이산형 H∞ 필터를 이용한 고정밀 GPS 모듈의 개발

The Development of Accurate GPS Module Using Discrete-Time H∞ Filter

원황휴*, 원필롱**, 이상훈**, 박옥득**, 김현수**, 김한실***
Nguyen Hoang Hieu*, Nguyen Phi Long**, Lee Sang-Hoon**, Park Ok-Deuk**, Kim Hyun-Su**, Kim Han-Sil***

Abstract – In this paper, we present the traditional GPS Position-Velocity (PV) model to apply for both Discrete-Time Kalman Filter and Discrete-Time H∞ Filter. The positioning algorithms of both filters are proposed for a stand-alone low-cost GPS module to increase its accuracy. For disturbance cancellation, the Kalman Filter requires the statistical information about process and measurement noises while the H∞ Filter only requires that these noises are bounded. Experiments show that with the same measurement data, H∞ Filter gives us better positioning results compared with Least-Squared method and Kalman Filter.

Key Words : Kalman Filter, H∞ Filter, GPS, discrete-time system, algorithm

1. INTRODUCTION

From the GPS receiver module, we obtain the raw measurement data (satellites' positions, satellites' velocities, pseudo-range and pseudo-range rate) required to compute the receiver position and velocity. However, these measurements are affected by several error sources such as measurement errors at recerver (0.5-1m), Ephemeris error (15-20m), Ionopheric delays (20-50m), Tropospheric delays (2-20m), Multipath (3-5m), Satellite clock error (1-5m) and other errors (<1m)... In summary, the total inaccuracy introduced to GPS positioning is about 100m. Although a part of these errors (consist of ionospheric, tropospheric and satellite clock bias) can be compensated partially using the information in broadcasted Almanac data, the remains still cause a big error. That is the reason for implementing estimate algorithms such as Least-squared, Extended Kalman Filter or H∞ Filter.

The Kalman Filter is the optimal filter when the measurement noise and process noise are zero mean white processes with known statistics. When there is significant uncertainty in the power spectral density of the exogenous signals a new measure of performance, referred to as H∞ -norm, is sometimes useful. In this paper, the H∞ Filter presented in [4] is applied to compare with the Kalman Filter and Original Least-Squared Algorithm. The structure of this paper is as follow. The discrete-time state space variable GPS model is described first; then the Kalman Filter and H∞ Filter Algorithm for that model; at the end are experiment results and conclusion.

2. SYSTEM MODELLING

2.1 Process model

Assume that the receiver only moves with nearly constant velocity, then the GPS dynamic process can be modeled as PV model in which the velocity is a random walk process, the acceleration and other disturbances are considered as process noise.

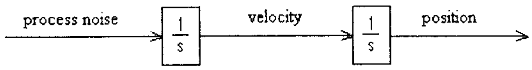

Fig. 1. Integrated random walk model for GPS receiver.

Besides the dynamic of the receiver itself, we must model the receiver clock error because it also distributes to the system dynamic process. When modeled correctly, the clock error can be solved and then it has no affect on positioning accuracy. Here we use a typical 2-states random-process to model the receiver clock error as in [2].

We define the receiver state-space variable included the three dimensions ECEF (Earth Centered Earth Fixed) positions and velocities combined with the receiver clock bias and clock drift.

저자 소개
* 원황휴 : 蔚山大學校 自動車船泊技術大學院 碩士課程
** 원필롱, 이상훈, 박옥득, 김현수:
蔚山大學校 電氣電子情報工學部 碩士課程
*** 김한실 : 蔚山大學校 電氣電子情報工學部 教授

$$x = \begin{bmatrix} X_r & \dot{X}_r & Y_r & \dot{Y}_r & Z_r & \dot{Z}_r & c\delta t & c\dot{\delta t} \end{bmatrix}^T \quad (1)$$

Where X_r, Y_r, Z_r are ECEF positions of receiver; $\dot{X}_r, \dot{Y}_r, \dot{Z}_r$ are the corresponding velocities; $c\delta t, c\dot{\delta t}$ are receiver clock bias and clock drift (in m and m/s).

The overall dynamic equation:

$$\dot{x} = Fx + w \quad (2)$$

with

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad w = \begin{bmatrix} 0 \\ u_2 \\ 0 \\ u_4 \\ 0 \\ u_6 \\ u_7 \\ u_8 \end{bmatrix} \quad (3)$$

Where u_2, u_4, u_6, u_7, u_8 are system driving noise.

The corresponding discrete-time dynamic equation of the above differential equation is:

$$x_{k+1} = \Phi x_k + w_k \quad (4)$$

Where x_k is the process state variable at time t_k, Φ is the transition matrix (relate x_k to x_{k+1} in the absence of a forcing function), w_k is the system driving noise at time t_{k+1} due to the presence of the noise input during (t_k, t_{k+1}) interval.

