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How Much Power can be Obtained from the Tides?

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Abstract

General formula are presented for the maximum power available from the tidal head in a closed basin and from the tidal currents in a channel connecting two large bodies of water. In the latter case, the available energy cannot be estimated from the kinetic energy flux in the undisturbed state, but can be obtained from knowledge of the tidal head between the ends of the channel and the maximum volume flux in the undisturbed state. The results are supported by detailed calculations for Johnstone Strait, British Columbia, using a two-dimensional finite element model. The model also allows an extension to the case of multiple channels. More work is needed to allow for partial tidal fences which do not occupy the whole cross-section of a channel.

Introduction

Using ocean tides to generate electrical power requires that there be both flow through marine turbines and a pressure head across them. There are two ways in which this can be achieved. In the first, a large tidal range in a coastal embayment is exploited, with water typically being trapped behind a barrage at high tide and then discharged through turbines when the water level outside the bay has fallen enough to provide an adequate head. This ebb generation may be supplemented or replaced by generation on the flood tide, and more elaborate schemes involving pumping or two basins are possible [1,2]. In the second approach, turbines can be placed in the strong tidal currents that are found in some channels, in much the same way as wind turbines can be used in the atmosphere. A pressure drop occurs across single turbines or across "fences" of turbines that might be deployed across a channel.

In the first approach, the pressure head (from the large tidal range) is primary, but a flow is required to generate power. In the second approach, the strong current is primary, but a loss of head will be associated with any exploitation. The two approaches are thus more similar than is sometimes appreciated.

The similarity of the two approaches is made clear if we think of the first approach as one in which the natural ebb and flow through the entrance to a bay is exploited in a manner similar to the second approach. Instead of considering a barrage in the entrance to a bay, one could evaluate the use of a fence of turbines, and represent the action of this in much the same way as in any other channel situation.

Small Bay

In one simple model [3], turbine fences were simulated in the entrance to a bay of surface area A, small enough that the surface elevation in it could be taken as spatially uniform, with a value $\zeta(t)$. The elevation outside was taken to be $a \cos \omega t$, unaffected by the presence of the turbines. Neglecting natural friction, the head difference $a \cos \omega t - \zeta$ is what drives the flow through the turbines, and the product of this flow and the head is the power generated. The average of this power over a tidal cycle was evaluated for various representations of the turbine drag. In all cases, and as expected, the power is small when there are few turbines, but also small when there are so many turbines that the flow is essentially blocked. There is an intermediate maximum which may be compared with the power

$$P_{\text{ref}} = \pi^{-1}\rho g A\omega a^2 \tag{1}$$

which would be generated if the potential energy of the water in the bay at high tide, with respect to low tide, were released once each tidal cycle. In practice, P_{ref} is a convenient metric for the power available from a conventional barrage scheme, though an overestimate as it would not be possible to exploit the full head.

For a linear frictional representation of the turbines, the maximum power is $(1/4)\rho g A\omega a^2$. This is a fraction $\pi/4$ of P_{ref}, and so not much less. Of course, fences of turbines might be much less efficient than the kind of turbine which could be used in a barrage.

For the more realistic representation of turbine friction as quadratic in the current, the maximum power drops by 3% from its value with linear friction (so that the coefficient 1/4 becomes 0.24). The maximum power is achieved with the tidal range in the bay being 74% of its natural value. The head difference driving the turbines comes largely from a phase difference between the tide inside and outside the bay, rather than by the tidal range in the bay being greatly reduced. Thus it seems that it is possible to manipulate the operating regime to allow ample flushing for aquaculture and pollution control as well as power generation.

Although we have expressed the maximum power here in terms of the area of the bay and the tidal amplitude, we note that we could write it, for quadratic turbine drag, as

$$P_{\max} = 0.24\rho g a Q_{\max} \tag{2}$$

where $Q_{\max} = \omega A a$ is the peak volume flux into the bay in the natural regime. In the natural regime, the volume flux is out of phase with the tidal elevation, so that there is no energy flux into the bay, but the product of the head ga and the natural volume flux does provide a useful measure of the power that might be available if the relative phase could be shifted.

