경사기능 복합재료 판의 기계적 강도해석

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Mechanical strength analysis for functionally graded composite plates

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ABSTRACT

Mechanical strength of functionally graded composite plates that composed of ceramic, functionally graded material and metal layers is investigated using 3-D finite element method. In FGM layer, material properties are assumed to be varied continuously in the thickness direction according to a simple power law distribution in terms of the volume fraction of a ceramic and metal. The 3-D finite element model is adopted by using an 18-node solid element to analyze more accurately the variation of material properties in the thickness direction. Numerical results are compared with those of the previous works. In addition, the displacements, the tensile stresses and the compressive stresses are analyzed for the variation of FGM thickness ratio and volume fraction distribution.

1. Introduction

Functionally graded materials (FGMs) have been designed and developed in many engineering parts that need to be super heat resistant, such as thermal barrier materials for aerospace structural applications and fusion reactors. In FGMs, material properties vary smoothly and continuously from one surface to the other, especially from metal to ceramic. From this smooth and continuous change in composition, FGMs can withstand extremely high temperature environments while maintain their structural integrity.

Jin and Batra [1] studied the effects of loading conditions, specimen size and metal particle size on the crack growth resistance curve and residual strength of a FGM based on the crack-bridging concept. It was found that the FGM exhibited strong R-curve behavior when a crack grew from the ceramic-rich region toward the metal-rich region and the residual strength of the FGM with an edge crack at the ceramic side was notch-insensitive. Tanaka et al. [2] formulated a method of macroscopic material tailoring in order to reduce globally the thermal stresses induced in the FGMs, with the help of the direct sensitivity analysis and the multiobjective optimization technique associated with the heat conduction/thermal stress analysis by means of incremental FGM. Na and Kim [3] analyzed nonlinear bending of functionally graded plates subjected to uniform pressure and thermal loads using 3-D finite element method. Oota et al. [4] applied a genetic algorithm to an optimization problem of minimizing the thermal stress distribution for a plate of step-formed

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FGMs. The step-formed FGM plate was analyzed as a laminated composite plate made of numerous layers with homogeneous and different isotropic material properties. Cho and Choi [5] explored the suitability of the yield-stress-calibrated objective function for maximizing the yield strength of heat-resisting FGMs. They used two-level finite element meshes, coarse mesh for the volume fraction field and fine mesh for the thermoelastic deformation field, in order to resolve the quality-time dilemma effectively.

In this work, the mechanical strength considering tensile and compressive stresses is analyzed for FGM composite plates using 3-D finite element method. An 18-node solid element is selected for more accurate modeling of material properties in the thickness direction. In FGM layer, material properties are assumed to be varied continuously in the thickness direction according to a simple power law distribution. In addition, the effective material properties are obtained according to the linear rule of mixtures. Numerical results are compared with those of the previous works. Furthermore, the displacement, the tensile stress and the compressive stress according to the FGM thickness ratio and volume fraction distribution are analyzed, in detail.

2. Modeling of FGM Composite Plates

A FGM composite plate, composed of ceramic, FGM, and metal layers, of length a, width b, and thickness h is considered. In FGM region, material properties are assumed to be varied in the thickness direction only. The ceramic and metal layers are assumed to be homogeneous and isotropic. The thickness ratios of ceramic, metal, and FGM layers are denoted by r_c, r_m and r_f, respectively, and they are expressed as

$$r_f = h_f / h,\ r_c = h_c / h = r_m = h_m / h = (1 - r_f)/2 \tag{1}$$

where h_c, h_m and h_f indicate the thicknesses of ceramic, metal and FGM layers, respectively. As the FGM thickness ratio r_f tends to 0, FGM composite plates approach the classical ceramic-metal layered composites, while they approach the fully FGM plates as r_f tends to 1. The volume fractions of metal V_m and ceramic V_c are given as follows by applying a simple power law distribution.

$$\begin{aligned} &0 \le z \le z_m && V_m(z) = 1 \\ &z_m \le z \le z_f && V_m(z) = \left(\frac{z_f - z}{z_f - z_m} \right)^n \\ &z_f \le z \le z_c && V_m(z) = 0 \\ &V_c(z) = 1 - V_m(z) \end{aligned} \tag{2}$$

where volume fraction index n indicates the material variation profile through the thickness direction and is a non-negative real number.

According to the linear rule of mixtures, the effective material properties P_{eff} can be obtained as following.

$$P_{eff}(z) = P_m V_m(z) + P_c V_c(z) \tag{3}$$

where P_m and P_c represent the material properties of the metal and ceramic, respectively.

3. 3-D Finite Element Method

A three-dimensional finite element model for thin and thick FGM plates is developed and an 18-node solid element is used to analyze more accurately the variation of material properties in the thickness direction of the system.

