

## Formulation and Identification of an Anisotropic Constitutive Model for Describing the Sintering of Stainless Steel Powder Compacts

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### Abstract

*Anisotropic constitutive equations for sintering of metal powder compacts have been formulated from a linear viscous transversely-isotropic model in which an anisotropic sintering stress has been introduced to describe free sintering densification kinetics. The identification of material parameters defined in the model, has been achieved from thermo-mechanical experiments performed on 316L stainless steel warm-compacted powder in a dilatometer allowing controlled compressive loading.*

**Keywords :** Sintering, compacts, anisotropy, modelling, constitutive equation

### 1. Introduction

The finite element simulation of the macroscopic behaviour of a complex green component during sintering can allow predicting its dimensional changes. Most constitutive models proposed for this purpose are isotropic. They are not relevant for describing the behaviour of metal powder compacts, which is usually strongly anisotropic due to prior die compaction. This anisotropy can be assumed to be of the transversely isotropic type with the pressing direction as the singular direction. In this paper, anisotropic constitutive equations for sintering have been formulated from a linear viscous transversely-isotropic model in which an anisotropic sintering stress has been introduced to describe free sintering densification kinetics. To identify the material parameters defined in the model, an extensive set of thermo-mechanical experiments has been performed on 316L stainless steel warm-compacted powder in a dilatometer allowing controlled compressive loading.

### 2. Formulation of Constitutive Equation

In classical isotropic constitutive equation proposed for sintering [1-2], the viscous strain rate tensor  $D_{ij}^v$  is written:

$$D_{ij}^v = \frac{\sigma'_{ij}}{2G} + \frac{(\sigma_m - \sigma_s)}{3K} \delta_{ij}$$

where  $\sigma'_{ij}$  is the stress deviator tensor,  $\sigma_m$  is the mean stress,  $\sigma_s$  is the sintering stress,  $\delta_{ij}$  is the unit tensor,  $K$  and  $G$  are the bulk and shear viscosities, respectively. To formulate an equation describing transverse isotropy we start from the general tensorial representation of Boehler [3]:

$$\underline{\underline{D}} = \alpha_1 \underline{\underline{I}} + \alpha_2 \underline{\underline{M}}_3 + \alpha_3 \underline{\underline{\sigma}} + \alpha_4 (\underline{\underline{M}}_3 \underline{\underline{\sigma}} + \underline{\underline{\sigma}} \underline{\underline{M}}_3)$$

$\underline{\underline{M}}_3$  is the structure tensor ( $\underline{\underline{M}}_3 = \underline{\underline{v}}_3 \otimes \underline{\underline{v}}_3$ ), where  $\underline{\underline{v}}_3$  is a unit vector in the anisotropy direction). For a transversely isotropic material,  $\alpha_i$  parameters can be expressed with only 5 independent material parameters:

$$\begin{cases} \alpha_1 = F \operatorname{tr}(\underline{\underline{\sigma}}) + (D - F) \operatorname{tr}(\underline{\underline{M}}_3 \underline{\underline{\sigma}}) & (5) \\ \alpha_2 = (D - F) \operatorname{tr}(\underline{\underline{\sigma}}) + (C + A - 2D - 4G) \operatorname{tr}(\underline{\underline{M}}_3 \underline{\underline{\sigma}}) & (6) \\ \alpha_3 = A - F & (7) \\ \alpha_4 = 2G - A + F & (8) \end{cases}$$

It is usual to replace these parameters to more comprehensive ones, such as viscosities and viscous Poisson's ratios:

$$A = \frac{1}{\eta_T} \quad C = \frac{1}{\eta_A} \quad D = \frac{-\nu_{AT}^{vp}}{\eta_A} \quad F = \frac{-\nu_{TT}^{vp}}{\eta_T} \quad G = \frac{1}{\eta_{AT}} \quad (9)$$

Thus  $\eta_A$  and  $\eta_T$  are the viscosities in axial and transverse direction respectively,  $\eta_{AT}$  is the shear viscosity,  $\nu_{AT}^{vp}$  and  $\nu_{TT}^{vp}$  are viscous Poisson's ratios. For describing the free sintering deformation, we add a sintering stress tensor  $\underline{\underline{\sigma}}^s$  to the external stress tensor. The final expression of the constitutive equation is thus

$$\underline{\underline{D}}^v = \alpha_1 \underline{\underline{I}} + \alpha_2 \underline{\underline{M}}_3 + \alpha_3 (\underline{\underline{\sigma}} + \underline{\underline{\sigma}}^s) + \alpha_4 (\underline{\underline{M}}_3 (\underline{\underline{\sigma}} + \underline{\underline{\sigma}}^s) + (\underline{\underline{\sigma}} + \underline{\underline{\sigma}}^s) \underline{\underline{M}}_3) \quad (10)$$

The components of the sintering stress tensor  $\underline{\underline{\sigma}}^s$  can be expressed from the free sintering strain rate in axial and transverse directions,  $\dot{\epsilon}_A$  and  $\dot{\epsilon}_T$ , and from the viscosity parameters ( $\eta_A$  and  $\eta_T$ ). To have a complete model we should add a thermo-elastic strain rate. The elastic part can be neglected whereas the thermal term is of course

absolutely required. It can be assumed to be isotropic (thus a single material parameter is required) or transversely isotropic if one wants to take into account the anisotropy observed during heating (then two thermal expansion coefficients are required).

