



경성대학교

보증기간이 종료된 이후의 최적의 예방보전정책:  
예방보전 비용이 예방보전 효과의 함수인 경우

2006. 5. 18.

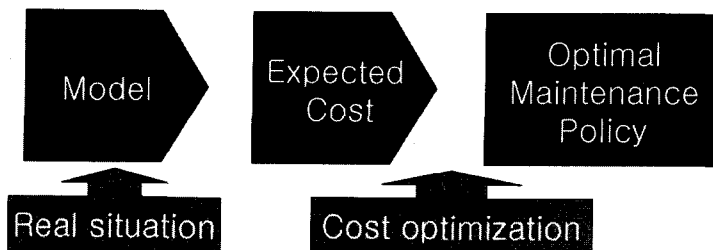
정 기 문

정보통계학과



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Optimal maintenance policy I



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## Optimal maintenance policy II

### ■ Without Warranty

- Barlow and Hunter(1960)
- Canfield(1986)
- Nakagawa(1986)
- Park, jung and Yum(2000)
- Park and Jung(2002)

### ■ Under Warranty

- Sahin and Polatoglu(1996)
- Jung and Park(2003)



*Optimal maintenance policy*  
the expected cost rate per unit time

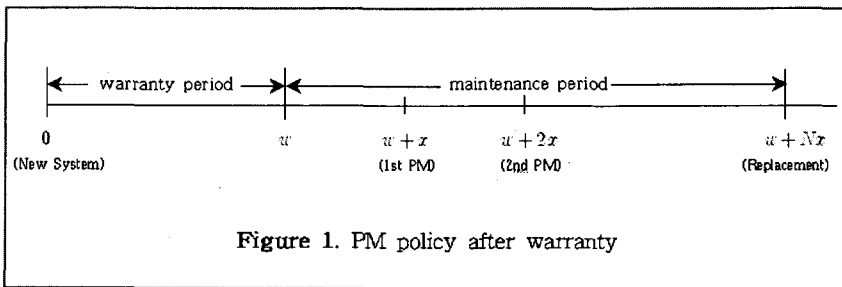


## Model and assumptions I

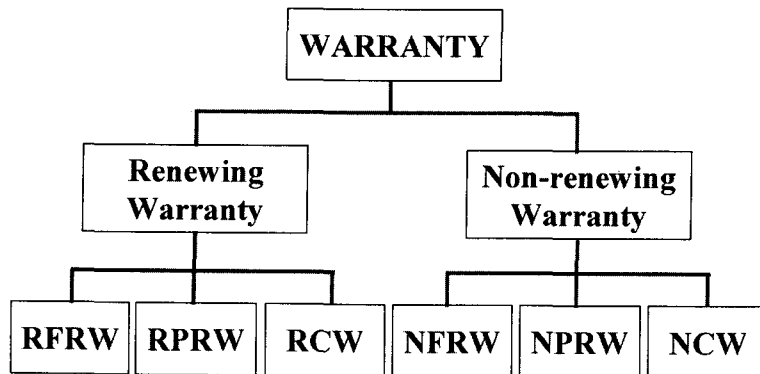
- Manufacturer provides a certain type of *warranty*
- System is maintained preventively at *periodic* times and is *replaced* by a new system at the Nth PM.
- If system fails between PMs, it undergoes only *minimal repair*
- PM slows the rate of system degradation
- *PM cost is an increasing function of the level of PM effect*



