Determination of joint production and delivery policy with multiple production lines for multiple products

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Abstract

Satisfying the customer orders with a short lead-time is one of the essential and competitive factors for business units under a mass customization environment. To shorten the lead-time for fulfilling various orders entails the production capacity expansion and efficient operational policy. Most business firms utilize the multiple production lines or facilities to cope with this business and manufacturing environment by making the manufacturing and distribution more flexible. In this study, we introduce the operational problem determining the joint production and delivery policy in an environment where multiple products are manufactured with more than one production lines. Also, we propose the heuristic solution approach for determining the product-line selection and joint lot size for this problem.

1. Introduction

Business environmental factors such as demand pattern, new product introduction (NPI), supply capacities which change dynamically in a continuous manner affect the supply chain strategy and operational procedure. To cope with these dynamic business environment, the flexible operation of production resource is one of the important managerial issues in global manufacturing firms which have multiple production sites around world. Moreover, the decision-making issues related to production management should be aligned with the distribution policy to deliver the customer's economic value. A lot of studies for deriving the optimal production - delivery policy have some limitations in that they assumed a single production unit, for example, plant, production line, machine, etc(Please refer to the papers [1]~[5] for readers' information.).

information. In this paper, we suppose the following business situation: This manufacturing firm utilizes multiple production site for multiple types of products. The global operation center tries to set-up the supply chain plan for efficient multi-plant operations. The basic decision problems are 1) which plants are chosen to cope with the demands? and then 2) How can we efficiently apportion the given demand among multiple selected plants?

2. Problem

As mentioned in previous section, we assume the global operation center whose major role is to efficiently operates the multiple production sites which are globally located. So, by introducing the following parameters, our decision problem can be formally described(Note that the holding costs for finished goods and raw materials measured by each unit per year.):

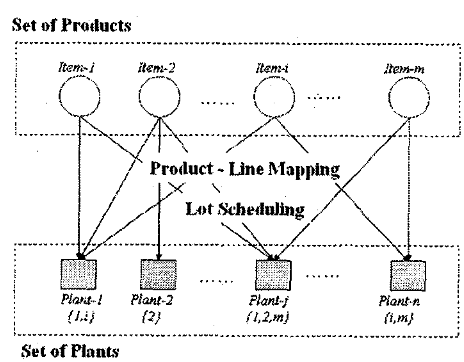

Figure.1 The Structure of Supply Chain

List of Parameters:

i: index for item, $i = 1, K, m$.

j: index for plant, $j = 1, \ldots, n$.

D_i: demand rate for item i .

A_i^R: ordering cost for raw material i.

H_i^R: holding cost for raw material i.

$p_{i,j}$: production rate at plant j for item i .

$d_{i,j}$: transfer rate from plant j for item i to central warehouse in units per year.

$S_{i,j}$: manufacturing setup cost at plant j.

$h_{i,j}$: manufacturer's holding cost.

A_i^W :material handling cost for delivery set-up.

H_i^W :inventory holding cost at warehouse.

A_i^C : retailer's ordering cost for item i.

H_i^C :retailer's holding cost item i.

For the relevance of this problem, two conditions, $p_j \geq d_j (j = 1,\ldots,n)$, $\sum_{j=1}^{n} d_{i,j} \geq D_i$ $(i = 1,\ldots,m)$, and $\sum_{i=1}^{m}(D_i / \sum_{j=1}^{n} p_{i,j}) \leq 1$ must be satisfied. And then, as roughly stated above, the following factors are determined according to the formalized manner. We assume that the deliveries of finished items to customers are executed while the production is running so as to minimize the inventories for finished items.

Decision Variables:

T: cycle time(years), $T = Q_i/D_i = k_i(q_i/D_i), \forall i$.

k_i: delivery frequency for item i during a single production period where $\mathbf{k} = (k_1,\ldots,k_m)$.

$\lambda_{i,j}$: apportioning ratio for item i at plant j where $\sum_{j=1}^{n} \lambda_{i,j} = 1, \forall i$.

