# 트레일러-트렉터문제에 대한 연구

# An analysis on Trailer and Tractor Problem (TTP)

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## Abstract

We consider the transportation problem of optimizing the use of trailers and tractors. Several variants with fixed loading and unloading time are discussed. We show that the variant requiring on-time delivery can be solved in polynomial time, whereas the other variant requiring time-window delivery is NP-hard. We also discuss that if the number of loading and unloading operations assigned to each trailer or tractor is limited, even the variant requiring on-time delivery becomes NP-hard.

## 1. Introduction

In a trailer and tractor problem (TTP), we are given a set of transportation requests to be fulfilled by trailers and tractors. For each transportation request, an empty container must be transported by a tractor to the specified origin where the trailer is loaded for a specified loading time and transported to its destination where the trailer is unloaded for a specified unloading time. The tractor can handle one trailer at a time behaving like a truck but it may begin moving other trailer once it completed the transportation of a trailer from one location to the other. After the completion of unloading, the trailer may be used again to accommodate another transportation request. The objective is to minimize the total number of trailers and tractors required to accommodate all transportation requests within a given planning time horizon.

The motivation of this research came from the outbound logistic planning within a steel mill. In the steel mill, the steel products known as *coils*, rolls of thin metal sheets, stored in warehouses (origin locations) are transported on a custom designed device called a *pallet* (trailer). A specific group, maximum of 100 tons, of coils to be transported by each pallet is call a *lot*. Each lot is designated to a dock (destination location). The loaded pallet is transported by a custom designed vehicle called *elevating truck* (tractor) from a warehouse to a dock where the coils are to be unloaded onto a ship.

We review existing research works that are relevant to our problem. The first one is the routing-scheduling problem, where jobs are located at different nodes of a transportation network and machines move around the network to process jobs [1]. Another one is the pickup and delivery problem with time window (PDPTW). PDPTW is a generalization of the vehicle routing problem with time window (VRPTW) seeking an optimal route for each vehicle to satisfy transportation requests, each requiring pickup at and delivery within a specified time window. In VRPTW, each route satisfies paring constraints since corresponding pickup and delivery must be serviced by the same vehicle [2]. All these problems are similar to TTP in that they seek to find some routes of vehicles to minimize relevant cost and involve loading and unloading at various locations. However, TTP differs from them since it involves both trailers and tractors to accommodate transportation requests. The problem that seems most similar to TTP is the truck and trailer routing problem [3]. In truck and trailer routing problem, however, the truck may itself carry payloads whereas the tractor in TTP cannot carry any payloads but simply transport trailers.

In the next section we develop a formal definition of TTP and present two variants that are treated in this paper. In section 3, we show that one restricted variant is NP-hard problem and the other is polynomial time solvable. In section 4, we show that if the number of loading and unloading operations assigned to each trailer or tractor is limited, even the variant requiring on-time delivery becomes NP-hard. Then a concluding remark follows.

## 2. Problem definition and restricted variants

Each instance of TTP consists of a set of transportation requests and a planning time horizon. Each transportation request specifies;

1) The origin and destination location
2) The duration of loading and unloading

The travel times between every pair of locations for a tractor to move loaded, empty, and without trailer are given, respectively. We assume that the time to couple and decouple a tractor with a trailer is included in the travel time. We also assume that all trailers and tractors are initially located at one specific location and must be returned to the initial location. The set of payloads to be transported by each trailer is call a *lot*. It is not so difficult to show that our problem becomes NP-compete if the times to start loading and unloading for each lot are treated as decision variables. Hence, we assume that the times to begin loading and unloading are determined in advance implying that the competition times of loading and unloading are also given. Under this assumption, we define two variants.

In the first variants, the departure time of loaded trailer at the origin should be same as the completion time of loading. Also the arrival time of loaded trailer at destination should be exactly same as the time to begin unloading. We name this variant 'on-time delivery variant'. In the other variant, the trailer can depart origin anytime after the completion of loading and arrive at destination anytime before the time to begin unloading. This variant is called the 'time-window delivery variant'.