The state transition matrix is given by:

$$\Phi = e^{F\Delta t} \approx I + F\Delta t = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

2.2 Measurement model

With the state variable defined above, the receiver must perform two measurements from at least four GPS satellites: the pseudoranges and pseudorange-rates.

Pseudoranges are the distances from GPS satellites to the receiver computed by the receiver GPS module. The equation relating the measurement of pseudoranges to the receiver position is:

$$p_i = \sqrt{(X_r - X_i)^2 + (Y_r - Y_i)^2 + (Z_r - Z_i)^2} + c\delta t + v_{pi} \quad (6)$$

i = 1, 2, ... n; $n \geq 4$

Where p_i is the measured pseudorange from satellite i, (X_i, Y_i, Z_i) is position of satellites i, v_{pi} is pseudorange measurement noise that affects p_i.

The equation relating pseudorange-rates to the receiver velocity is:

$$D_i = \frac{(X_r - X_i)(\dot{X}_r - \dot{X}_i) + (Y_r - Y_i)(\dot{Y}_r - \dot{Y}_i) + (Z_r - Z_i)(\dot{Z}_r - \dot{Z}_i)}{\sqrt{(X_r - X_i)^2 + (Y_r - Y_i)^2 + (Z_r - Z_i)^2}} + c\dot{\delta t} + v_{Di}$$

i = 1, 2, ... n; $n \geq 4$ (7)

Where D_i is the pseudorange-rate from satellite i, $(\dot{X}_r, \dot{Y}_r, \dot{Z}_r)$ is velocity of satellite i, v_{Di} is pseudorange-rate measurement noise that affect D_i.

In case of n satellites are tracked, we define the measurement vector:

$$z_k = \begin{bmatrix} p_1 & p_2 & \cdots & p_n & D_1 & D_2 & \cdots & D_n \end{bmatrix}^T \quad (8)$$

The linearized measurement equation is as follow:

$$\Delta z = z_k - h(x_k^*) = H_k \Delta x + v_k \quad (9)$$

Where $\Delta x = x_k - x_k^*$ is the incremental quantity in state variable, v_k is the measurement noise vector (include pseudorange and pseudorange-rate measurement noise), H_k is the linearized measurement matrix as in [2].

$$H_k = \left[\frac{\partial h}{\partial x}\right]_{x = x^*} = \begin{bmatrix} \frac{\partial p_1}{\partial X} & 0 & \frac{\partial p_1}{\partial Y} & 0 & \frac{\partial p_1}{\partial Z} & 0 & 1 & 0 \\ \cdots \\ \frac{\partial p_n}{\partial X} & 0 & \frac{\partial p_n}{\partial Y} & 0 & \frac{\partial p_n}{\partial Z} & 0 & 1 & 0 \\ \frac{\partial D_1}{\partial X} & \frac{\partial D_1}{\partial \dot{X}} & \frac{\partial D_1}{\partial Y} & \frac{\partial D_1}{\partial \dot{Y}} & \frac{\partial D_1}{\partial Z} & \frac{\partial D_1}{\partial \dot{Z}} & 0 & 1 \\ \cdots \\ \frac{\partial D_n}{\partial X} & \frac{\partial D_n}{\partial \dot{X}} & \frac{\partial D_n}{\partial Y} & \frac{\partial D_n}{\partial \dot{Y}} & \frac{\partial D_n}{\partial Z} & \frac{\partial D_n}{\partial \dot{Z}} & 0 & 1 \end{bmatrix} \quad (10)$$

3. KALMAN FILTER & H∞ FILTER ALGORITHM

We have the discrete-time time-varying state-space model for GPS system :

$$x_{k+1} = \Phi x_k + w_k$$
$$\Delta z_k = H_k \Delta x + v_k \quad (11),(12)$$

Both Kalman Filter and H Filter described in this paper are used to solve state-space variable of the above state-space model.

3.1 Kalman Filter Algorithm

To apply Kalman Filter and solve for x_{k+1}, we assume that w_k and v_k are zero mean independent white-noise with the covariance matrices given by:

$$Q_k = E(w_k w_k^T)$$
$$R_k = E(v_k v_k^T) \quad (13),(14)$$

Where E denotes the expectation, Q_k is the covariance matrix of the process noise w_k, R_k is the covariance matrix of the measurement noise v_k.. The precise value of Qqk and rk must be known to obtain the optimal solution of Kalman Filter.

The Kalman Filter recursive equations are given below:

- Compute Kalman Gain:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R)^{-1} \quad (15)$$

- Update state estimate:

$$\hat{x}_k = \hat{x}_k^- + K_k [z_k - \hat{z}_k^-] \quad (16)$$

- Compute error covariance for updated estimate:

$$P_k = (I - K_k H_k) P_k^-$$ (17)

- Project ahead:

$$\hat{x}_{k+1}^- = \Phi.\hat{x}_k$$ (18)

$$P_{k+1}^- = \Phi P_k \Phi^T + Q$$ (19)

$$\hat{z}_{k+1}^- = h(\hat{x}_{k+1}^-)$$ (20)

To enter the Kalman Filter recursive loop, we must provide it with the initial value $\widehat{x_0^-}$ (the initial state vector), $\widehat{z_0^-}$ (the initial predict measurement) and P_0^- (the initial error covariance). These selections are very important to Kalman Filter to assure its convergence and some suggests can be found in [3].