$\boldsymbol{\lambda}_i$: a vector of apportioning ratio for item i, where $\boldsymbol{\lambda}_i = (\lambda_{i,1},\ldots,\lambda_{i,n})$, $\boldsymbol{\Lambda} = (\boldsymbol{\lambda}_1,\ldots,\boldsymbol{\lambda}_m)$.

ς_i: a vector of selected plants, where $\varsigma_i = (\varsigma_{i,1},\ldots,\varsigma_{i,n})$, $\varsigma_{i,j}$=1 if $\lambda_{i,j}$>0, otherwise $\varsigma_{i,j}$=0.

3. Solution Approach

The total relevant cost for operating the supply chain system can be represented by

$$TRC(T,\mathbf{k},\boldsymbol{\Lambda}) = \sum_{i=1}^{m}(\alpha_{1,i} + \alpha_{2,i}k_i)/T + T\sum_{i=1}^{m}\left[(\beta_{1,i} + \beta_{2,i}/k_i) - \sum_{j=1}^{n}\theta_{i,j}\lambda_{i,j}^2\right] \quad (1)$$

where

$$\alpha_{1,i} = (A_i^R + \sum_{j=1}^{n} S_{i,j}\varsigma_{i,j}), \alpha_{2,i} = (A_i^W + A_i^C)$$
$$\beta_{1,i} = H_i^W D_i/2, \beta_{2,i} = (H_i^W + H_i^C)D_i/2$$
$$\theta_{i,j} = \left((h_{i,j} - H_i^R)/2p_{i,j} + (H_i^W - h_{i,j})/2d_{i,j}\right)D_i^2$$

Hence, our problem can be re-formalized as follows:

$$\textit{Minimize } TRC(T,\mathbf{k},\boldsymbol{\Lambda}) \quad (2.1)$$

subject to

$$\sum_{j=1}^{n} \lambda_{i,j} = \lambda_{i,a} (\leq 1), i = 1,\ldots,m \quad (2.2)$$
$$0 \leq \lambda_{i,j} \leq \overline{\lambda_{i,j}}, i = 1,\ldots,m; j = 1,\ldots,n \quad (2.3)$$

, where $\overline{\lambda_{i,j}} \leq d_{i,j}/D_i$

After performing some algebraic manipulation, we get the following simplified problem, that is, the above problem is transformed into the problem whose unique decision variable is the apportioning policy, i.e., $\boldsymbol{\Lambda}$.

$$TC(\boldsymbol{\Lambda}) \equiv TRC(T^*(\mathbf{k}^0(\boldsymbol{\Lambda}),\boldsymbol{\Lambda}),\mathbf{k}^0(\boldsymbol{\Lambda}),\boldsymbol{\Lambda})$$
$$= 2\sqrt{\left[\sum_{i=1}^{m}(A_i^R + \sum_{j=1}^{n} S_{i,j}\varsigma_{i,j})\right]\left[\sum_{i=1}^{m}(\beta_{1,i} - \sum_{j=1}^{n}\theta_{i,j}\lambda_{i,j}^2)\right]} + 2\sum_{i=1}^{m}\sqrt{\alpha_{2,i}\beta_{2,i}} \quad (3)$$

Additionally, the optimal production cycle length described for the given $\boldsymbol{\Lambda}$ can be rearranged into the following Eq.(4).

$$T^*(\boldsymbol{\Lambda}) = \sqrt{\frac{\sum_{i=1}^{m}(A_i^R + \sum_{j=1}^{n} S_{i,j}\varsigma_{i,j})}{\sum_{i=1}^{m}(\beta_{1,i} - \sum_{j=1}^{n}\theta_{i,j}\lambda_{i,j}^2)}} \quad (4)$$

