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On the possibility of freak wave forecasting

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Abstract. Modern Ocean wave forecasting systems predict the mean sea state, as characterized by the wave spectrum, in a box of size $\Delta x \Delta y$ surrounding a grid point at location $\mathbf{x}$. It is shown that this approach also allows the determination of deviations from the mean sea state, i.e. the probability distribution function of the surface elevation. Hence, ocean wave forecasting may provide valuable information on extreme sea states.

Introduction

I will start with a brief discussion of the progress we have made in ocean wave forecasting at ECMWF during the past 10 years or so. Then, I will discuss to what extend we may contribute to the problem of extreme wave height climatology and to the prediction of extreme events, such as for example freak waves.

The programme of the talk is as follows.

- Forecasting of the mean sea state: Ocean wave forecasting is about forecasting of the mean sea state in a grid box. The fundamental evolution equation for the wave spectrum is the energy balance equation and its solution is the basis of operational forecasting at ECMWF since June 1992.

  We show progress by validation of analysis and forecast against observations.

- Climatology of extremes: Based on a reanalysis effort of the weather over the period 1958-2002: ERA40. Try to make a homogeneous analysis of the weather using one analysis system, but distribution of observations is inhomogeneous. Also, a reanalysis is expensive so weather and wave analysis is only done on a coarse $\simeq 160$ km grid.

  Discuss work by Caires and Sterl to obtain 100 year return values using peaks-over-threshold method.

- Theory of FREAK WAVES: On the open ocean extreme waves are generated by **nonlinear focussing**, a four-wave interaction process that also causes the Benjamin-Feir (1967) Instability. We obtain, for given wave spectrum, the pdf of the wave height for surface gravity waves. In particular, kurtosis is an important parameter in the estimation of extreme events.

- Verification: Much progress in the lab, some progress in the field, but a global validation is desirable.

- Operational Implementation: A simplified, deep-water version of the theory is operational since October 2003.

Forecasting of the mean sea state

Ocean waves obey a set of deterministic evolution equations. For operational forecasting, solving these equations is not practical because, apart from the initial amplitudes, knowledge of the phase of the waves is required. This information is not available. Furthermore, just as in the atmospheric problem, there is chaotic behaviour.

Therefore, consider the evolution of the **mean sea state** in a box with width $\Delta \mathbf{x}$ at location $\mathbf{x}$. The ensemble average is essentially an average over the phases of the waves.

The mean sea state is then given by the wavenumber spectrum $F(\mathbf{k};\mathbf{x},t)$, while the action density spectrum $N(\mathbf{k};\mathbf{x},t)$ is defined as

$$N = \frac{gF}{\sigma}$$

with $\sigma = \sqrt{gk\tanh(kD)}$. The action density is the number density of waves, hence the energy E of the waves is given by $E = \sigma N$, while the wave momentum $\mathbf{P}$ is given by $\mathbf{P} = \mathbf{k}N$.

Averaging the deterministic evolution equations then gives for waves on a slowly varying current $\mathbf{U}$ the energy balance equation

$$\frac{\partial N}{\partial t} + \nabla_{\mathbf{x}}.(\nabla_{\mathbf{k}}\Omega\, N) - \nabla_{\mathbf{k}}.(\nabla_{\mathbf{x}}\Omega\, N) = S.$$

Here, Ω represents the dispersion relation

$$\Omega = \mathbf{k}.\mathbf{U} + \sigma.$$

The source function S on the right hand side represents the physics of wind-wave generation (S_{in}), dissipation by wave breaking and other causes (S_{dissip}) and four-wave interactions (S_{nonlin}). In other words,

$$S = S_{in} + S_{nonlin} + S_{dissip}.$$

In the 1980's there was a dedicated effort to develop efficient parametrisations of all the source functions, which still is the basis of present day wave forecasting and the two-way interaction of ocean waves and atmosphere.

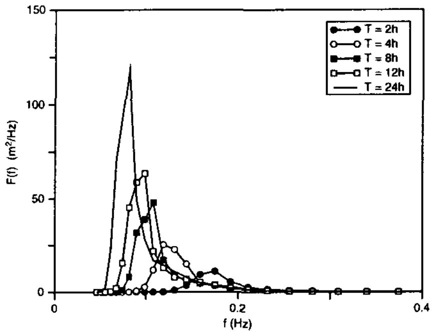

Figure 1: Evolution in time of the one-dimensional frequency spectrum for a wind speed of 18 m/s

The result was a realistic simulation of the spectrum of wind-generated ocean waves, including the well-known overshoot phenomenon. (see Fig. 1)

Progress in Wave Prediction

Discuss progress in ocean wave forecasting at ECMWF during the past 15 years or so.

