PDP 열화분석 예제를 통한 랜덤계수모델에서의 고장시간분포 추정

배 석 주　　김 성 준
한양대학교 산업공학과

Failure-Time Estimation from Nonlinear Random-Coefficients Model: PDP Degradation Analysis

Suk Joo Bae　　Seong-Joon Kim
Department of Industrial Engineering
Hanyang University, Seoul, Korea

Abstract

As an alternative to traditional life testing, degradation tests can be effective in assessing product reliability when measurements of degradation leading to failure can be observed. This article proposes a new model to describe the nonlinear degradation paths caused by nano-contamination for plasma display panels (PDPs): a bi-exponential model with random coefficients. A sequential likelihood ratio test was executed to select random effects in the nonlinear model. Analysis results indicate that the reliability estimation can be improved substantially by using the nonlinear random-coefficients model to incorporate both inherent degradation characteristics and contamination effects of impurities for PDP degradation paths.

1. Introduction

Due to extremely high product reliability, failure information is often sparse in the manufacture of display devices, such as plasma display panels (PDPs), light emitting diodes (LEDs), and vacuum fluorescent displays (VFDs). However, covariate information indicating degradation has become increasingly available in all facets of reliability modeling. Not only can degradation measurements lead to improved reliability analysis over standard failure time analysis (Lu and Meeker, 1993), but they provide additional information related to failure mechanisms of test units. Meeker and Escobar (1998) provided a comprehensive guide to degradation analysis for various life tests including accelerated life test (ALT).

The general approach used to derive lifetime distribution from degradation testing is to first estimate the coefficients in the pre-specified degradation model using all of the degradation data, then predict the lifetime distribution for the products based on the estimated model coefficients linked with the failure-times. In some simple cases where the degradation path is linear, we can easily compute the lifetime distribution expressed as a closed-form. However when the degradation path is nonlinear and more than one of the coefficients are characterized as random, a closed form expression will not exist, hence the lifetime distribution must be evaluated numerically using simulation and intensive resampling methods such as bootstrap procedures (Lu and Meeker, 1993)

In this article, we investigate how to accurately estimate failure-time distribution from nonlinear degradation paths caused by nano-contamination for plasma display panels (PDPs), where the coefficients in the degradation model vary significantly between item to item.

The paper is organized as follows. In Section 2 the degradation physics of PDP brightness are illustrated and in Section 3 a nonlinear random-coefficients model is introduced. Estimation is based on approximation methods for maximizing the likelihood function of the degradation data. In Section 4 a failure-time distribution is derived based on the estimated degradation paths. In Section 5 the PDP degradation data are analyzed following the proposed approaches. Some concluding remarks are presented in Section 6.

2. Degradation Physics of the PDP

The plasma display panel (PDP) is a flat, self-emissive panel display with excellent image quality. Due to its wide-viewing angle and large sized screen, the PDP is

becoming a leading display device for both business and consumer display applications. The most common PDP in production today is the two-substrate, three-electrode surface discharge alternating-current (AC) structure, which is shown in Figure 1.

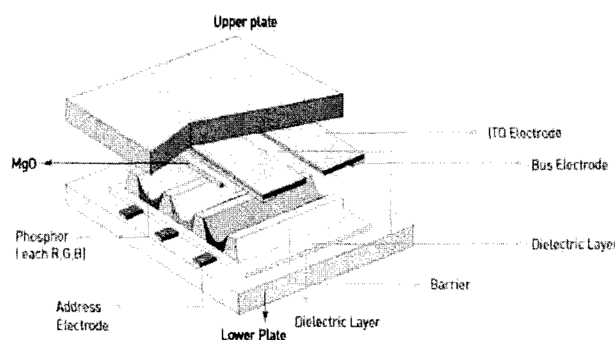

Figure 1 : Basic Structure of an alternating-current(AC) plasma display panel(PDP).