This analysis neglects the head loss that might occur in the natural regime because of acceleration in the entrance channel and naturally occurring friction, but this head loss is likely to be much smaller than the tidal amplitude in bays with large enough tides to be worth exploiting.

Channel

A different situation occurs in numerous locations where the tidal range is not particularly big, but strong tidal currents, and large volume fluxes, occur in channels between two large bodies of water. The similarity with the bay situation is that, once again, too few turbines will generate little power, but too many turbines will block the flow and also lead to little power.

One difference from the previous situation of a bay is that it may be reasonable now to take the tidal elevation at both ends of the channel to be unaffected by changes within the channel itself, unlike the bay situation in which the level in the bay is dependent on conditions in the entrance channel. Another difference is that it is now important to take into account the geometry of the channel and the frictional forces in it before turbines are added.

At first sight, it seems that determination of the maximum power available will depend on the development and use of a detailed numerical model to solve the partial differential equations governing the flow. However, reasonable assumptions have been used to develop an integral approach which led to revealing general conclusions [4]. The assumptions were

(i) as already mentioned, the elevations at the ends of the channels are taken as given and do not change (the difference was taken in [4] as $\zeta_0(t) = a\cos\omega t$),

(ii) the current is uniform across the channel, though varying with distance along the channel and with time,

(iii) the channel is short compared with a tidal wavelength, so that, as shown by scale analysis (for a progressive tidal wave) the volume flux along the channel can then be taken as $Q(t)$, a function of time but not position along the channel.

Integrating along the channel, of length L, led to the governing equation

$$c\frac{dQ}{dt} - g\zeta_0 = -\int_0^L F_{\text{turb}}\,dx - \alpha Q|Q|, \tag{3}$$

where $c = \int_0^L A^{-1}\,dx$, with $A(x)$ now the cross-sectional area of the channel as a function of the along-channel coordinate x, and

$$\alpha = \int_0^L C_d(hA^2)^{-1}\,dx + \frac{1}{2}A_e^{-2}. \tag{4}$$

Here C_d is the quadratic drag coefficient and A_e is the cross-sectional area at the exit end of the channel where flow separation may occur. F_{turb} is the force exerted on the turbines per unit mass of water. There is a balance at any instant between the acceleration of the mass of fluid in the channel, the pressure gradient associated with surface slope, the integrated drag $\int_0^L F_{\text{turb}}\,dx$ associated with the turbines, and the internal friction and separation. The average power generated is given by

$$P = \overline{\rho Q\int_0^L F_{\text{turb}}\,dx}, \tag{5}$$

where the overbar represents an average over a tidal cycle.

The situation is most easily understood and evaluated if, as is often the case, the acceleration term is negligible. The flow at any instant is then quasi-steady and the power produced at any instant is

$$\rho Q\int_0^L F_{\text{turb}}\,dx = \rho Q(g\zeta_0 - \alpha Q|Q|). \tag{6}$$

The right hand side is zero when the term in brackets is zero (the natural regime with no turbines) and also when there are so many turbines that the flow is completely blocked and $Q = 0$. There is a maximum when Q is reduced to $3^{-1/2} = 0.58$ of what it would be in the natural regime. In that case and assuming that the turbine drag is quadratic in the current, the maximum average power may be written as

$$P_{\text{max}} = 0.21\rho g a Q_{\text{max}} \tag{7}$$

where, as before, Q_{max} is the peak volume flux in the natural state. The formula here is remarkably close to that of (2). Moreover, in the other limit of the channel case, with a balance in the natural situation between head and acceleration rather than friction, the multiplier in (7) becomes 0.24. For intermediate situations, with both acceleration and friction important, the coefficient varies between 0.24 and 0.21, with an intermediate dip to 0.20. As pointed out in [4], using a coefficient of 0.22 in (7) should give the power potential of a channel to within 10%, though the precise value can be established by examining the phase lag between head and current in the natural state, as this is determined by the mix of acceleration and friction. This result for the maximum power is independent of the location of turbine fences in a channel, though it is clearly more efficient to place them in a constriction where the currents are strong.

At maximum power in the quasi-steady situation, two-thirds of the original head along the channel is transferred to the turbine fences.