- considerable improvement in forecasting parameters such as H_S and T_p.
- there have been considerable improvements in the wave model. Another important reason is better quality of analyzed and forecast **wind**.

Measure progress by comparing first-guess wave height with Altimeter wave height data and by comparing forecast wave height with independent buoy observations. Only show wave height against buoy data (see Fig.2).

Climatology of Extremes

Based on a reanalysis effort of the weather over the period 1958-2002: ERA40. Try to make a homogeneous analysis of the weather using one analysis system, but distribution of observations is inhomogeneous.

Also, a reanalysis is expensive so weather and wave analysis is only on a coarse $\simeq$ 160 km grid. Therefore pressure gradients and winds are underestimated, and as a consequence also wave height. For proper wave

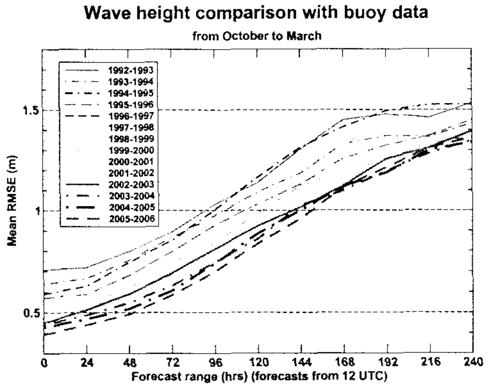

Figure 2: Validation of model significant wave height against buoy data as a function of forecast time. Curves are for different Winterseasons

height climatology from ERA40 results corrections are needed.

Caires and Sterl (2005), compared 100-year return values from buoy observations (during period 1980-1999) with those from model data and validated the resulting correction with 100-year returns from Topex Altimeter data. Correction is as follows:

$$X_{100}^{buoy} = 0.52 + 1.30 X_{100}^{ERA40} \tag{1}$$

The resulting 100-year return map is shown in Fig. 3.

Theory of Freak Wave Generation

There are now a number of possible explanations available to explain extreme events, such as diffraction and focussing by currents. However, in large areas of the ocean currents are relatively small, hence another explanation, based on nonlinear interactions has emerged.

- Linear theory: No wave-wave interaction. Focussing of wave energy only occurs when the phases of the waves are favourable (**constructive interference**). Gives at best a doubling of wave height → Gaussian pdf for elevation η.

- Nonlinear Waves: Now there are **four-wave interactions**. Thus, a wave may borrow energy from its neighbours. Because of this extra focussing wave height may become at most $3-6$ times as large as the average wave height→ Large deviations from Gaussian.

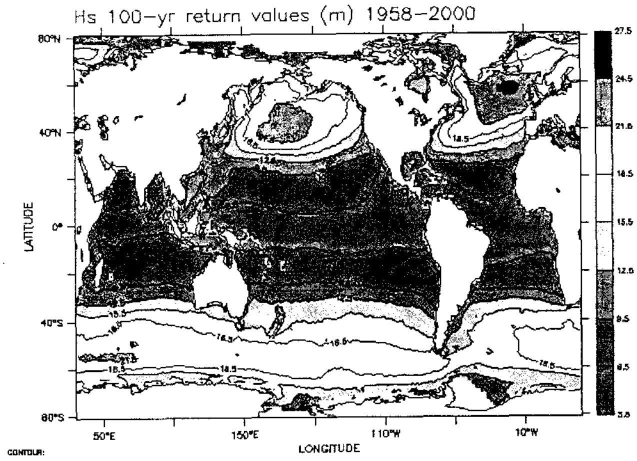

Figure 3: 100-year return value estimates of H_S based on ERA-40 data as obtained from www.knmi.nl/waveatlas

The fun is that these deviations can be obtained by means of the usual **statistical mechanics** approach for wave-wave interactions (Janssen, 2003).

Zakharov Equation

This has been shown by doing Monte Carlo Forecasting with the 1D version of the Zakharov equation, which describes the evolution of the complex amplitude $a(\mathbf{k})$ of the free gravity waves:

$$\frac{\partial a_1}{\partial t} + i\omega_1 a_1 = -i \int d\mathbf{k}_{2,3,4} T_{1,2,3,4} a_2^* a_3 a_4 \delta_{1+2-3-4},$$

where $\mathbf{k}$ is the wave number and $\omega = \sqrt{gk}$. $T_{1,2,3,4}$ is a complicated function of frequency and wavenumber, and has a number of symmetries which garantee that the system is **Hamiltonian**.