To find $\boldsymbol{\Lambda}^*$ minimizing $TRC(T^*(\mathbf{k}^0(\boldsymbol{\Lambda}),\boldsymbol{\Lambda}),\mathbf{k}^0(\boldsymbol{\Lambda}),\boldsymbol{\Lambda})$ in Eq. (3), we should solve two sub-problems simultaneously for multiple items: one is the selection problem to choose the plants utilized and the other is to determine apportioning ratios for those selected plants. $TRC(T^*(\mathbf{k}^0(\boldsymbol{\Lambda}),\boldsymbol{\Lambda}),\mathbf{k}^0(\boldsymbol{\Lambda}),\boldsymbol{\Lambda})$ has a complex structure incurred by the multiplication effects between $(A_i^R + \sum_{j=1}^{n} S_{i,j}\varsigma_{i,j})$ and $(\beta_{1,i} - \sum_{j=1}^{n}\theta_{i,j}\lambda_{i,j}^2)$ for any arbitrary two different items. Hence, it is hard to determine $\boldsymbol{\Lambda}^*$simultaneously for all items because of these multiplication effects among items. Assume that T is fixed in Eq. (1). Then, $\mathbf{k}^* = (k_1^*,\ldots,k_m^*)$ minimizing $TRC(\mathbf{k},\boldsymbol{\Lambda} \mid T)$ can be derived from $\partial TRC(\mathbf{k}^*,\boldsymbol{\Lambda} \mid T)/\partial k_i^* = 0 (i = 1,\ldots,m)$ as seen below.

$$k_i^* = T\sqrt{\beta_{2,i}/\alpha_{2,i}}\ ,\quad i=1,\ldots,m \qquad (5)$$

And then, $TRC(\mathbf{k}^*, \mathbf{\Lambda} \mid T)$ can be rearranged as a function of $\mathbf{\Lambda}=(\lambda_1,\ldots,\lambda_m)$ for a fixed cycle length T.

$$TRC(\mathbf{k}^*(\mathbf{\Lambda}),\mathbf{\Lambda}\mid T) = \frac{\sum_{i=1}^m (A_i^R + \sum_{j=1}^n S_{i,j}\varsigma_{i,j})}{T} + \left[\sum_{i=1}^m \beta_i - \sum_{i=1}^m\sum_{j=1}^n \theta_{i,j}\lambda_{i,j}^2\right]T + \frac{\sum_{i=1}^m \alpha_{2,i}k_i^*}{T} + \sum_{i=1}^m \frac{\beta_{2,i}}{k_i^*}T$$
$$= \frac{\sum_{i=1}^m R_i(\lambda_i \mid T)}{T} + 2\sum_{i=1}^m \sqrt{\alpha_{2,i}\beta_{2,i}} \qquad (6)$$

where $R_i(\lambda_i \mid T) = \left[(A_i^R + \sum_{j=1}^n S_{i,j}\varsigma_{i,j}) + T^2(\beta_{1,i} - \sum_{j=1}^n \theta_{i,j}\lambda_{i,j}^2)\right]$.

We can minimize the $TRC(\mathbf{k}^*, \mathbf{\Lambda} \mid T)$ by minimizing $R_i(\lambda_i \mid T)(i=1,\ldots,m)$ for each item for a fixed T since $\sum_{i=1}^m \sqrt{\alpha_{2,i}\beta_{2,i}}$ is constant regardless of the values of decision variables. The proposed solution procedure is executed iteratively as follows: We set an arbitrary cycle length T. And then, determine the optimal apportioning ratios λ_i^* for each item under the assumption that T is predetermined. After getting $\mathbf{\Lambda}^*$, we calculate T^* from the current $\mathbf{\Lambda}^*$ and then determine $\mathbf{\Lambda}^*$ iteratively. We repeat this iterative procedure until the convergence occurs. At the following Eq.(7), $T_{(l-1)}$ represents the cycle length derived at $(l-1)^{th}$ iteration.

$$TRC(\mathbf{k}^*(\mathbf{\Lambda}),\mathbf{\Lambda}\mid T_{(l-1)}) = \frac{\sum_{i=1}^m f_{i(l)}(\lambda_i) + T_{(l-1)}^2 \sum_{i=1}^m g_{i(l)}(\lambda_i)}{T_{(l-1)}} + 2\sum_{i=1}^m \sqrt{\alpha_{2,i}\beta_{2,i}} \qquad (7)$$

where

$f_{i(l)}(\lambda_i) = (A_i^R + \sum_{j=1}^n S_{i,j}\varsigma_{i,j}), g_{i(l)}(\lambda_i) = (\beta_{1,i} - \sum_{j=1}^n \theta_{i,j}\lambda_{i,j}^2)$.

