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Evaluation of Surrogate Models for Shape Optimization of Compressor Blades

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Abstract

Performances of multiple surrogate models are evaluated in a turbomachinery blade shape optimization. The basic models, i.e., Response Surface Approximation, Kriging and Radial Basis Neural Network models as well as weighted average models are tested for shape optimization. Global data based errors for each surrogates are used to calculate the weights. These weights are multiplied with the respective surrogates to get the final weighted average models. The design points are selected using three level fractional factorial D-optimal designs. The present approach can help address the multi-objective design on a rational basis with quantifiable cost-benefit analysis.

1. INTRODUCTION

In recent years, the shape optimization based on three-dimensional flow analysis has been performed in the design process of turbomachinery blades as an efficient tool to increase the performance. Three-dimensional, unsteady and vortical nature of flow inside the turbomachine makes flow prediction difficult. Recent use of optimization methods replacing trial and error approach has helped to enhance the performance of turbomachine.

Recently, the use of lean (dihedral), sweep, and skew (stacking line in rotational direction) in axial flow compressor rotor has become a matter of interest in the design of turbomachinery blades. Fischer et al. [1] represented that the separation was reduced in the bowed stator blade leading to increase in the stagnation pressure ratio and efficiency for a 4-stage high speed compressor. Sweep and lean in an axial flow compressor rotor blade for optimization is used for optimization by Gallimore et al. [2] and they showed that hub corner and tip leakage losses was reduced due to optimization. Response surface method is applied to increase compressor blade efficiency [3-5]. Papila et al. [6] has performed shape optimization using RBNN enhanced RSA method for optimization to improve the performance of supersonic turbine blade. Surrogate models are being used widely in multidisciplinary optimizations [7-9]. Goel et al. [10] developed weighted surrogate model using RSA, Kriging and neural network models. They concluded that weighted average surrogate model is more reliable prediction method than individual surrogates.

In this paper, transonic axial compressor blade shape optimization has been performed using multiple surrogate models and three-dimensional RANS (Reynolds-averaged Navier-Stokes equations) analysis. Sweep, lean and skew of the blade are selected as design variables. The flow field is resolved, the prediction capabilities of different models are compared, and advantages of multiple surrogate models are highlighted.

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2. PROBLEM DESCRIPTION AND NUMERICAL PROCEDURE

Blade Geometry and Flow Parameters

NASA rotor 37 [11], an axial-flow compressor rotor with a low-aspect ratio, is used for blade shape optimization in the present study. The rotor tip clearance is 0.356 mm (0.45 percent span). The measured choking mass flow rate is 20.93 kg/s, which corresponds to 103.67% of the design flow rate (20.19kg/s).

The three-dimensional thin-layer Navier-Stokes and energy equations are solved on body-fitted grids using an explicit finite-difference scheme. Artificial dissipation terms have been added to resolve shocks. The algebraic turbulence model of Baldwin and Lomax [12] has been employed to estimate the eddy viscosity.

A composite grid system with structured H-, C-, and O-type grids is adopted to represent the complicated configuration of the axial compressor. H-type grid consists of $60\times36\times63$ grids (in the streamwise, pitchwise and spanwise directions, respectively) and is introduced for the inlet flow region. C-type grid consists of $350\times46\times63$ grids, and is used for the blade passage. The grid embedded in the tip clearance consists of $182\times13\times13$ grids. The whole grid system has about 1,181,000 grid points.

Mach numbers in each direction, total pressure, and total temperature are given at the inlet. At exit, the hub static pressure ratio has been specified, and the radial equilibrium equation is solved along the blade span. A periodic tip clearance model is used to resolve the tip clearance flow explicitly. No-slip and adiabatic wall conditions are used at all the wall boundaries. For reducing the computational load, flow field in a single blade passage is simulated by applying periodic boundary condition in the tangential direction.

Objective Functions and Design Variables

In the present study, efficiency, total temperature ratio and total pressure ratio are selected as objective functions of the shape optimization of the rotor blades. In order to improve the overall efficiency, the objective function is defined as:

$$F_1 = 1 - \eta_{ad},$$
$$\eta_{ad} = \frac{\left(P_{0exit}/P_{0inlet}\right)^{(\gamma-1)/\gamma} - 1}{T_{0exit}/T_{0inlet} - 1} \quad (1)$$

where, η_{ad} is adiabatic efficiency, P_0 and T_0 are total pressure and total temperature, respectively. Total temperature ratio (F_2) and total pressure ratio (F_3) are also tested as objective functions for shape optimization, as follows:

$$F_2 = T_{0\,exit}/T_{0\,inlet} \quad (2)$$

where, the location of inlet is not same as in equation (1), and corresponds to the location of inlet of C-grid. The optimizations are to increase the efficiency (therefore to reduce efficiency based objective function, F_1) and total temperature ratio.

