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Recent Progress of Freak Wave Prediction

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Abstract. Based on a weakly non-Gaussian theory the occurrence probability of freak waves is formulated in terms of the number of waves in a time series and the surface elevation kurtosis. Finite kurtosis gives rise to a significant enhancement of freak wave generation in comparison with the linear narrow banded wave theory. For fixed number of waves, the estimated amplification ratio of freak wave occurrence due to the deviation from the Gaussian theory is 50% – 300%. The results of the theory are compared with laboratory and field data.

Introduction

The last decade freak waves have become an important topic in engineering and science and are sometimes featured by a single and steep crest causing severe damage to offshore structures and ships. Freak wave studies started in the late 80's [1] and the high-order nonlinear effects on the freak waves were discussed in the early 90's [2, 3]. Due to the many research efforts, the occurrence of freak waves, their mechanism and detailed dynamic properties are now becoming clear [4, 5, 6, 7, 8, 9]. The state of the art on freak waves was summarized at the *Rogue Wave Conference*, held in 2000 and 2004 [10, 11]. It was concluded that the third order nonlinear interactions enhance freak wave appearance and are the primary cause of freak wave generation in a general wave field except for the case of strong wave-current interaction or wave diffraction behind the islands.

Numerical and experimental studies have demonstrated that freak-like waves can be generated frequently in a two-dimensional wave flume without current, refraction or diffraction [2, 4, 7, 12]. Moreover, the numerical studies clearly indicate that a freak wave having a single, steep crest can be generated by the third order nonlinear interactions in deep-water [2]. Also, the theoretical background of freak wave generation has become more clear [6], but the quantitative occurrence probabilities in the ocean remain uncertain. In addition, it is still questionable how to characterize the dominant statistical properties of the freak wave occurrence in terms of nonlinear parameters, spectral shape, water depth and so on.

Nevertheless, although there is no doubt that the third order nonlinear interactions relate with the steep wave generation in the random wave train, the theoretical background of the relationship between the freak wave generation and the third order nonlinear interactions is not well-established. Freak wave generation is sometimes discussed in the context of the Benjamin-Feir instability in deep-water waves because of the similarity of the steep wave profile itself [2, 7]. The last two decades, the Benjamin-Feir type instability of the deep-water gravity waves has been studied by many researchers using the nonlinear Schrödinger type of equations [13, 14, 15], mode-coupling equations [16], pseudo-spectral methods [17] and experiments [18]. However, there is disparity between the periodic wave instabilities and random wave behavior, because the broad banded spectra and random phase approximation are essential describing the ocean waves in nature [17, 19]. Thus, the energy transfer of random waves due to four-wave interactions has been studied for describing spectral evolutions [20, 21]. By means of a series of numerical investigations Yuen and Lake [22] stated that the instability is confined within an initially unstable range and becomes weak if the spectral bandwidth broadens. Alber [23] mathematically demonstrated that for a random sea the Benjamin-Feir instability vanishes if the wave spectrum is sufficiently broad. Therefore, there is discrepancy between the nonlinear behavior of periodic waves and random waves.

Janssen [24] investigated the freak wave occurrence as a consequence of four-wave interactions including the effects of non-resonant four-wave interactions. He found that the homogeneous nonlinear interactions give rise to deviations from the Gaussian distribution for the surface elevation on the basis of the Monte Carlo simulations of the Zakharov equation. Surprisingly, inhomogeneities only play a minor role in the evolution of the wave spectrum. He also formulated the analytical relationship between spectral shape and the kurtosis of the surface elevation. These results have the potential to unify previous freak wave studies covering nonlinear interactions, spectral profiles to nonlinear statistics and etc.

The purpose of this study is to investigate the relationship between kurtosis and the occurrence probability of freak waves through the nonlinear four-wave interactions. First, for a nonlinear stochastic wave field the relationship between high-order moments including kurtosis of surface elevation and nonlinear transfer function is derived. Second, the wave height and maximum wave height distributions are formulated as a simple function of kurtosis by the non-Gaussian theory. Third, the wave height distribution is compared with laboratory experiments and the occurrence probabilities of freak waves are compared with field observations. Finally, the dependence of the occurrence of freak waves on the number of waves and kurtosis will be analyzed and discussed.