The model finally includes 8 material parameters, which are *a priori* functions of the temperature, T, the actual relative density,  $\rho$  and the green density,  $\rho_0$ . They have been identified from the experiments performed on stainless steel warm-pressed compacts of relative green density 0.9.

### 3. Model Parameter Identification

The standard sintering conditions for 316L steel powder compacts have been chosen to be the following : heating rate 30°C/min, delubrication plateau at 700°C during 15 min, sintering plateau at 1250°C during 30min, 10% H<sub>2</sub> / 90% Ar atmosphere. Dilatometry measurements have been performed in axial and transverse directions (Fig. 1). It appears that density changes due to thermal expansion are larger than the changes due to sintering. Thus the actual relative density seems not to be the right parameter to characterize the progress of sintering. Hence we define the following parameter for the description of sintering advancement:

$$\chi = \rho - \alpha \Delta T - \rho_0$$

where  $\alpha$  is the bulk thermal expansion coefficient and  $\Delta T$  is the difference between the actual temperature and room temperature.

Dilatometry tests also proved that the final anisotropy was mainly due to the axial shrinkage suddenly occurring around 1000° (Fig. 1), which has been interpreted as the effect of contact pore closing [4]. The deformation is thus supposed to be due to two mechanisms:

- Mechanism 1: packing pore shrinkage, which is almost isotropic ;
- Mechanism 2: contact pore closing, which results only in axial shrinkage.

Then the sintering advancement parameter,  $\chi$ , is divided in two terms,  $\chi = \chi_1 + \chi_2$ , where  $\chi_1$  describes packing pore changes and  $\chi_2$  describes contact pore changes.

The free sintering strain rate in axial direction is next expressed as the addition of the contributions of both mechanisms, whereas it is assumed that only Mechanism 1 contributes to the transverse strain rate:

$$\dot{\epsilon}_A = \dot{\epsilon}_A^1(\chi^1, T) + \dot{\epsilon}_A^2(\chi^2, T) \quad (16)$$

$$\dot{\epsilon}_T = \dot{\epsilon}_T^1(\chi^1, T) \quad (17)$$

An analytical expression has been fitted to each function by using the strain rates changes deduced from dilatometry measurements:

$$\dot{\epsilon}_A^1(T, \chi^1) = \Omega_A^1(T) \cdot (\chi^1)^{n_A^1(T)} \quad (18)$$

$$\dot{\epsilon}_A^2(T, \chi^2) = \Omega_A^2(T) \cdot (\chi^2)^{n_A^2(T)} \quad (19)$$

$$\dot{\epsilon}_T^1(T, \chi^1) = \Omega_T^1(T) \cdot (\chi^1)^{n_T^1(T)} \quad (20)$$

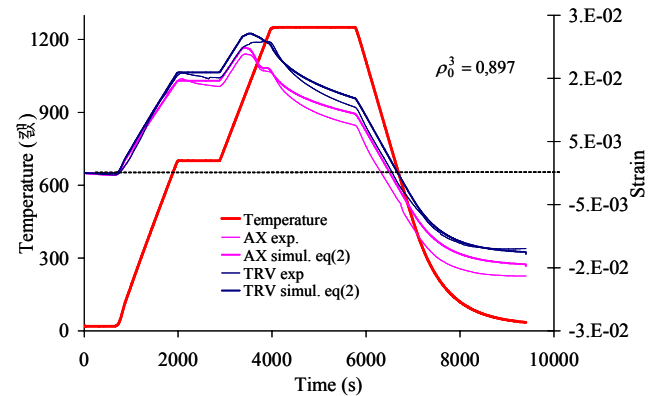
The deformation due to packing pore closing has been supposed to be isotropic during heating.

Viscosity changes as function of T and  $\chi$  have been fitted with the following relation:

$$\eta_{A,T} = \phi_{A,T}(T) \cdot (\chi)^{\gamma_{A,T}(T)}$$

Viscous Poisson's ratios, which are difficult to measure, are assumed to be constant and equal to 0.3.

As a control of model parameter identification, the strains during a standard sintering cycle has been calculated with the equations described above and compared with dilatometry measures (Fig. 1). The adjustment is rather satisfactory. In particular the drop in axial direction due to contact pore closed is correctly described. The deformation during sintering plateau is not perfectly modelled, which results in a significant difference in final axial strain.



**Fig. 1. Comparison of experimental and predicted strains during free sintering test.**

### 4. References

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