The display characteristics of an AC PDP are defined by the degradation of the phosphors and surface properties of the MgO film. In particular, a PDP''s luminosity is mainly attributed to the phosphors that degrade exponentially over most of the usage period. As a result, the luminosity at time t can be expressed as:

$$\eta(t) = \phi \exp(-\gamma t), \quad t \geq 0,$$

where ϕ is the initial luminosity, and $\gamma(>0)$ is the rate of degradation. Surface propoerties of the MgO film are mainly affected by a discharge aging process. During sustained discharge periods, impure gases (e.g., O_2, CO_2, H_2) produced during the manufacturing process can seep into the Xe gas mixture, increasing the discharge voltage, decreasing the UV intensity, and contaminating the MgO layer. Sometimes, the efficiency of the phosphors can also be reduced by the gas discharge. Manufacturers must exhaust harmful impurities through an "outgassing" process, then execute accelerated stress tests to burn off remaining impurities in order to provide a better displaying image before shipping them to customers. This burn-in procedure (called "aging" in the industry) is considered to be essential in the manufacturing process. In light display testing, burn-in connotes a different process of reliability degradation; infant mortality is not a chief concern in terms of degradation in the luminosity. However, if the burn-in procedure fails to burn off these impure gases completely, impurities remaining inside will poison the phosphors and decrease emission efficiency of the PDP. In particular, the blue-emitting phosphor, europiumactivated barium magnesium aluminate ($BaMgAl_{10}O_{17}$: Eu^{2+}

(BAM)) consisting of nano-particles is highly vulnerable to the harmful impurities produced during the baking and the panel sealing processes which are parts of PDP manufacturing.

After several hundred hours of operation in testing environments, those impurities are burned out at rate λ and the remaining amount of impurities at time t can be expressed as:

$$\vartheta(t) = \rho \exp(-\lambda t),$$

where ρ is the initial amount of impurities remaining after the outgassing process. By taking both inherent degradation characteristics of phosphors and contamination effects of the impurities on the PDP into account, the PDP luminosity at time t can be expressed as the following *bi-exponential model*

$$y(t) = \phi_1 \exp(-\gamma_1 t) + \phi_2 \exp(-\gamma_2 t), \quad t \geq 0 \tag{1}$$

based on the 2nd order reaction process. Some coefficients in nonlinear model (1) vary significantly from unit to unit, hence they should be considered as random. Degradation models that do not consider random effects will compound these effects with error estimates, thereby grossly decreasing the precision of the reliability estimates.

In light display devices, luminosity is the most important performance characteristic and failure is defined by how much the luminosity decreases over time. The variable of interest is the amount of change from an initial luminosity level while industry standards define failure (called *soft failure*) at the time when a device luminosity falls below 50% of its initial luminosity (see Tannas (1985)). In order to optimize testing time by considering the trade-off between the accuracy of PDP reliability and testing costs, advances in accurate modeling of AC PDP performance degradation are required.

3. Nonlinear Ranom-Coefficients Model

Random-coefficients models provide a powerful tool for analyzing repeated measurement data that arise in various fields of application, such as economics and pharmacokinetics (Davidian and Giltinan, 1995). Random-coefficients models are intuitively appealing because they allow for flexible variance-covariance structures of the response vector.

A general nonlinear random-coefficients (NRC) model for the jth response on the ith individual test item can be defined as

$$y_{ij} = f(\beta_i, t_{ij}) + \varepsilon_{ij} \quad i = 1, \ldots, m, \; j = 1, \ldots, n_i \tag{2}$$

where y_{ij} is the jth response on the ith individual, t_{ij} is the covariate vector for the jth measurement time on the ith individual, $f(\cdot)$ is a nonlinear function of t_{ij} and parameter vector β_i and ε_{ij} is a normally distributed random error term.

Modeling the ith individual response is accomplished by letting y_i and ε_i be the $(n_i \times 1)$vectors of responses and random within-individual errors for individual i, respectively. Define the $(n_i \times 1)$ mean response vector $f(\beta_i, t_i) = (f((\beta, b_i), t_{i1}), \ldots, f((\beta, b_i), t_{in_i}))'$ for the ith individual test item, depending on the $(p \times 1)$ individual-specific regression parameter β_i, which is represented with $(p \times 1)$ vector of fixed effects β and $(q \times 1)$ vector of random effects b_i.