Three design variables; one for sweep, another for lean, and the other for skew. Blade sweep, α is defined at the rotor tip, and normalized by the axial tip chord (= 27.77 mm). The airfoil sections are moved towards downstream direction for positive sweep (α). Blade lean is defined as the movement of aerofoil normal to the chord line. Here, lean, β is taken as positive if the aerofoil sections are moved towards blade suction surface side. Lean (β) is taken as zero at hub and is linearly connected from hub to tip and. Skew angle (γ) is defined only at rotor tip. If the blade bends towards pressure surface side, skew angle is taken as positive. The skew line is defined as a second order polynomial. The constants and the coefficients are found by the constraints; skew angle is zero at both hub and mid span, and γ at tip of the blade.

The boundary and the mid points of the range are prescribed for each design variable as in Table 1.

3. OPTIMIZATION METHODOLOGY

In optimization procedure, initially the variables are selected, and the design space is decided for improvement of system performance. Using DoE, the design points are selected, and at these design points the objective functions are calculated using flow solver. Next step is to construct the surrogates and find optimal points.

Table 1 Design ranges of blade sweep, lean, and skew

Variables	Lower Bound	Middle	Upper Bound
Sweep (%)	0.0	12.6	25.2
Lean (%)	-3.6	-1.8	0.0
Skew(radian)	0.0	0.05	0.1

Surrogate Models

Response surface approximation (RSA) method [13] is curve fitting by regression analysis. If the regression coefficients are β's, the polynomial function for response surface analysis becomes:

$$\hat{F} = \beta_0 + \sum_{j=1}^{n}\beta_j x_j + \sum_{j=1}^{n}\beta_{jj}x_j^2 + \sum \sum_{i\neq j}^{n}\beta_{ij}x_i x_j \quad (3)$$

where n is number of design variables, and x's are the design variables.

For radial basis neural network (RBNN) [14], the design parameters are spread constant (SC) and a user defined Error Goal (EG). In MATLAB [15], *newrb* is the function for RBNN network design.

In Kriging model [16], linear polynomial function with Gauss correlation function is used for model construction. Kriging postulation is the combination of global model and departures of the following form:

$$\hat{F}(x) = f(x) + Z(x) \quad (4)$$

where $\hat{F}(x)$ represents the unknown function, $f(x)$ is the known function of x, and $Z(x)$ is the realization of a stochastic process with mean zero and non-zero covariance.

For two weighted average models propose by Goel et al. [12], WTA2 and WTA3, predicted response is defined as follows:

$$\hat{F}_{wt.avg}(x) = \sum_{i}^{N_{SM}} w_i(x)\hat{F}_i(x) \quad (5)$$

where, N_{SM} is the number of basic surrogate models used to construct weighted average model. i^{th} surrogate model at design point x produces weight $w_i(\mathbf{x})$, and $\hat{F}_i(x)$ is the predicted response by i^{th} surrogate model.

Weights are decided using the concept that the surrogate which produces high error is having low weight, and thus low contribution to the final weighted average surrogate. The error is calculated using generalized mean square cross-validation error (GMSE) or PRESS (for RSA). In this work, a global weights selection scheme based on global data-based measure of goodness is used for error calculation.

WTA2 or "best PRESS model" is the scheme where the best surrogate is selected for optimization. This is regarded as weighted scheme, since the weight associated with minimum PRESS is unity, and other weights are made zero.

In WTA3 scheme, weighting scheme calculation is based on the theory that weights should reflect our confidence in the surrogate model and weights should filter out adverse effects of the model which represents the data well but performs poorly in unexplored regions. In this case weights are calculated as:

$$w_i^* = \left(E_i + \alpha E_{avg}\right)^{\beta}, \quad w_i = w_i^* \Big/ \sum_i w_i^*$$

$$E_{avg} = \sum_{i=1}^{N_{SM}} E_i \Big/ N_{SM} \; ; \beta < 0, \alpha < 1$$

$$E_i = \sqrt{GMSE_i}, i = 1,2,\ldots, N_{SM} \tag{6}$$

Two constants α and β are calculated on the basis of sensitivity of parameters, and these values are decided as α = 0.05 and β = −1. GMSE is the leave-one-out cross validation (or PRESS in polynomial Response Surface Approximation). The surrogate formulation is same as equation (6).