Benjamin-Feir Index

Benjamin and Feir (1967) were the first to show that a nonlinear uniform wave train evolves towards a strongly modulated state (nonlinear focussing). Later work has shown that for a continuous wave spectrum one needs, apart from **steep waves**, a **narrow spectrum**. Define as integral measure of steepness

$$\epsilon = (k_0^2\langle\eta^2\rangle)^{\frac{1}{2}} \tag{2}$$

where $\langle\eta^2\rangle$ is the average surface elevation variance and k_0 the peak wave number). Then, the **Benjamin-Feir Index** is defined as

$$BFI = \epsilon\sqrt{2}/\sigma'_\omega,$$

where $\sigma'_\omega = \sigma_\omega/\omega_0$ is the relative width of the frequency spectrum. A similar quantity involving the directional width is relevant as well.

Stochastic Approach

In wave forecasting we are interested in predicting quantities such as the second moment

$$B_{i,j} =< a_i a_j^* >,$$

where angle brackets denote an ensemble average. Follow methods employed in Statistical Mechanics (Liouville → Boltzmann).

For a homogeneous sea state the action density $N(\mathbf{k})$ is defined as

$$B_{i,j} =< a_i a_j^* >= N_i\delta(\mathbf{k}_i - \mathbf{k}_j),$$

and the task is to derive an evolution equation for N from the Zakharov equation. Because of nonlinearity, the equation for the second moment couples to the fourth moment, etc, resulting in an infinite hierarchy of equations, known as the **BBGKY** hierarchy. Closure is achieved by assuming that the waves are weakly nonlinear so that the pdf of the surface elevation is close to a Gaussian (Random-Phase Approximation (RPA)).

For example, the fourth moment is

$$< a_j a_k a_l^* a_m^* > = B_{j,l}B_{k,m} + B_{j,m}B_{k,l} + D_{j,k,l,m},$$

where D is the so-called fourth cumulant, which vanishes for a Gaussian sea state. A similar relation applies for the 6th moment, and application of RPA closes the BBGKY hierarchy. As a consequence, the fourth cumulant D, subject to the initial value $D(t=0)=0$, becomes

$$D_{i,j,k,l} = 2T_{i,j,k,l}\delta_{i+j-k-l}G(\Delta\omega, t)$$
$$[N_iN_j(N_k+N_l) - (N_i+N_j)N_kN_l]$$

where $\Delta\omega = \omega_i + \omega_j - \omega_k - \omega_l$. Requires extensive use of the symmetries of T. In addition, the action density N is assumed to evolve on the slow time scale. The function G is defined as

$$G(\Delta\omega, t) = i\int_0^t d\tau e^{i\Delta\omega(\tau-t)} =$$
$$R_r(\Delta\omega, t) + iR_i(\Delta\omega, t).$$

Knowledge of the fourth cumulant is essential for (Janssen, 2003)

- evolution of N caused by four-wave interactions
- determination of deviations from Normality.

Substitution of D in the equation for the second moment gives the Hasselmann (1962) equation

$$\frac{\partial}{\partial t}N_4 = 4\int d\mathbf{k}_{1,2,3}T^2_{1,2,3,4}\delta_{1+2-3-4}R_i(\Delta\omega, t)$$
$$\times [N_1N_2(N_3+N_4) - N_3N_4(N_1+N_2)].$$

Note there are now two timescales implied by $R_i(\Delta\omega, t) = \sin(\Delta\omega t)/\Delta\omega$:

- short times: $\lim_{t\to 0} R_i(\Delta\omega, t) = t$, hence $T_{NL} = O(1/\epsilon^2\omega_0)$, the Benjamin-Feir timescale (non-resonant!)
- large times: $\lim_{t\to\infty} R_i(\Delta\omega, t) = \pi\delta(\Delta\omega)$, corresponding to resonant wave-wave interactions, hence $T_{NL} = O(1/\epsilon^4\omega_0)$

Deviations from Normality are most conveniently expressed by means of the kurtosis,

$$C_4 =< \eta^4 > /3 < \eta^2 >^2 - 1.$$

Using D the kurtosis becomes

$$C_4 = \frac{4}{g^2m_0^2}\int d\mathbf{k}_{1,2,3,4}T_{1,2,3,4}\delta_{1+2-3-4}\left(\omega_1\omega_2\omega_3\omega_4\right)^{\frac{1}{2}}$$
$$\times R_r(\Delta\omega, t)N_1N_2N_3.$$

As $\lim_{t\to\infty} R_r(\Delta\omega, t) = P/\Delta\omega$, the kurtosis is determined by both resonant and non-resonant interactions.

The expression is too involved in an operational context, and we are still exploring the detailed consequences of this general result. Here, the case of a narrow-band wave train is briefly discussed.

Narrow-band approximation

We took a first step in validating the formula for the kurtosis by considering the simple case of a uni-directional, narrow-band spectrum.

Also, including the effects of the bound waves one then finds the following result for the kurtosis.

$$C_4 = 8\epsilon^2 + \frac{\pi}{3\sqrt{3}} \times BFI^2,$$

hence the kurtosis depends on the square of the BF index, because the first term, the contribution by the bound waves, is small.