Considering a three-dimensional solid body in equilibrium as,

$$\int \delta \mathbf{E}^T \mathbf{S} dV - \delta \mathrm{W} = 0 \tag{4}$$

where $\delta\mathbf{E}$, $\mathbf{S}$, $\delta\mathrm{W}$ and V indicate the virtual strain vector expressed in terms of the displacement vector $\mathbf{u}$, the 2nd Piola-Kirchhoff stress vector, the external virtual work and the volume of the undeformed configuration, respectively. The stress vector $\mathbf{S}$ is related to the strain vector $\mathbf{E}$ through the following equations.

$$\mathbf{S} = \mathbf{CE} \tag{5}$$

where $\mathbf{C}$ is the 6×6 elastic matrix of material stiffnesses, defined in the local coordinate system. For the finite element discretization, the displacement vector $\mathbf{u}$ can be given as

$$\mathbf{u} = [u \quad v \quad w]^T = \mathbf{N}\mathbf{q}_e \tag{6}$$

where u, v and w denote the displacements in x-, y- and z-directions. Furthermore, $\mathbf{N}$ and $\mathbf{q}_e$ are the shape function matrix and the element nodal displacement vector, respectively. The strain vector $\mathbf{E}$ and the virtual strain vector $\delta\mathbf{E}$ can be written as

$$\mathbf{E} = \mathbf{B}\mathbf{q}_e,\quad \delta\mathbf{E} = \mathbf{B}\delta\mathbf{q}_e \tag{7}$$

where $\mathbf{B}$ is a matrix of derivatives of the shape functions. The external virtual work $\delta\mathrm{W}$ is related to the element nodal load vector $\mathbf{Q}_e$ as following.

$$\delta\mathrm{W} = \delta\mathbf{q}_e^{\ T}\mathbf{Q}_e \tag{8}$$

By substituting Eqs. (5-8) into Eq. (4), the following equilibrium equation can be obtained.

$$\sum_e \delta\mathbf{q}_e^{\ T}\left[\mathbf{K}_e\mathbf{q}_e - \mathbf{Q}_e\right] = 0 \tag{9}$$

where the element stiffness matrix $\mathbf{K}_e$ is expressed as

$$\mathbf{K}_e = \mathbf{B}^T\mathbf{C}\mathbf{B} \tag{10}$$

After assembling over all elements, Eq. (9) becomes

$$\mathbf{Kq} - \mathbf{Q} = 0 \qquad (11)$$

where **K**, **q**, and **Q** denote the global stiffness matrix, the global nodal displacement vector, and the global nodal load vector, respectively. Eq. (11) can be solved for **q**. The stress vector can be obtained by substituting **q** into Eqs. (5) and (7).

4. Numerical Results and Discussions

In order to verify the performance of present code, numerical results are compared with those of the previous works for the case of a clamped isotropic square plate under uniform pressure. Further, the mechanical strength of clamped square FGM composite plates under mechanical load is investigated. Silicon nitride (Si_3N_4) and stainless steel (SUS304) are chosen to be the constituent materials of the FGM composite plates.

In numerical results, the following dimensionless values are applied.

$$\bar{x} = x/a, \quad \bar{y} = y/b, \quad \bar{z} = z/h, \quad \bar{w} = -w/h$$
$$\bar{\sigma}_x = (\sigma_{xx}/E_m)(a/h)^2 \qquad (12)$$

where E_m represents Young's modulus of metal.

4.1 Isotropic plates

To check the validity of the present result, the maximum displacement and stress of a clamped isotropic square plate under uniform pressure are analyzed. The uniform pressure q_1 applied on the top surface of the plate is expressed as

$$q_1 = -\bar{q}\, E(h/a)^4 \qquad (13)$$

The numerical results are compared with analytical solutions [6]. Table 1 presents the maximum displacement $(\bar{w})_{max}$ and the maximum stress $(\bar{\sigma}_x)_{max}$ under uniform pressure. This shows good agreement between the present work and the previous result.

Table 1. Dimensionless maximum displacement and stress of a clamped isotropic square plate under uniform pressure (ν=0.3, $\bar{q}$=5).

Dimensionless quantities	Source	
	Analytical [6]	Present
$(\bar{w})_{max}\ (\times 10^{-2})$	6.8796	6.8735
$(\bar{\sigma}_x)_{max}$	1.5390	1.4975

4.2 FGM composite plates

In this section, the mechanical strength of fully clamped square Si_3N_4-SUS304 FGM composite plates subjected to mechanical load is investigated. In order to evaluate the mechanical strength, the tensile stress and the compressive stress are investigated, in detail. The sinusoidal load q_2 distributed over the top surface of the plate is given by the expression

$$q_2 = -\bar{q}\, E_m (h/a)^4 \sin \pi\bar{x} \sin \pi\bar{y} \qquad (14)$$

The tensile and compressive strengths of FGM composite plates at each point can be calculated according to the linear rule of mixtures through

$$\sigma_{Bt}(\bar{z}) = \sigma_{Btm} V_m(\bar{z}) + \sigma_{Btc} V_c(\bar{z})$$
$$\sigma_{Bc}(\bar{z}) = \sigma_{Bcm} V_m(\bar{z}) + \sigma_{Bcc} V_c(\bar{z}) \qquad (15)$$