3.2 H∞ Filter Algorithm

Different from Kalman Filter, the H∞ Filter only requires that the process noise and measurement noise be bounded. The objective is to estimate x_k that satisfy the H∞ Filter criterion:

$$\sup J = \sup \frac{\sum_{k=0}^{N-1} \|x(k) - \hat{x}(k)\|_{\hat{U}}^2}{\|x(0) - \hat{x}(0)\|_{\hat{P}_0^{-1}}^2 + \sum_{k=0}^{N-1} \left(\|w(k)\|_{\hat{Q}^{-1}}^2 + \|v(k+1)\|_{\hat{R}^{-1}}^2 \right)} < \frac{1}{\gamma} = \gamma$$ (21)

Where γ is a pre-specified value; ||.|| represents a vector norm; $\hat{U}, \widehat{P_0^{-1}}, \widehat{Q^{-1}}, \widehat{R^{-1}}$ are matrices used in the weighted norms in J, x(0) is the initial guesstimate.

The main H∞ Filter equations are proposed in [4]. To apply to our GPS system (10) and (11), we perform linearizing and obtain the following equations:

$$\theta = (\hat{Q}^{-1} + H_k^T \hat{R}^{-1} H_k)^{-1}$$ (22)

$$\phi_{11} = \Phi - \theta H_k^T \hat{R}^{-1} H_k \Phi$$ (23)

$$\phi_{12} = \theta$$ (24)

$$\phi_{21} = \hat{\gamma}\hat{U} - \Phi^T H_k^T \hat{R}^{-1} H_k \Phi + \Phi^T H_k^T \hat{R}^{-1} H_k \theta H_k^T \hat{R}^{-1} H_k \Phi$$ (25)

$$\phi_{22} = \phi_{11}^T$$ (26)

$$P_{k+1} = \phi_{11} P_k (I - \phi_{21} P_k)^{-1} \phi_{11}^T + \theta, \qquad P(0) = 0$$ (27)

The H∞ Filter gain is computed as follow:

$$K_{k+1} = P_{k+1} H_k^T \hat{R}^{-1}$$ (28)

The H∞ Filter update equation:

$$\hat{x}_{k+1} = \hat{x}_{k+1}^- + K_{k+1} \left[z_{k+1} - \hat{z}_{k+1}^- \right]$$ (29)

Where $\hat{x}_{k+1}^- = \Phi.\hat{x}_k$ and $\hat{z}_{k+1}^- = h(\hat{x}_{k+1}^-)$ (30),(31)

Note that when $\gamma = \infty$, P_{k+1} is the same as the Kalman Filter error covariance matrix and the gain K_{k+1} is also the same as the Kalman Filter gain.

4. EXPERIMENT RESULTS

To perform experiment, we fix the receiver's antenna at an already known position in Ulsan University campus (the 30 minutes average position coordinates from Novatel GPS module). Using a low cost GPS module made by Kiryung company, we collect raw measurement data as input to our filters running on a computer. Results between the filters are presented in Figure 2.

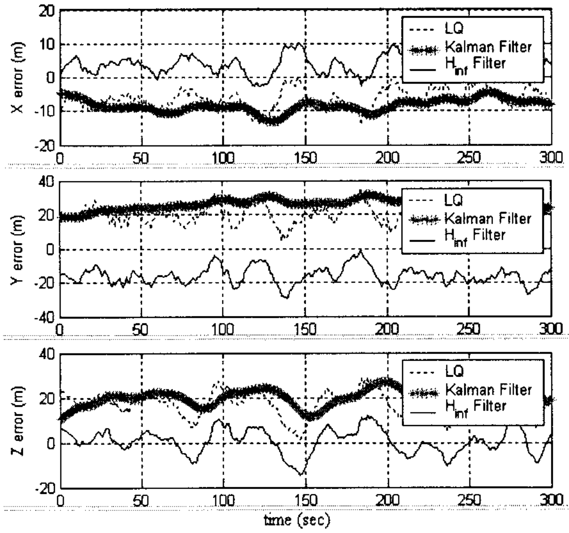

Fig. 2. Experiment results

5. CONCLUSION

An application of H∞ Filter for GPS system has been developed and compared with original Least-Squared method and Kalman Filter. The results show that the result from H∞ Filter is rather good. Especially in case when we have no ideas about the noise statistical characteristic, the H∞ Filter is a reasonable choice because of its' robustness and ease in implementation.

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