Verification

The present findings have been verified in the laboratory and in the sea of Japan (see for more details a contribution by Mori and Janssen (2006) at this meeting. However, a more global validation of all this is clearly desirable.

In the case of the wave height distribution we start from the probability distribution for the envelope and phase of a narrow band wave train and we make use of the property that wave height is twice the envelope. As a result, the probability that wave height exceeds $h \times H_S$ is:

$$P_H(h) = e^{-2h^2}\left[1 + C_4 B_H(h)\right],$$

where

$$B_H(h) = 2h^2\left(h^2 - 1\right).$$

This result verifies well with observations (see Figs. 4 and 5).

Operational Implementation

At ECMWF, we have implemented the following scheme:

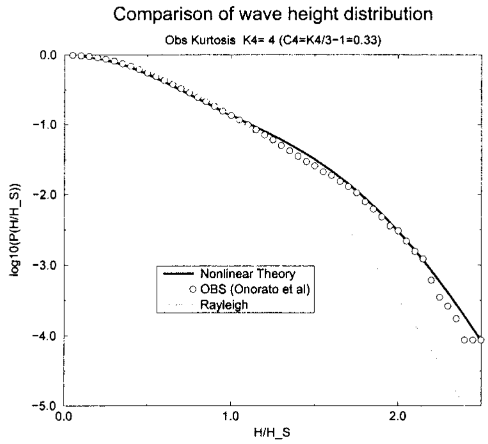

Figure 4: Comparison of theoretical and observed (Onorato et al, 2005) wave height distribution. For reference, the linear Rayleigh result is shown as well.

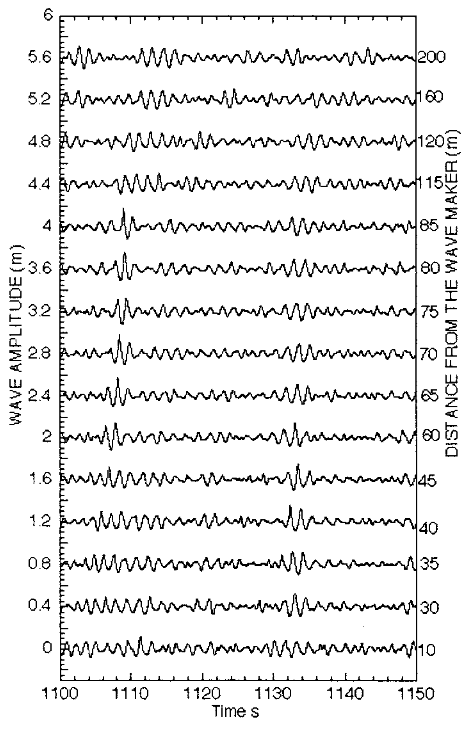

Figure 5: Evolution of surface elevation in space and time from the big wave tank in Trondheim (from Onorato et al, 2004). The formation of Freak Waves is clearly seen.

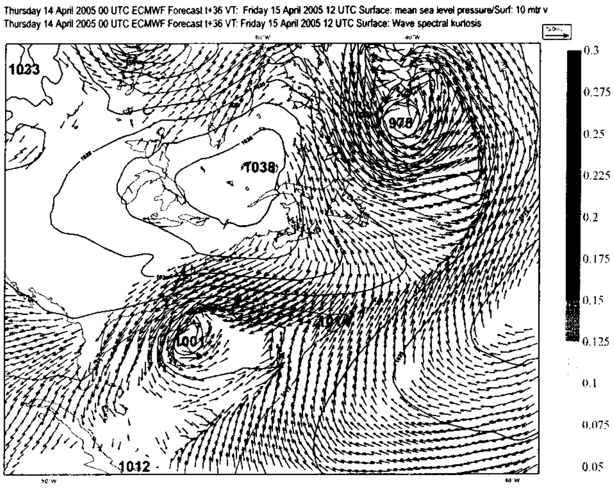

Figure 6: Kurtosis field during the period the cruise ship Dawn experienced some damage.

From the predicted wave spectrum we infer the B.F. Index. From the B.F Index we obtain the deviations from the Normal distribution, e.g. as measured by the Kurtosis. Given the kurtosis and the significant wave height, we are able to answer question such as what is the enhanced probability on extreme events. An example of operational output is given in Fig. 6.

Conclusions

Present-day wave forecasting systems give an accurate estimate of the seastate. Much progress because of improved models for atmosphere and ocean-waves, and the use of satellite data in our analysis system.

Modelling effort might be of help in estimating climatology of extremes, but be aware of large variability in these estimates. The wave climate is not stationary on a time scale of 100 years.

There is perspective in prediction extreme sea states, but work is still required to explore further consequences of the general result.

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