In this work, design points are selected using three-level fractional factorial design. And, number of design points for construction of models with three design variables is reduced to 24 by using D-optimal design [13]. Evaluations of the objective functions at these design points are carried out by three-dimensional RANS analysis. At the next step, surrogates are constructed using objective function values at design points. Optimal points are searched by Sequential Quadratic Programming (SQP) from these constructed surrogates.

4. RESULTS AND DISCUSSION

Tables 2 and 3 show different surrogate models and their optimization results. The optimal points given in these tables are normalized, and have the values between 0 and 1. Tables 2(a) and 3(a) compare the predictions of optimal objective function values by different surrogate models with those by RANS calculation at corresponding optimal design points. It is shown that in this aspect Kriging model is the worst as the error associated with this model is the highest in all of the cases. In case of F_1, RBNN is the best, and WTA3 follows the next. But, in case of F_2, WTA2 (RSA) is the best, and RBNN follows the next. However, in the RANS calculations, RBNN shows the best overall performance to predict the optimum points, and WTA3 also shows reasonable predictions.

The Tables 2(b) and 3(b) show the cross validation errors and calculated weights for the weighted average models. The results show that the RSA gives the best performance among three individual surrogates. Thus, only the weight associated with RSA is unity.

Table 2(a) Optimal points and respective predicted and calculated values of objective function, F_1.

Models	α_{opt} *(sweep)*	β_{opt} *(lean)*	γ_{opt} *(skew)*	$F_{predicted}$	$F_{calculated}$	$F_{calculated} - F_{predicted}$	*Reduction in* F_1 *(%)*
WTA2	0.401	0.748	0.632	0.0993	0.1015	2.20E-03	10.57
WTA3	0.442	0.905	0.651	0.0995	0.1010	1.50E-03	11.01
RBNN	0.411	1.000	0.615	0.0994	0.1008	1.40E-03	11.19
RSA	0.401	0.748	0.632	0.0993	0.1015	2.20E-03	10.57
KRG	0.465	0.971	0.635	0.0990	0.1018	2.80E-03	10.31

Table 2(b) Weights for different models to construct weighted average models for F_1.

MODEL	*Cross Validation Errors,* E_{cv}	*Weight (WTA2)*	*Weight (WTA3)*
RBNN	3.09E-03	0	0.322
RSA	2.40E-03	1	0.409
KRG	3.71E-03	0	0.270

Table 3(a) Optimal points and respective predicted and calculated values of objective function, F_2.

Models	α_{opt} *(sweep)*	β_{opt} *(lean)*	γ_{opt} *(skew)*	$F_{predicted}$	$F_{calculated}$	$F_{calculated}-F_{predicted}$	*Total Temp. ratio ncreased %*
WTA2	0.000	0.769	0.165	1.2744	1.2748	4.00E-04	0.22
WTA3	0.000	0.676	0.044	1.2730	1.2749	1.90E-03	0.22
RBNN	0.000	0.903	0.237	1.2738	1.2748	1.00E-03	0.22
RSA	0.000	0.769	0.165	1.2744	1.2748	4.00E-04	0.22
KRG	0.000	1.000	0.392	1.2705	1.2738	3.30E-03	0.14

Table 3(b) Weights for different models to construct weighted average models for objective function, F_2.

MODEL	*Cross Validation Errors, E_{cv}*	*Weight (WTA2)*	*Weight (WTA3)*
RBNN	7.39E-03	0	0.359
RSA	6.72E-03	1	0.393
KRG	1.09E-02	0	0.248

5. CONCLUSIONS

Two multiple surrogate models as well as three basic models are tested in the optimization of a transonic axial compressor blade. The weighted averaged multiple surrogate models shows good overall performance to predict the optimum values of the objective functions in comparison with RANS calculations, while KRG model shows the worst. However, in the RANS calculations, RBNN predicts the best optimum points, and multiple surrogate model, WTA3 also shows reasonable predictions. Over 11 % of relative increase in blade efficiency is obtained with the best model.

It is found from the results that the advantage of using multiple surrogate models is to protect the designer from the results of selecting a poor surrogate. And, these models also give the advantage of increasing the chance of getting substantial improvement by generating multiple optima. The present approach can help address the multi-objective design on a rational basis with quantifiable cost-benefit analysis.

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