Based on the set-up in Davidian and Giltinan (1995), within-individual variation and between individual variation in the NRC model can be written respectively as

$$\mathbf{y}_i = \mathbf{f}(\beta_i, \mathbf{t}_i) + \varepsilon_i \quad \text{and} \quad \beta_i = \mathbf{A}_i \beta + \mathbf{B}_i \mathbf{b}_i \tag{3}$$

where A_i, B_i are known design matrices of size $(p \times p)$ and $(p \times q)$ for the fixed and random effects respectively, which are used to simplify model specification. We assume that ε_i is independently and identically distributed as Normal$(0, \sigma^2 I_{n_i})$ and b_i as Normal$(0, D)$, where D is a positive definite $(q \times q)$ variance-covariance matrix. Here, β_i is specific to the ith test item through b_i, and b_i and ε_i are assumed to be independent for $i = 1, \ldots, m$.

Within the framework of (3), since the random effects are unobserved quantities, maximum likelihood estimation in the NRC model is based on the marginal density of $\mathbf{y}$

$$p(\mathbf{y} \mid \beta, \sigma^2, \mathbf{D}) = \int p(\mathbf{y} \mid \mathbf{b}, \beta, \sigma^2, \mathbf{D}) p(\mathbf{b}) d\mathbf{b} \tag{4}$$

where $p(\mathbf{y} \mid \beta, \sigma^2, \mathbf{D})$ is the conditional density of $\mathbf{y}$ given the random effects b having the marginal distribution $p(b)$.

In general, since this integral does not have a closed-form expression when the model function f is nonlinear in b, approximation methods can be used to estimate the marginal density (4). Bae and Kvam (2004) introduced various approximation methods to numerically optimize the loglikelihood corresponding to (4) and evaluated them in terms of comparison criteria for nonmonotonic degradation paths of vacuum fluorescent displays. In this paper the parameters in the NRC model are estimated using Lindstrom and Bates' (LB) algorithm (1990), which is available in the S-Plus/R nlme function (Pinheiro and Bates, 1995).

4. Failure-Time Distribution

To derive the failure-time distribution and its quantiles, define failure time T as the time that the actual degradation path $\tau(t;\beta,b,\varepsilon)$ reaches the prespecified degradation level τ_f. Then the distribution of the failure time is

$$F_T(t) = P(T \le t) = P[\tau(t;\beta,\mathbf{b},\varepsilon) \le \tau_f] \tag{5}$$

The failure-time distribution depends on the distribution of the random coefficient b which is determined by the variance-covariance matrix D. Let $y(t)$ be the response at time t and denote the true value of $\Theta \equiv (\beta, D, \sigma^2)$ as Θ_0, then the failure time can be expressed as

$$F_T(t;\Theta_0) = P_{\Theta_0}(y(t) \le \tau_f) = \int P(y(t) \le \tau_f \mid \mathbf{b}) p(\mathbf{b}) d\mathbf{b} \tag{6}$$

In practice, the parameters Θ_0 are estimated with ML estimators $\hat{\Theta} \equiv (\hat{\beta}, \hat{D}, \hat{\sigma}^2)$ obtained through approximation methods such as the LB algorithm presented in the preceding section. Then ML estimates of the $F_T(t)(\hat{F}_T)$ and p percentile $t_p(\hat{t}_p)$ can be computed by replacing Θ_0 with their estimates $\hat{\Theta}$. However, if there is no closed-form expression for $\hat{F}_T$ or if the inverse transformation with respect to t is overly complicated as in the degradation model (1), we can choose to evaluate $\hat{F}_T$(or $\hat{t}_p$) using Monte Carlo simulation. For this evaluation, we first use the model parameter estimates $\hat{\beta}$ and $\hat{D}$ (obtained from the m sample paths) to generate the N simulated realizations $\check{\beta}$, $\check{b}$. From N values of $\check{\beta}$ and $\check{b}$, compute the N failure times $\check{t}$ by substituting $\check{\beta}$ and $\check{b}$ into $\tau(t;\beta,b)$, and then solve for τ_f. For any desired values of t, $F_T(t)$ is estimated from the simulated empirical distribution

$$\widehat{F_T(t)} \approx \frac{\mathrm{Number of}(\check{t} \le t)}{N} \tag{7}$$

The procedure for constructing parametric bootstrap confidence intervals is implemented via the procedure in Bae and Kvam (2004).