It is important to recognize that (7) is very different from the frequently-used estimate of the power potential as the average, over a tidal cycle, of the kinetic energy flux $\frac{1}{2}\rho u^3 A$, where u is the current and A the cross-sectional area. This formula had no real basis, and seemed dubious anyway as it gives a result which is very dependent on the location where it is evaluated. Nonetheless, if the flow is quasi-steady and dominated by exit separation rather than friction within the channel, the power may be written as $0.38\overline{\rho u_e^3} A_e$, where the subscript e implies evaluation at the exit [4]. If internal friction matters, the coefficient 0.38 should be increased. Thus, depending on the situation, the kinetic energy flux at the exit may over- or underestimate the power potential. The kinetic energy flux at the most constricted section is likely to be an overestimate.

The simple theory described above assumes just a single tidal constituent. If there are many other constituents, with heads having ratios $r_1, r_2, \ldots$ to the main constituent (usually M_2), then the power calculated with the main constituent alone must be multiplied by a factor $1 + (9/16)(r_1^2 + r_2^2 + \ldots)$ if the dynamics is quasi-steady, but $1 + r_1^2 + r_2^2 + \ldots$ if the force balance in the natural state is dominated by acceleration [4].

Johnstone Strait, British Columbia

Johnstone Strait, between Vancouver Island and mainland British Columbia, has strong tidal currents, driven by the out-of-phase tidal elevation between the ends.

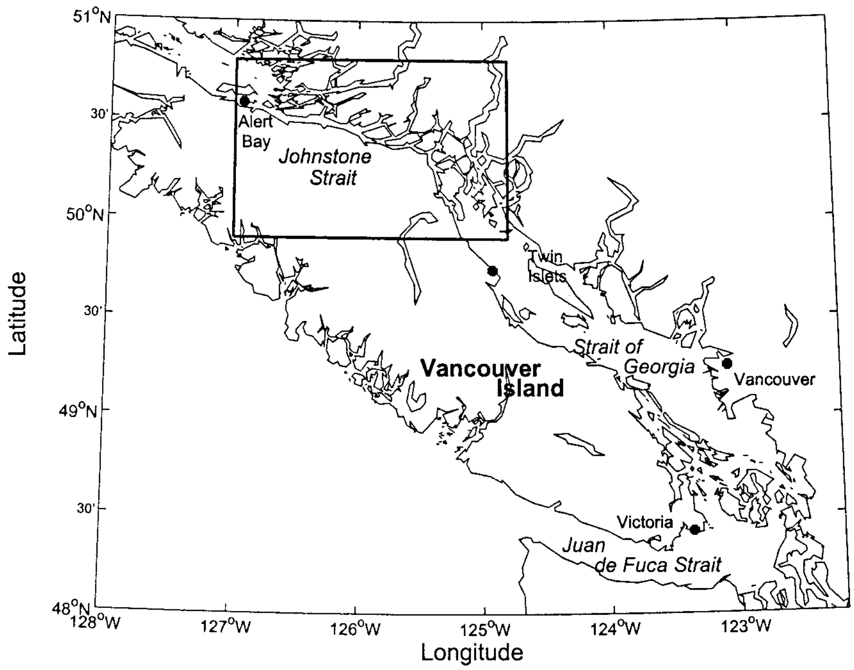

Figure 1. Vancouver Island and surrounding waters. The channels of Johnstone Strait inside the black box are shown in more detail in Figure 2.

The dominant constituent is M_2 which has a head difference amplitude a of 2.11 m. The volume flux varies a few % over the length of the strait, with an average value of 3.11×10^5 m^{3}s^{-1}. The phase lag of the current behind the forcing is approximately 35°, so that the dynamics is closer to a frictional than an accelerative balance. A coefficient of 0.20 is appropriate in (7) so we expect a maximum power of 1.32 GW.

To check on this, a 2D finite element model has been run for a domain extending out into the Pacific Ocean, and then, to simulate an array of tidal turbines, the bottom friction in each of the three shaded areas of Figure 2 in turn was slowly increased [5].