The objective function $TRC(\mathbf{k}^*(\mathbf{\Lambda}),\mathbf{\Lambda}\mid T_{(l-1)})$ in Eq. (7) can be reduced into the Eq. (3) and it has a minimum value when $T_{(l-1)}^2 = \dfrac{\sum_{i=1}^m f_{i(l)}(\lambda_i)}{\sum_{i=1}^m g_{i(l)}(\lambda_i)} = T_{(l)}^2$, which is also described in Eq. (4). This equation can be used as a convergence condition explained in the detail solution procedure.

At first, it is necessary to make sets of minimal paths under the property of optimal solutions for $g_i(\lambda_i)$. To make up alternative path sets in a k-out-of-n structure, we at first sort the transfer rates in non-decreasing order to maximize the number of alternative path sets by considering the property of optimal solution for $g_i(\lambda_i)$. From this ordered sequence, we build up a directed graph containing those nodes. And then, we can iteratively generate an alternative path sets satisfying the following condition: $\sum_{j=1}^{n(\varphi_{i,w})} \bar{\lambda}_{i,j} \geq 1$ and $\sum_{j=1}^{n(\varphi_{i,w})} \bar{\lambda}_{i,j} - \bar{\lambda}_{i,l} < 1$ for any $l \in \varphi_{i,w}$, where $\boldsymbol{\varphi}_i$ is a set of alternative path sets for item i and $\varphi_{i,w}$ is an arbitrary path set included in $\boldsymbol{\varphi}_i$, i.e., $\varphi_{i,w} \subset \boldsymbol{\varphi}_i$ and $w \in \{1,\ldots,n(\boldsymbol{\varphi}_i)\}$.

Suppose that a manufacturer already determined a set of plants to be utilized and $\varphi_{i,w}$ is an arbitrary path set for item i, i.e., $\varphi_{i,w} \subset \boldsymbol{\varphi}_i$. Then, the value of $(A_i^R + \sum_{j=1}^n S_{i,j}\varsigma_{i,j})$ in $g_i(\lambda_i)$ is fixed regardless of the determination of apportioning ratios λ_i for item i. For the arbitrarily given path set $\varphi_{i,w}$, we can get the economical apportioning ratios λ_i^* by maximizing $\sum_{j=1}^n \theta_{i,j}\lambda_{i,j}^2$. Define $G_i(\lambda_i \mid \varsigma_i) = \sum_{j\in\phi_{i,w}} \theta_{i,j}\lambda_{i,j}^2$, then $G_i(\lambda_i \mid \varsigma_i)$ is a convex function in λ_i since it is a quadratic function of λ_i, and the set of λ_i maximizing $G_i(\lambda_i \mid \varsigma_i)$ also minimizes $\sum_{i=1}^n g_i(\lambda_i)$ in Eq. (7). Define the Lagrangian function by $L_i(\lambda_i, \eta_i \mid \varsigma_i) = G_i(\lambda_i \mid \varsigma_i) + \eta_i(\sum_{j\in\phi_{i,w}} \lambda_{i,j} - \lambda_{i,a})$ for each item. And then, define $\Delta L_i(\lambda_i \mid \varsigma_i) = \left[G_i(\lambda_i \mid \varsigma_i) - G_i(\lambda_i^0 \mid \varsigma_i)\right] + \eta_i^0(\sum_{j\in\phi_{i,w}} \lambda_{i,j} - \lambda_{i,a})$ for an arbitrary λ_i while keeping the equality constraint of $\sum_{j=1}^n \lambda_{i,j} = \lambda_{a,i}$, then $\Delta L_i(\lambda_i \mid \varsigma_i)$ can be arranged as seen in Eq. (8).