5. PDP Example

The degradation test for AC PDPs was executed to assess the reliability of AC PDPs at a constant stress level. In this experiment, PDP degradation was accelerated by using a higher than normal frequency level. Six individual PDP degradation paths consist of

measurements of luminosity inspected regularly. As shown in Figure 2, the luminosity of six PDPs decreases rapidly at the initial stage of degradation testing. This phenomenon is largely caused by impurities remaining inside the PDPs after the burn-in process. Bae and Kvam (2006) analyzed PDP data tested at a lower frequency level with a change-point model to estimate the unknown time change-point within the degradation path to provide information related to burn-in in the manufacturing process. However, in order to model the degradation paths more fully, we consider a nonlinear model with random coefficients.

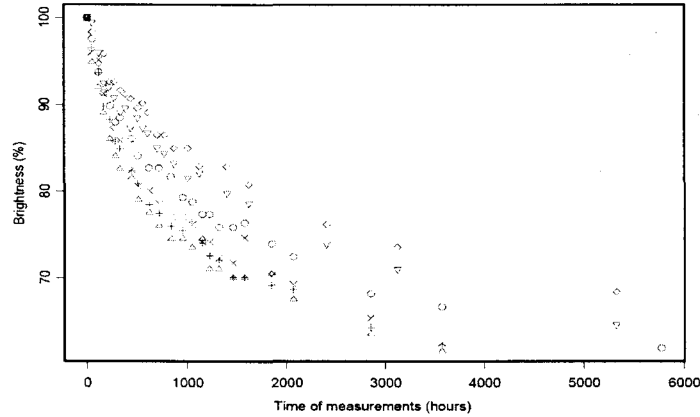

Figure 2 : Degradation paths of sic PDPs.

5.1 PDP Degradation Analysis

The lifetime of PDPs is limited by the degradation of phosphors on the rear plate of the panel. The degradation rate will be constant over time, that is, $dA/dt=-\gamma$, where A is the concentration of phosphors and $\gamma>0$ is the degradation rate constant. Consequently, the amount of degradation in luminosity proportional to phosphors degradation at time t is: $\eta(t)=\phi\exp(-\gamma t)$, where $\phi(>0)$ denotes initial luminosity. We seek to find a model for relative luminosity by dividing each of the luminosity measurements by initial luminosity to easily derive failure-time of the PDPs, where the failure is defined at the time when the relative luminosity

$$y(t)=\eta(t)/\eta(0)=\exp(-\gamma t) \tag{8}$$

falls below 0.5 or $100y(t)$ below 50%. Percentage values for relative luminosity will be used for convenience hereafter. Because the degradation rate varies from item to item, we consider the rates as random coefficients, then estimate the parameters using S-Plus/R nlme function. However, the model based on inherent

degradation mechanism of phosphors fails to adequately describe the PDP degradation paths in Figure 2. Residual analysis (not given here) shows that the residuals are highly skewed negative with a median equal to −5.2747.

Alternatively, we seek to combine simultaneously the phosphors degradation and the effect of burn−in into a degradation model of the luminosity. The impurity burn−in rate and the phosphors degradation rate, however, are not separable; to model this conflictive behavior in luminosity degradation explicitly, we introduce the bi−exponential model (1) to the relative luminosity data. First, we consider a simplest model without random−effects and estimate the parameters using the nonlinear least squares (NLS) method (Seber and Wild, 1989). The fixed−effects model also fails to adequately describe the degradation by ignoring the grouping of brightness measurements according to individual units and fitting a single model to the collective PDP paths.

To recap, the data suggest that the PDP degradation model should include variability among and within individual units, which is modeled most effectively using random coefficients. The likelihood ratio test (LRT) was sequentially executed to compare nonlinear random−coefficients (NRC) models fit by maximum likelihood to decide which of the coefficients in the model require random effects to account for between−unit variation. The model−building procedure is summarized in Table 1. We additionally computed the Akaike Information Criterion (AIC) (Sakamoto et al., 1986) and the Bayesian Information Criterion (BIC) (Schwarz, 1978) to support the best model selection procedure. If we are using the AIC and the BIC to compare two models for the same data, we prefer the model with lower AIC and BIC.

