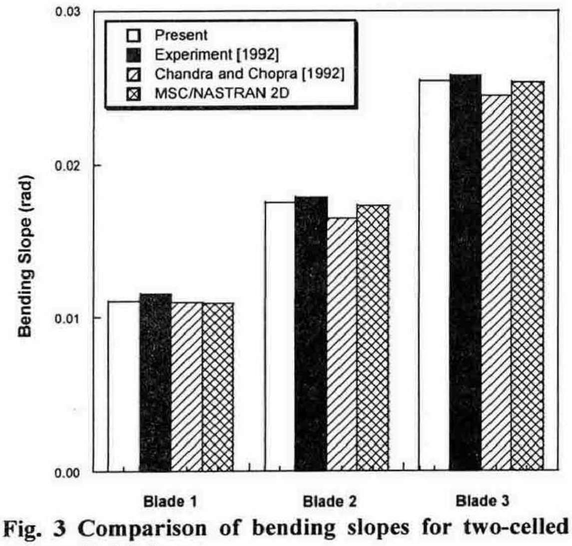

Fig. 3 Comparison of bending slopes for two-celled composite blades.

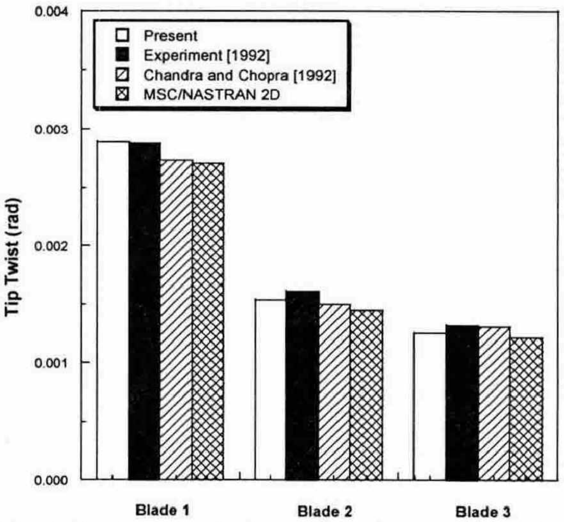

Fig. 4 Comparison of tip twist angles for two-celled composte blades.

Table 1 Geometry and material properties of two-celled composite blades.

Properties	Values
E_{11}	131 GPa (19 x 10^6 psi)
E_{22}	9.3 GPa (1.35 x 10^6 psi)
G_{12}	5.86 GPa (0.85 x 10^6 psi)
ν_{12}	0.40
Ply thickness	0.127 mm (0.005 in)
Airfoil	NACA 0012
Length	641.4 mm (25.25 in)
Chord	76.2 mm (3 in)
Airfoil thickness	9.144 mm (0.36 in)

Table 2 Layup cases of extension-torsion coupled blades.

Cases	Spar	Web	Skin
Blade 1	$[0/15]_2$	$[0/15]_2$	[15/-15]
Blade 2	$[0/30]_2$	$[0/30]_2$	[30/-30]
Blade 3	$[0/45]_2$	$[0/45]_2$	[45/-45]