The compressive strength of a ceramic is larger than that of a metal, on the other hand, the tensile strength of a ceramic is smaller than that of a metal. So as to evaluate the mechanical strength, the stress ratio σ^* is introduced by using the tensile stress ratio $\bar{\sigma}_t$ and the compressive stress ratio $\bar{\sigma}_c$, as following [4].

$$\sigma^* = \begin{cases} \bar{\sigma}_t = \sigma_{xx}/\sigma_{Bt} & \sigma_{xx} \geq 0 \\ \bar{\sigma}_c = \sigma_{xx}/\sigma_{Bc} & \sigma_{xx} \leq 0 \end{cases} \qquad (16)$$

In this equation, to avoid failure, the condition $|\sigma^*| < 1$ should be fulfilled and when $|\sigma^*|$ becomes small, the structure gets better mechanical strength.

Fig. 1 presents the maximum displacement $(\bar{w})_{max}$ with variation of FGM thickness ratio and volume fraction index. When the volume fraction index n is increased, the displacement decreases. This is because as the volume fraction index is increased, the content of ceramic increases. When n<1, the displacement increases as the FGM thickness ratio r_f is increased. On the contrary, when $n \geq 1$, it decreases as r_f is increased.

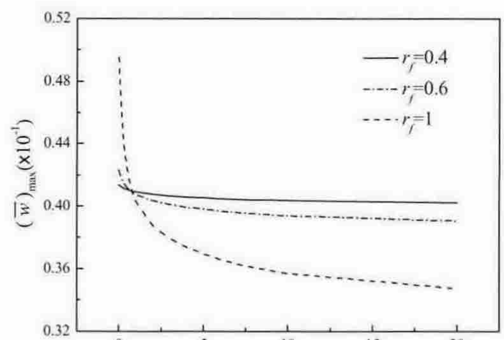

Fig. 1. Maximum displacement of FGM composite plates (a/h=50, $\bar{q}$=5)

The maximum tensile stress $(\bar{\sigma}_t)_{max}$ of FGM composite plates according to FGM thickness ratio and volume fraction index is shown in Fig. 2. In FGM composite plates, as the volume fraction index is increased, the tensile stress decreases. However, in fully FGM plates (r_f=1), it shows a different response. That is, the tensile stress increases generally, as n is increased but when $n \geq 2$, the tensile stress decreases as the FGM thickness ratio r_f is increased. In all cases of n, the tensile stresses have the smallest values when r_f is 1.

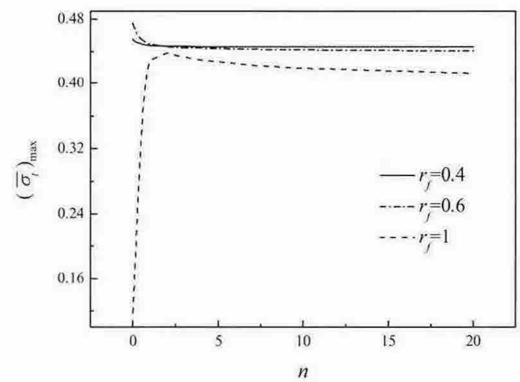

Fig. 2. Maximum tensile stress of FGM composite plates (a/h=50, $\overline{q}$=5).

Fig. 3 illustrates the maximum compressive stress $|\overline{\sigma}_c|_{max}$ with respect to FGM thickness ratio and volume fraction index. The responses are very similar to those of maximum displacement. From Figs. 2-3, in overall cases, the tensile stresses have larger values than the compressive stresses, that is $|\sigma^*|_{max} = (\overline{\sigma}_t)_{max}$. Thus, the tensile stress is the most important factor for the mechanical strength of the FGM composite plates under mechanical load.

5. Conclusions

The mechanical strength considering tensile and compressive stresses are investigated for clamped Si_3N_4–SUS304 FGM composite. The maximum displacement and compressive stress decrease when the volume fraction index is increased. However, the maximum tensile stress decreases as the volume fraction index is increased for the FGM composite plates, but it increases and decreases for the fully FGM plates. In overall cases, the tensile stresses have larger values than the compressive stresses. Thus, the tensile stress is most important value for evaluation of mechanical strength.

As a result, FGM thickness ratio and volume fraction distribution have great effects on the mechanical strength of the FGM composite plates.

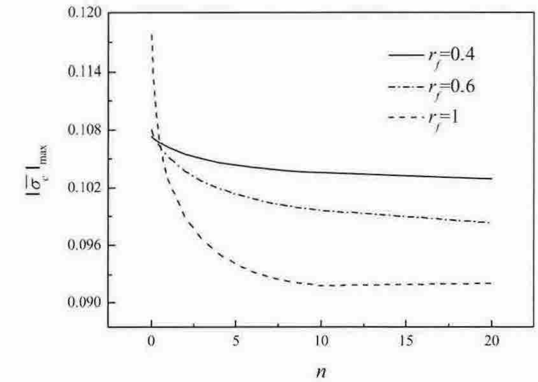

Fig. 3. Maximum compressive stress of FGM composite plates (a/h=50, $\overline{q}$=5).

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