High-order Moments in the Nonlinear Stochastic Wave Field

Our starting point is the Zakharov equation [25], which is a deterministic nonlinear evolution equation for surface gravity waves in deep water. Zakharov obtained from the Hamilton equations an approximate evolution equation for the amplitude of the free surface gravity waves [25], that contained the third order non-resonant and resonant four-wave interactions. In order to eliminate the effects of bound waves he applied on B a canonical transformation of the type

$$B = B(b, b^*), \tag{1}$$

where b is the normal variable of the free gravity waves. The evolution equation for b, called the Zakharov equation, becomes

$$\frac{\partial b_1}{\partial t} + i\omega_1 b_1 = -i \int \mathrm{d}\vec{k}_{2,3,4} T_{1,2,3,4} b_2^* b_3 b_4 \delta_{1+2-3-4}, \tag{2}$$

where, for brevity we have introduced the notation $b_1 = b(\vec{k}_1)$, etc, and the nonlinear transfer function $T_{1,2,3,4}$ as found by Krasitskii[21] enjoys a number of symmetries which guarantee that the Zakharov equation is Hamiltonian and conserves wave energy. In Janssen's Paper [24] it was shown that in the context of the deep-water version of the Zakharov equation extreme surface gravity waves are generated by nonlinear focusing in a random wave field. This process also causes the Benjamin-Feir instability of a uniform wave train. As a consequence, for deep-water waves a considerable enhancement of the probability for extreme waves is found.

The nonlinear term in Eq.(2) will generate deviations from the normal, Gaussian probability distribution function (PDF) for the surface elevation. It is of interest to determine these deviations because it gives us information on the occurrence of extreme sea states. According to Eq.(29) of Janssen (2003), the fourth moment $\langle\eta^4\rangle$ and kurtosis μ_4 can be obtained in terms of the action density N and of the nonlinear transfer function $T_{1,2,3,4}$. The result is

$$\kappa_{40} = \frac{\langle\eta^4\rangle}{m_0^2} - 3 \tag{3}$$

$$= \mu_4 - 3 \tag{4}$$

$$= \frac{12}{g^2 m_0^2} \int \mathrm{d}\vec{k}_{1,2,3,4} T_{1,2,3,4} \sqrt{\omega_1\omega_2\omega_3\omega_4}\, \delta_{1+2-3-4} R_r(\Delta\omega, t) N_1 N_2 N_3 \tag{5}$$

where κ_{40} is the fourth order cumulant of the surface elevation η and is equivalent to $\mu_4 - 3$, where μ_4 is the normalized fourth order moment, kurtosis of the surface elevation. The transfer function $R_r = (1 - \cos(\Delta\omega t))/\Delta\omega \rightarrow \mathcal{P}/\Delta\omega$ for large time t, where $\Delta\omega = \omega_1 + \omega_2 - \omega_3 - \omega_4$ and $\mathcal{P}$ denotes the principle value of the integral to avoid singularity in the integral.

Wave Height Distribution

Following a central limit theorem, linear, dispersive random waves have a Gaussian PDF for the surface elevation. Finite amplitude effects result, however, in deviations from the Normal distribution, as measured by a finite skewness and kurtosis. We assume that waves to be analyzed here are unidirectional with narrow banded spectra and satisfy the stationary and ergodic hypothesis. Let $\eta(t)$ be the sea surface elevation as a function of time t and $\zeta(t)$ an auxiliary variable such that $\eta(t)$ and $\zeta(t)$ are not correlated. Assuming both $\eta(t)$ and $\zeta(t)$ are real zero-mean function with variance σ, we have