$$\Delta L_i(\lambda_i \mid \varsigma_i) = \lambda_{a,i}\left(\sum_{j\in\phi_{i,w}} \frac{1}{\theta_{i,j}}\right)^{-1} \sum_{j\in\phi_{i,w}} \frac{(\lambda_{i,j} - \lambda_{i,j}^0)^2}{\lambda_{i,j}^0} \qquad (8)$$

At first, we can determine λ_i for individual item by the heuristic procedure which was already proposed. As previously stated, in that heuristic procedure, the feasibility condition of λ_i taking into account the transfer rates, i.e., $\lambda_{i,j} \le d_{i,j}/D_i, \forall i, j \in \phi_{i,w}$ for the given alternative path set $\phi_{i,w}$ for item i. For all possible path sets for item i, we can evaluate $R_i(\lambda_i \mid T)$ by determining λ_i for the given path set $\varphi_{i,w}$. And then, we choose the best path set yielding the lowest $R_i(\lambda_i \mid T)$ among the multiple path sets for item i.

<u>Solution Procedure</u>

<u>Stage-1) Determine individual apportioning policy for each item</u>

Step.1.1) $l=1$. Set $T_{(l)}$ arbitrarily. Derive the path sets for each item to satisfy the given demand by considering the upper bound of apportioning ratio of each flexible plant for the corresponding item's demand rate.

Step.1.2) And then, determine apportioning ratios for each alternative path set. Perform the *Algorithm ISA* for each alternative path set (Refer to the Kim et al. (2005) for more detailed information for Algorithm ISA.). Choose the best path set among the $n(\boldsymbol{\varphi}_i)$ alternative path sets for the given cycle length $T_{(l)}$.

$$\boldsymbol{\lambda}_i^* = \arg_{\boldsymbol{\lambda}_{i(w)}} \min\left\{R_i(\boldsymbol{\lambda}_{i(w)} \mid T_{(l)}) \mid \varphi_{i,w} \subset \boldsymbol{\varphi}_i, w = 1, \ldots, n(\boldsymbol{\varphi}_i)\right\} \tag{9}$$

where

$$R_i(\boldsymbol{\lambda}_{i(w)} \mid T_{(l)}) = \left[(A_i^R + \sum_{j=1}^{n(\varphi_{i,w})} S_{i,j}\varsigma_{i,j}) + (T_{(l)})^2(\beta_{1,i} - \sum_{j=1}^{n(\varphi_{i,w})} \theta_{i,j}\lambda_{i,j}^2)\right].$$

Step.1.3) $l = l+1$. Calculate $T_{(l)} = \sqrt{\dfrac{\sum_{i=1}^{m}(A_i^R + \sum_{j=1}^{n} S_{i,j}\varsigma_{i,j}^*)}{\sum_{i=1}^{m}(\beta_{1,i} - \sum_{j=1}^{n}\theta_{i,j}(\lambda_{i,j}^*)^2)}}$. If $T_{(l)} = T_{(l-1)}$, then best path sets for all items don't change and stop Stage.1).

<u>Stage-2) Check the feasibility of aggregated policy</u>

Step.2.1) If $\rho(\boldsymbol{\Lambda}) = \sum_{i=1}^{m} \max_{j \in \phi_i^*}\{\lambda_{i,j}D_i/p_{i,j}\} \le 1$, then aggregated production schedule is feasible one and stop this procedure. Otherwise, determine δ_i^*.

Step.2.2) Perform Stage.1) by the modified upper bound of apportioning ratio, i.e.,

$$\overline{\lambda_{i,j}} = \min\left\{\lambda_{i,a}, \delta_i^* \frac{p_{i,j}}{D_i}\right\}, \forall i, j \tag{10}$$

4. Concluding Remarks

We investigated the decision-making issue where the global manufacturing firm tries to optimize its supply chain network facilitating multiple production sites or plants. From the inherent structure of this problem, we should solve two problems simultaneously: one is site selection problem; the other is how to apportion the production lot among the multiple sites(or plants). An effective solution mechanism is needed to improve the performance of supply chain network. To construct a more sophisticated supply chain model, the incorporation of distribution policy is also desirable extension of the proposed model in this paper.

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