### Model and assumptions II



### Warranty Policy





## Expected cost rate per unit time I

### (1) Renewing Warranty

$$C_R(x, N) = \frac{c_1 + \bar{F}(w)(N-1)C_{pm}(x, \tau) + (c_m + c_{fw})\bar{F}(w)c_2}{I(w) + (w + Nx)\bar{F}(w)}$$

$$c_1 = \begin{cases} \frac{c_r}{w} I(w) + c_r \bar{F}(w) + c_{fw} F(w), & \text{under RPRW} \\ c_r \bar{F}(w) + c_{fw} F(w), & \text{under RFRW} \end{cases}$$

$$c_2 = \sum_{i=1}^{N-1} \sum_{r=1}^i \{h((i-1)(x-r) + (x+w)) - h(i(x-r) + w)\}x + \sum_{k=1}^{N-1} \int_{kx+w}^{(k+1)x+w} h(t-k\tau) dt.$$



## Expected cost rate per unit time II

### (2) Non-renewing Warranty

$$C_N(x, N) = \frac{c_3 + (N-1)C_{pm}(x, \tau) + (c_m + c_{fm})c_4}{w + Nx}$$

$$c_3 = \begin{cases} c_r \frac{(w-y)}{w} + c_r + c_{fw}, & \text{under NPRW} \\ c_r + c_{fw}, & \text{under NFRW} \end{cases}$$

$$c_4 = \sum_{i=1}^{N-1} \sum_{r=1}^i \{h((i-1)(x-r) + (x+y)) - h(i(x-r) + y)\}x + \sum_{k=1}^{N-1} \int_{kx+y}^{(k+1)x+y} h(t-k\tau) dt.$$



### Optimal PM policy I

#### (1) Renewing Warranty

##### ■ Step 1

$$(I(w) + w \bar{F}(w))((N-1)C_{pm}(x, \tau) + (c_m + c_{fm})(a_1 + xa_2 + a_3)) + N\bar{F}(w)((N-1)(xC_{pm}(x, \tau) - C_{pm}(x, a)) + (c_m + c_{fm})(x^2 a_2 + xa_3 - a_4)) = Nc_1$$

where

$$a_1 = \sum_{i=1}^{N-1} \sum_{j=1}^k (h((i-1)(x-\tau) + (x+w)) - h(i(x-\tau) + w))$$

$$a_2 = \sum_{i=1}^{N-1} \sum_{j=1}^k (h'((i-1)(x-\tau) + (x+w))i - h'(i(x-\tau) + w)i)$$

$$a_3 = \sum_{k=0}^{N-1} \{((k+1)h((k+1)x + w - kz) - kh(kx + w - kz))\}$$

$$a_4 = \sum_{k=0}^{N-1} \int_{x+w}^{(k+1)x+w} h(t - kz) dt$$



### Optimal PM policy II

##### ■ Step 2 $C_R(x_N, N)$

##### ■ Step 3

$$N^* = \min_N C(x_N, N), \quad N = 1, 2, 3, \dots$$

#### (2) Non-renewing Warranty



### Numerical examples

- Weibull distribution
- $h(t) = \beta\lambda t^{\beta-1}$  for  $\beta > 1$  and  $t \geq 0$
- $\beta = 3$  and  $\lambda = 1$
- $C_{pm}(x, z) = c_0 + c_1(x-z)^{-1}$
- $C_{pm}(x, z) = c_0 + c_1 \exp\{-(x-z)\}$



Table 1. Optimal PM policy:  $C_{pm}(x, z) = c_0 + c_1(x-z)^{-1} = c_0 + c_1(x-\alpha x)^{-1}$   
 ( $\beta = 3, \lambda = 1, c_0 = 1, C_{rev} = 1, C_{re} = 30, \omega = 0.5, \gamma = 0.3, l = 1$ )