$$Z(t) = \eta(t) + i\zeta(t) = A(t)e^{i\phi(t)}, \tag{6}$$

$$A(t) = \sqrt{\eta^2(t) + \zeta^2(t)}, \tag{7}$$

$$\phi(t) = \tan^{-1}\left(\frac{\zeta(t)}{\eta(t)}\right), \tag{8}$$

where A is the envelope of the wave train and ϕ the phase. Mori and Yasuda [26] investigated the wave height distribution as a function of kurtosis and skewness using the joint probability density function of $\eta(t)$ and $\zeta(t)$ for a narrow banded weakly nonlinear wave train. We will follow this approach closely. For weakly nonlinear waves deviations from the Normal distribution are small. In those circumstances the PDF of the surface elevation can be described by the Edgeworth distribution. As there is no-correlation between $\eta(t)$ and $\zeta(t)$, the joint probability density function of $\eta(t)$ and $\zeta(t)$ becomes

$$p(\eta,\zeta) = \frac{1}{2\pi}\exp\left[-\frac{1}{2}(\eta^2+\zeta^2)\right] \times\left[1 + \frac{1}{3!}\sum_{n=0}^{3}\frac{3!}{(3-n)!n!}\kappa_{(3-n)n} H_{3-n}(\eta)H_n(\zeta) + \frac{1}{4!}\sum_{n=0}^{4}\frac{4!}{(4-n)!n!}\kappa_{(4-n)n} H_{4-n}(\eta)H_n(\zeta)\right] \tag{9}$$

where H_n is the nth order Hermite polynomial. All variables will be normalized by the variance of the surface elevation $\sigma = m_0^{1/2}$ (where m_0 is the zero moment of the wave spectrum) and have zero-mean.

The PDF of the envelope A follows now immediately from an integration of the joint probability distribution

$$p(A,\phi) = A\, p(\eta,\zeta) \tag{10}$$

over ϕ, hence

$$p(A) = \int_0^{2\pi} \mathrm{d}\phi\, p(A,\phi). \tag{11}$$

Performing the integration over ϕ it is found that the first term of (9) gives the usual Rayleigh distribution for wave amplitude. Assuming the narrow band spectra, wave height H equals $2A$ and hence the wave height PDF becomes

$$p(H) = \frac{1}{4} H e^{-\frac{1}{8}H^2} \left[1 + \kappa_{40} A_H(H)\right], \tag{12}$$

where

$$A_H(H) = \frac{1}{384}\left(H^4 - 32H^2 + 128\right). \tag{13}$$

The comparison of exceedance probability of wave heights with experimental data is shown in Fig.1. The filled circles • denote experimental data, the Rayleigh distribution is represented by the dotted line, Eq.(12) corresponds to the solid line, and the wave height distribution including skewness effects proposed by Mori and Yasuda [26] (denoted as ER, Edgeworth-Rayleigh, in the figure) corresponds to the dashed line. For simplicity we refer to Eq.(12) as Modified ER (MER) hereafter. Due to the nonlinear effects, the exceedance probability obtained from the laboratory data departs for large wave height from the Rayleigh distribution. Both the MER and ER distributions for the exceedance probability of wave heights follow this separation at large amplitude region. Surprisingly, the MER distribution shows better agreement with the laboratory data than the ER distribution, although the corrections to the Rayleigh distribution only stem from effects of finite kurtosis.

Maximum Wave Height Distribution and Freak Wave Occurrence

The PDF of maximum wave height p_m in wave trains can be obtained once the PDF of wave height $p(H)$ and exceedance probability of wave height $P(H)$ is known [27], thus

$$p_m(H_{max})dH_{max} = N[1 - P(H_{max})]^{N-1} p(H_{max})dH_{max} \tag{14}$$

with N the number of waves. For sufficiently large N one may use the approximation

$$\lim_{N\to\infty}^{N} [1 - P(H_{max})]^N \simeq \lim_{N\to\infty}^{N} \exp\left[-NP(H_{max})\right], \tag{15}$$

Substituting Eq.(12) into Eq.(14), gives the PDF of the maximum wave height, p_m,

$$\begin{aligned} p_m(H_{max})dH_{max} &= \frac{N}{4} H_{max}\, \mathrm{e}^{-\frac{H_{max}^2}{8}} \left[1 + \kappa_{40} A_H(H_{max})\right] \\ &\times \exp\left\{-N\mathrm{e}^{-\frac{H_{max}^2}{8}} \left[1 + \kappa_{40} B_H(H_{max})\right]\right\} dH_{max} \end{aligned} \tag{16}$$