Model	Random-coefficients included	Covariance structure	d.f.	AIC	BIC	Test	LRT statistic	p-value
1	(b_1, b_2, b_3, b_4)	diagonal	9	473.248	499.722			
2	(b_1, b_2, b_3)	diagonal	8	471.247	494.781	1 vs. 2	5.23×10^{-5}	0.9942
3	(b_1, b_3, b_4)	diagonal	8	471.247	494.781	1 vs. 3	6.54×10^{-5}	0.9935
4	(b_1, b_2)	general	8	560.536	584.069	1 vs. 4	89.289	< 0.0001
5	(b_3, b_4)	general	8	531.913	555.446	1 vs. 5	60.665	< 0.0001
6	(b_1, b_3)	general	8	469.247	489.839	1 vs. 6	26.252	< 0.0001
7	(b_2, b_4)	general	8	468.026	491.559	1 vs. 7	3.221	0.0727
8	(b_1, b_3)	diagonal	7	469.247	489.839	1 vs. 8	7.23×10^{-5}	1.000

Table 1 : Likelihood ratio tests comparing different random−coefficients models for the PDP data.

The final fitted NRC model is

$$y_{ij}(t) = (18.5286 + b_{1i})\exp(-0.00263t_j) + (80.9413 + b_{3i})\exp(-0.000053t_j) \quad (9)$$

where $(b_{1i}, b_{3i})' \sim Normal((0,0)', (4.1736^2, 0; 0, 4.6186^2))$ and $\sigma^2 = 1.0008$. The final NRC model, fitted with solid lines in Figure 3, provides more reliable results in predicting true values of the PDPs luminosity than the model (11). For the NRC model (9), the average of absolute relative error, $|y - \hat{y}|/y$ is 9.79×10^{-3}, much smaller than model (8) (0.9866).

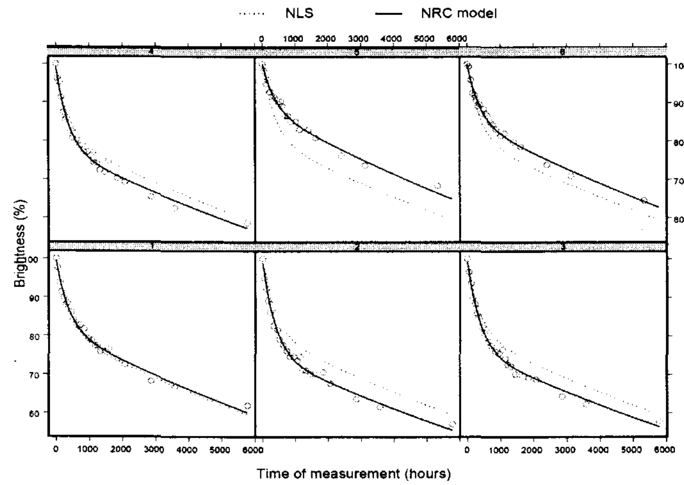

Figure 3: NRC model and NLS fit for the PDPs.

5.2 PDP Failure-Time Analysis

Based on all the estimates of fixed and random effects, the distribution $F_T(t)$ of the failure time (the time to reach 50% of initial luminosity) and the quantile t_p are estimated by applying the procedure in Section 4. The failure-time distribution is generated using Monte Carlo simulation with $N = 50,000$. To introduce Monte Carlo evaluation of $F_T(t)$, we first use the ML estimates $\hat{\beta}$ and $\hat{D}$ obtained from the six sample paths to generate the N simulated realizations $\check{\beta}$ and $\check{b}$. From the N values of $\check{\beta}$ and $\check{b}$, we compute the N failure times $\check{t}$ which are cross-times between the realized degradation paths and τ_f , and derive $\hat{F}_T(t)$ based on N simulated failure-times for any desired values of t.