Warranty Type	$c_1$	$\alpha$	$x^*$	$N^*$	$C(x^*, N^*)$
RPRW	0	0.1	1.8045308160	1	20,71236288
		0.3	1.8045308160	1	20,71236288
		0.5	0.9699940159	2	20,46671367
		0.7	0.7079218257	3	20,00742389
		0.9	0.4994735261	5	19,27667469
	0.2	0.1	1.8045308160	1	20,71236288
		0.3	1.8045308160	1	20,71236288
		0.5	0.9814147734	2	20,63063121
		0.7	1.0153048120	2	20,43087647
		0.9	1.0792839850	2	20,57370360
BPRW	0	0.1	1.7321387270	1	19,43152886
		0.3	1.7321387270	1	19,43152886
		0.5	0.9320419488	2	19,25572750
		0.7	0.6807384641	3	18,87967420
		0.9	0.5724068304	4	18,27435506
	0.2	0.1	1.7321387270	1	19,43152886
		0.3	1.7321387270	1	19,43152886
		0.5	0.9446982377	2	19,43140935
		0.7	0.9781337259	2	19,26339771
		0.9	1.7321387270	1	19,43152886



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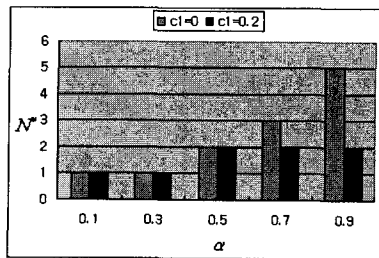
Table 1. Optimal PM policy:  $C_{PM}(x, z) = c_0 + c_1(x-z)^{-1} = c_0 + c_1(x-\alpha x)^{-1}$   
( $\beta=3, \lambda=1, c_0=1, C_{RR}=1, C_{RE}=30, \omega=0.5, \nu=0.3, l=1$ )

Warranty Type	$c_1$	$\alpha$	$x^*$	$N^*$	$C(x^*, N^*)$
NPRW.	0	0.1	2.1370400750	1	23.16274087
		0.3	2.1370400750	1	23.16274087
		0.5	0.8036595870	3	22.36546264
		0.7	0.5496431418	5	21.39174749
		0.9	0.4214412258	8	19.84292981
	0.2	0.1	2.1370400750	1	23.16274087
		0.3	1.1258884520	2	23.05772670
		0.5	1.1596878970	2	22.67593030
		0.7	0.8706459155	3	22.13623628
		0.9	0.9665035065	3	22.10246449
RPRW.	0	0.1	1.8706067040	1	18.37498050
		0.3	0.9839965492	2	18.36151415
		0.5	0.7081280049	3	18.03369622
		0.7	0.5864222829	4	17.44107862
		0.9	0.4670092284	6	16.47684262
	0.2	0.1	1.8706067040	1	18.37498050
		0.3	1.8706067040	1	18.37498050
		0.5	1.0228096920	2	18.20639057
		0.7	1.0607486600	2	17.96434157
		0.9	1.1294370780	2	18.03645453

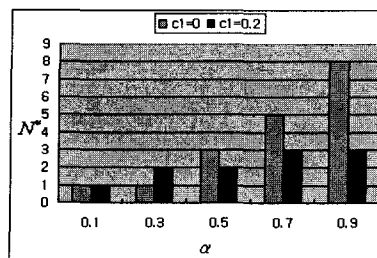
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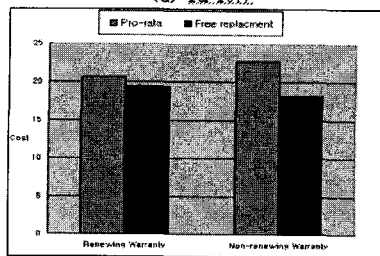
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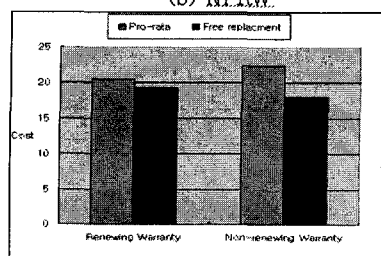
(a) RPRW.



(b) NPRW.



(a)  $C_{PM}(x, z) = c_0 + c_1(x - \alpha x)^{-1}$



(b)  $C_{PM}(x, z) = c_0 + c_1 \exp(-(x - \alpha x))$

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