Equations Eq.(16) are evaluated as a function of N and κ_{40} (or μ_4). For $\kappa_{40} = 0$ results are identical to the ones following from the Rayleigh distribution. For simplicity it will be assumed

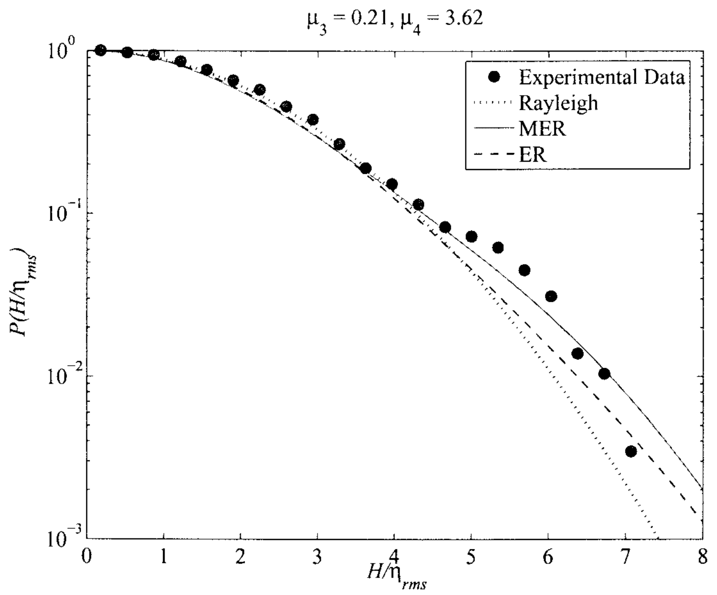


Figure 1: Comparison of wave height distribution from laboratory data and theory ($\mu_3 = 0.21, \mu_4 = 3.62$; •: laboratory data, solid line: Eq.(12), dashed line: [26], dotted line: Rayleigh distribution).

that $H_{1/3} = 4m_0^{1/2}$, although it is $H_{1/3} = 4.004m_0^{1/2}$ in an exact linear random wave theory. The freak wave condition in this study therefore becomes $H_{max}/m_0^{1/2} > 8$, and we obtain from Eq.(16) the following simple formula to predict the occurrence probability of a freak wave as function of N and κ_{40},

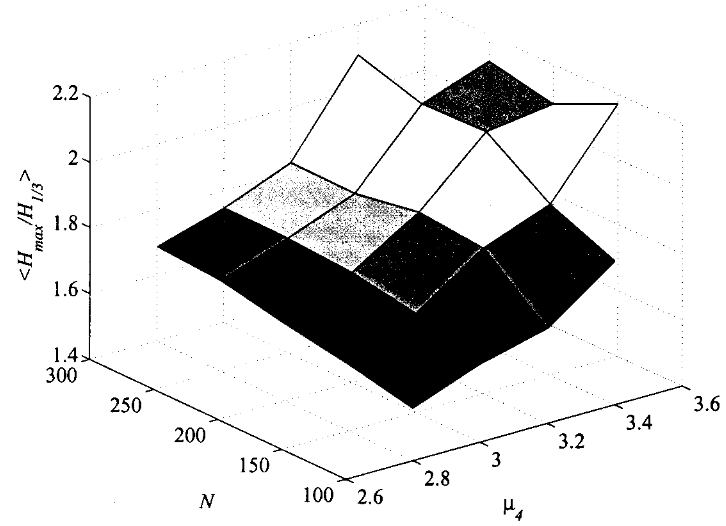
$$P_{freak} = 1 - \exp\left[-\beta N(1 + 8\kappa_{40})\right] \tag{17}$$

where $\beta = \mathrm{e}^{-8}$ is constant.