Optimizing the use of trailers and tractors at the same time is still under investigation and it is not treated in this paper. Instead, we take an approach to minimize the number of required trailers first and find minimum number of tractors while keeping the number of required trailers.

## 3. Two restricted variants

As stated in last section, our objective is to find the minimum number of required trailers and then find the minimum number of required tractors while the number of required trailer stays the same. As a consequence, we first schedule trailers and then schedule tractors. That is, we first determine the sequence of lots that each trailer handles. Then we schedule the movements of tractors. By assigning each lot loaded on a trailer to a tractor, we generate a complete schedule for both trailers and tractors. Clearly, if either the scheduling of trailers or that of tractors is NP-hard, so it becomes the overall problem. Hence, to solve the overall problem in polynomial time, we need to formulate algorithm to schedule both trailers and tractors in polynomial time.

### 3.1 On-time delivery variant

We present trailer scheduling algorithm first. Obviously, each trailer is tied up to a lot from the starting time of loading at its origin till the completion time of unloading at its destination. According to these times, we can define trailer graph (G-trailer) like below.

**G-trailer(N,A)**

N: $n \in N$, Node n, corresponding to each lot, has 4 attribute;

$n_{lp}$: origin

$n_{up}$: destination

$n_{st}$: start time of loading

$n_{et}$: completion time of unloading

A: arc(i,j) Possibility of reusing

Then we define a set A of arc(i,j) indicating the possibility of reusing the same trailer for the j-th lot after using it for the i-th lot. Namely, arc(i,j) is defined if a tractor can move the trailer from the destination of the i-th lot to the origin of the j-th lot starting from the completion time of unloading the i-th lot till the time to begin loading the j-th lot. By construction, the trailer graph is acyclic digraph and finding the minimum number of trailers to cover all lots is equivalent to finding the minimum number of directed paths which cover all the nodes in the trailer graph. Since, it is acyclic, finding the minimum number of directed paths can be done in polynomial time[7].

Once all the lots are assigned to trailers, we need to schedule tractors to accommodate the movements of trailers with the minimum number of tractors. To this end, we define the tractor graph (G-tractor) similar to the trailer graph.

**=====================From here===**

**G-tractor(N,A)**

N: $n \in N$, Node n, corresponding to each movement of trailers.

$n_{lp}$: start location

$n_{up}$: end location

$n_{st}$: start time

$n_{et}$: end time

A: arc(i,j) Possibility of reusing

Make arc from node i to node j if tractor can move from movement i's end location to movement j's start location between movement i's end time and movement j's start time.

Resulting graph is also acyclic and the same method applied trailer graph solve tractor schedule problem in polynomial time

### 3.2 Time-window delivery variant

For time window case, trailer scheduling is same with on-time delivery case. But tractor scheduling problems differs. In this case each activity can be done anytime between corresponding lot's loading end time and unloading start time, which is time-window. As a result tractor schedule problem becomes NP-hard. For more detailed description, see [6].

**4. Cardinality restricted case**

As described in last section trailer scheduling problem is equivalent to fining minimum cardinality directed path which cover given acyclic graph. Again it is equivalent to finding minimum cardinality chain decomposition in partially ordered set [4]. According to [5], finding cardinality restricted chain decomposition in partially ordered set is NP-hard problem. As a result, even on-time delivery variant becomes NP-hard problem when we restrict number of lots each trailer can load during scheduling horizon

**5. Concluding remarks**

Two restricted variants for trailer and tractor problems are analyzed. We showed that one variant is polynomial time solvable and the other is NP-hard. It is also shown that polynomial time solvable variant also becomes NP-hard when the cardinality of lots each trailer loads or cardinality of trailer each tractor can move during schedule horizon.

An obvious direction for future research is to develop heuristics for NP-hard variants with worst-case performance analysis.

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