The point estimates and $100(1-\delta)\%$ confidence intervals of the pth quantiles for the failure time distribution were derived. The intervals are based on Bae and Kvam' s (2004) procedure for constructing parametric bootstrap confidence

intervals of the pth quantiles for two empirical failure time distributions. In this case B=4,000 bootstrap samples were generated. $\hat{F}_T(t)$ and its 90%, 95% bootstrap confidence intervals are plotted in Figure 4. The point estimates and 90%, 95% confidence intervals (in parentheses) of the pth quantiles of failure time distribution are summarized in Table 2, for p = 0.01, 0.1, 0.5, and 0.9. The lifetime estimates based on model (9), which ignores the burn-in effect, are much shorter than those derived from the NRC model (8).

	Quantile			
	0.01	0.1	0.5	0.9
bi-exp. model (13)	6,414.9	7,693.5	9,103.9	10,416.0
90% CI	(6,339.3, 6,490.5)	(7,644.8, 7,742.2)	(9,059.3, 9,148.5)	(10,372.0, 10,460.0)
95% CI	(6,299.1, 6,530.7)	(7,619.0 , 7,768.0)	(9,035.6 , 9,172.2)	(10,349.0, 10,483.0)
exp. model (11)	4,206.5	4,273.7	4,359.7	4,451.3
90% CI	(4,195.2, 4,217.8)	(4,264.9, 4,282.5)	(4,352.8, 4,366.6)	(4,441.9, 4,460.7)
95% CI	(4,189.2, 4,223.8)	(4,260.3, 4,287.1)	(4,349.1, 4,370.3)	(4,436.9, 4,465.7)

Table 2 : Quantiles and their 95% bootstrap confidence intervals : PDP example.

6. Conclusion

In this article, we introduced a bi-exponential model with random coefficients to characterize nonlinear degradation paths caused by contamination effects of (nano-sized) impurities remaining after the burn-in process of PDPs. The bi-exponential model fits the nonlinear degradation data well by incorporating both inherent degradation characteristics of phosphors and contamination effects of impurities. Not only do models ignoring or failing to correctly capture these nonlinear characteristics lose the efficiency of prediction, but they also risk grossly underestimating the true reliability. If an initial unstabilized degradation stage caused by incomplete burn-in can be eliminated by extending the burn-in time or using a higher stress, the reliability estimation can be greatly improved by mainly extrapolating lifetimes from the second term of the bi-exponential model with a slower degradation rate.

An effective burn-in strategy that considers burn-in cost and reliability simultaneously can be developed for this kind of degradation data. Optimal burn-in policies in the past have been based on ordinary failure-time data, but are relatively undeveloped for use with degradation data.

7. Reference

[1] Bae, S. J., and Kvam, P. H. (2004), “"A Nonlinear Random−Coefficients Model for Degradation Testing” ", Technometrics, Vol. 46, No. 3, 460-469.

[2] Bae, S. J., and Kvam, P. H. (2003), “"A Change−Point Analysis for Modeling Incomplete Burn−In for Light Displays” ", IIE Transactions, To appear.

[3] Davidian, M., and Giltinan, D. M. (1995), Nonlinear Models for Repeated Measurement Data,Chapman and Hall, London.

[4] Lindstrom M. J., and Bates, D. M. (1990), “"Nonlinear Mixed Effects Models for Repeated Measures Data” ", Biometrics, Vol. 46, 673-687.

[5] Lu, C. J., and Meeker, W. Q. (1993), “"Using Degradation Measures to Estimate of Time−tofailure Distribution” ", Technometrics, Vol. 35, 161−176.

[6] Meeker, W. Q., and Escobar, L. A. (1998), Statistical Methods for Reliability Data, Wiley, NewYork.

[7] Pinheiro, J. C., and Bates, D. M. (1995), “"Mixed Effects Models, Methods, and Classes for S and Splus” ", University of Wisconsin, Dept. of Statistics (available from Statlib).

[8] Sakamoto, Y., Ishiguro, M., and Kitagawa, G. (1986), Akaike Information Criterion Statistics, Reidel, Dordrecht, Holland.

[9] Schwarz, G. (1978), “"Estimating the Dimension of a Model” ", Annals of Statistics, Vol. 6, 461−464.

[10] Seber, G. A. F., and Wild, C. J. (1989), Nonlinear Regression, Wiley, New York.

[11] Tannas, L. E. (1985), Flat−Panel Displays and CRTs, Van Nostrand Reinhold, New York.