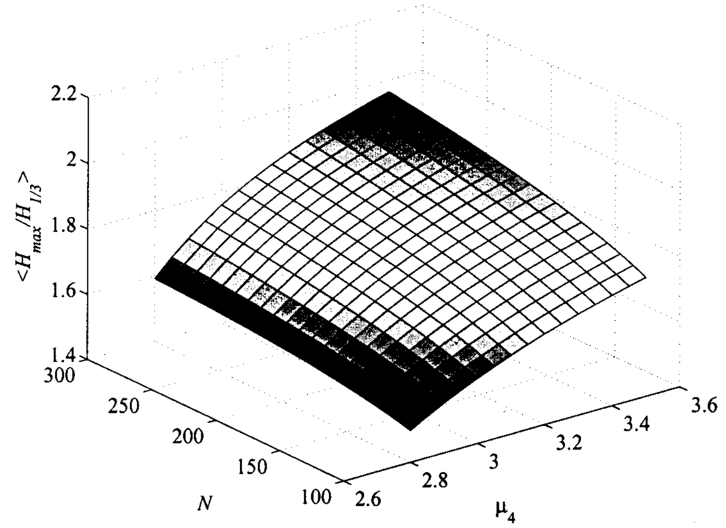
Using Eq.(17) it is seen that the effect of kurtosis becomes already of the same order as linear theory for $\kappa_{40} = 1/8$. This corresponds to $\mu_4 = 3.125$, and is not a strong nonlinear condition. Hence, both the effects of finite kurtosis and the number of waves N are important for determining the probability of maximum wave height in the nonlinear wave train.

The observed data was originally collected using an ultra sonic wave gauge at a depth of 30m, off the coast of the Pacific Ocean. The length of each record was 20min and the data were collected every hour from March 1 to the end of June in 2001. The wave statistics such as H_{max}, $H_{1/3}$, $T_{1/3}$, N, μ_3, and μ_4 were operationally calculated and archived. Note that the water depth of 30m is relatively shallow water. Therefore, to eliminate shallow water effects, the data are excluded if the dimensionless water depth $k_p h$ is less than 2.0 (it corresponds to $T_{1/3} \geq 8$s). The total number of valid data was about 2546.

We discuss the general behavior of the PDF of maximum wave height in the nonlinear wave field, by showing the ensemble averaged $H_{max}/H_{1/3}$ of each bin as a function of μ_4 and N in Fig.2. The brackets <> indicate the ensemble averaged value. Fig.2 (a) is observed data and (b) is the expected value of Eq.(16) through numerical integration. The dependence of $H_{max}/H_{1/3}$ on N is weaker than expected from Eq.(16). This is because the length of observed time series was fixed to 20min, so we cannot discuss the dependence of $H_{max}/H_{1/3}$ on number of waves in detail. On the other hand, the dependence of $\langle H_{max}/H_{1/3}\rangle$ on μ_4 is clear. The theoretically predicted $\langle H_{max}/H_{1/3}\rangle$ is underestimated compared to the observed data but it



(a) Observed data



(b) Theory

Figure 2: Dependence of $\langle H_{max}/H_{1/3}\rangle$ on μ_4 and N.

agrees with the observed data in a qualitative sense. The observed $\langle H_{max}/H_{1/3}\rangle$ monotonically increases for increasing μ_4, but for high values of kurtosis the theoretically estimated value of $\langle H_{max}/H_{1/3}\rangle$ is lower.

Conclusion

For a narrow-band, random wave train we have shown that the kurtosis of the surface elevation is mainly determined by resonant and non-resonant wave-wave interactions, while bound waves only give a small contribution. Thus, the kurtosis and related high-order cumulants can be evaluated on the basis of Janssen's work[24]. Second, we have shown that for a narrow-band wave train the wave height and the maximum wave height probability distribution depends to a good approximation on the wave variance and the kurtosis. As a consequence it is possible to formulate the freak wave occurrence probability in terms of the kurtosis and the number of waves in a time series. From the comparison with laboratory and field data, we conclude the following:

- The second order cross-cumulant κ_{22} is $1/3$ of the fourth cumulant, κ_{40}, of the surface elevation.
- The weakly non-Gaussian theory shows the dependence of the expected maximum wave height on kurtosis, which is supported by the observed data.
- The occurrence probability of freak waves is significantly enhanced by the kurtosis increase caused by four-wave interactions.

In order to check the validity of the approach developed here, in particular the dependence of freak wave occurrence on the kurtosis and the number of waves, systematic and continuous field measurements of freak waves including directional wave spectra and nonlinear statistics will be critically required.

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