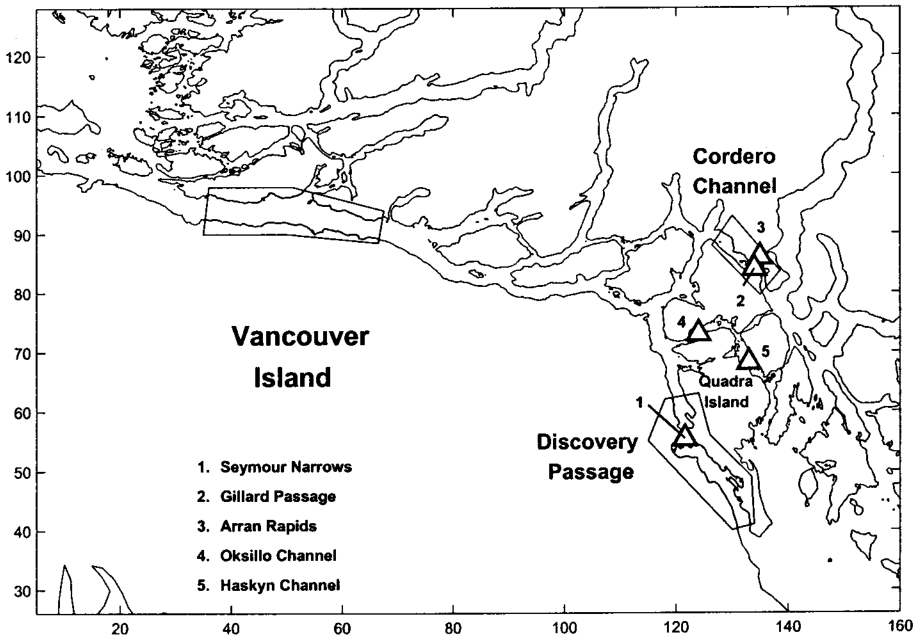

Figure 2. Three subregions of Johnstone Strait in which the bottom friction is increased to simulate arrays of tidal turbines.

In each case the dissipation associated with the extra friction at first increases and then decreases, while at the same time the strength of the tidal current decreases. For Johnstone Strait, the upper left shaded area in Figure 2, the maximum dissipation, which we take as an upper bound for the available power, is found to be 1.34 GW, close to the value of 1.32 GW expected from [4]. At maximum power the current is reduced to 58% of its natural value, also in agreement with [4]. The head along the whole strait increases by less than 3%.

Further southeast, the channel splits into two main channels: Discovery Passage and Cordero Channel (the other passages are very minor). The theory of [4] no longer applies, as turbines in one channel can cause diversion of the flow into the other. The numerical model predicts power maxima of 401 MW and 277 MW for Discovery Passage and Cordero Channel respectively. In each case, the current in the exploited channel drops to approximately 57% of the original value, while the current in the other channel increases by 14%. Cordero is the more likely to be exploited as Discovery Passage is used extensively for shipping.

Discussion

Recent theory, supported by numerical modelling, has provided new insight and general formulae for the power available from strong tidal currents if they are exploited by fences of turbines occupying the whole cross-section of a channel. For both a small bay and a channel connecting two large bodies of water, there is an optimum number of turbines (too many block the flow) and the maximum power available is proportional to the product of the tidal head and the peak tidal flow rate in the natural regime.

There will be energy losses associated with internal efficiency factors for the turbines, and part of the driving head will be lost to drag on supporting structures. Also, if water is allowed to flow between turbines, or if, to allow for shipping, the fences only partially fill the channel either vertically or horizontally, there will be a loss of head as flows with different speeds merge downstream. The loss may be significant. A very simple model for this was explored in [3], but further work is required.

Our own initial experience with a 3D finite element model has shown that, if the friction increase is spread smoothly over an area of the bottom or one side of a channel, then the water just tends to avoid that region! In other words, if only partial tidal fences are used, they will have to be sufficiently separated in the downstream direction that internal turbulence in the flow can restore a velocity profile between each encounter. Alternatively, maybe successive fences can be on opposite sides of a channel, with each exploiting the faster flow that has bypassed an upstream fence. There is much still to be done.

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