다양한 수송수단을 고려하는 (1:N) 공급체인에 대한 재고-수송의 통합계획수립.

Integrated inventory-distribution planning in a (1:N) supply chain system with heterogeneous vehicles incorporated

김은석*, 성창섭**, 양우석***

* 한국과학기술원 산업공학과 (silverstone@kaist.ac.kr)
** 한국과학기술원 산업공학과 (cssung@kaist.ac.kr)
*** 한국과학기술원 산업공학과 (parani@kaist.ac.kr)

Abstract

This paper considers an integrated inventory-distribution system with a fleet of heterogeneous vehicles employed where a single warehouse distributes a single type of products to many spatially distributed retailers to satisfy their dynamic demands and the product is provided to the warehouse via procurement ordering from any manufacturing plant or market. The problem is formulated as an Mixed Integer Programming with the objective function of minimizing the sum of inventory holding cost (at the warehouse and retailers), and transportation cost and procurement ordering cost at the warehouse, subject to inventory-balancing constraints, ordering constraints, vehicle capacity constraints and transportation time constraints. The problem is proven to be NP-hard. Accordingly, a Lagrangean heuristic procedure is derived and tested for its effectiveness through computational experiments with some numerical instances.

1. Introduction and Problem description

The proposed problem is associated with an (1:*N*) supply chain system which is composed of one warehouse and spatially distributed retailers. In order to satisfy any given retailer demands, the warehouse has to place orders for some amount of the product from a higher echelon, say a manufacturing plant or a supplier (market). The warehouse employs a fleet of heterogeneous vehicles(different loading capacities) to distribute the product to multiple retailers. Each vehicle is allowed to make round trips between the warehouse and retailers. The associated ordering planning and vehicle scheduling are made for each period. The time required for a round trip between the warehouse and each retailer is assumed to be less than each period. Each vehicle is allowed to make several round trips to the same retailer or other retailers in a single period if the total travel time does not exceed the length of each period. Demand is assumed to be known for the entire planning horizon. Both the warehouse and retailers can hold inventory and there is no limit on the storage capacity for inventory. Each demand can be delivered early but not late (i.e. shortage is not allowed).

Transportation cost is composed of distance-based cost and quantity-based cost. The distance-based transportation cost is incurred when a vehicle makes a round trip to a retailer, in proportion to the associated distance between the warehouse and the retailer, while the quantity-based transportation cost can be computed as the delivery quantity multiplied by unit variable cost. The distance-based cost depends on vehicles used for delivery, but the quantity-based cost is independent of vehicles. The fixed ordering cost at the warehouse is independent of any amount ordered, but it may differ period by period. The inventory holding cost is charged according to inventory quantities at the end of each period. The unit holding cost may also differ at the warehouse and each retailer period by period.

The associated parameters and variables are listed as follows.

Parameters

- t : index for time periods $(1,\ldots,T)$
- i : index for vehicles $(1,\ldots,M)$
- j : index for retailers $(1,\ldots,N)$ with 0 representing the warehouse
- c_{jt}^{Q} : quantity-based transportation cost for delivering a unit to retailer j in period t
- c_{ijt}^{Q} : distance-based transportation cost of vehicle i for a round trip between the warehouse and retailer j in period t
- h_{jt} : per-unit inventory holding cost for inventory at the end of period t at retailer j
- d_{jt} : demand quantity at retailer j during period t
- o_t : fixed cost incurred at the warehouse for the product to be ordered from a higher echelon in period t
- V_i : loading capacity of vehicle i
- τ_j : transportation time between the warehouse and retailer j
- τ : length of each period

Variables

- y_{ijt} : number of round trips made by vehicle i to serve retailer j in period t
- I_{jt} : inventory level at retailer j in period t
- x_{jt} : amount delivered to retailer j in period t
- z_t : 0-1 variable representing whether the warehouse places an order for the product in period t
- q_t : amount ordered at the warehouse in period t

The integrated inventory-distribution planning problem can be mathematically expressed as in the following mathematical programming:

Problem P :

$$Z_{opt} = Min\sum_t\sum_j\sum_t c_{ijt}^{D} y_{ijt} + \sum_j\sum_t c_{jt}^{Q} x_{jt} + \sum_j\sum_t h_{jt} I_{jt} + \sum_t o_t z_t$$

subject to

$$I_{0t} = I_{0t-1} + q_t - \sum_j x_{jt} \qquad \forall t \qquad (2)$$

$$I_{jt} = I_{jt-1} + x_{jt} - d_{jt} \qquad \forall j,t \qquad (3)$$

$$q_t \le M_t z_t \qquad \forall t \qquad (4)$$

$$x_{jt} \le \sum_i V_i y_{ijt} \qquad \forall j,t \qquad (5)$$

$$\sum_j \tau_j y_{ijt} \le \tau \qquad \forall i,t \qquad (6)$$

$$y_{ijt} \in \{0,1,2,\cdots\} \qquad \forall i,j,t \qquad (7)$$

$$z_t = \{0,1\} \qquad \forall t \qquad (8)$$

$$x_{jt}, I_{jt}, q_t \ge 0 \qquad \forall j,t \qquad (9)$$

Eqs. (2) and (3) represent the inventory balance constraints at the warehouse and retailers, respectively. Constraint (4) enforces binary variable z_t to become 1 whenever the warehouse places an order for the product from the higher echelon. M_t is a very large positive number. Constraint (5) ensures that the quantity delivered to a retailer in a period does not exceed the sum of the loading capacities of the vehicles visiting the retailer. Constraint (6) ensures that the total travel time of each vehicle during each period does not exceed the length of each period.

2. Solution approach

Problem P is NP-hard, because it has the general knapsack problem structure, so that a heuristic algorithm is derived to get a good, near-optimal solution. In this paper, the Lagrangean relaxation approach is used to develop the heuristic solution procedure for the proposed mixed integer programming problem.

2.1 Lagrangean Relaxation

The original problem [P] is relaxed by dualizing constraint (5) with Lagrangean multipliers $\lambda_{jt} \ge 0$. The resulting relaxed problem is $L(\lambda)$.

Problem $L(\lambda)$:

$$Z_L(\lambda) = Min\sum_t\sum_j\sum_t (c_{ijt}^{D} - V_i\lambda_{jt}) y_{ijt} + \sum_j\sum_t h_{jt} I_{jt} + \sum_j\sum_t (c_{jt}^{Q} + \lambda_{jt}) x_{jt} + \sum_t o_t z_t$$

subject to

(2), (3), (4), (6), (7), (8), (9) and $\lambda_{jt} \ge 0$ (10)

The relaxed problem $L(\lambda)$ can be decomposed into two independent subproblems, $L_1(\lambda)$ and $L_2(\lambda)$.

Subproblem $L_1(\lambda)$:

$$Z_{L_1}(\lambda) = Min\sum_j\sum_t (c_{jt}^{Q} + \lambda_{jt}) x_{jt} + \sum_j\sum_t h_{jt} I_{jt} + \sum_t o_t z_t$$

subject to

(2), (3), (4), (8), (9) and (10)

Subproblem $L_2(\lambda)$:

$$Z_{L_2}(\lambda) = Min\sum_t\sum_j\sum_t (c_{ijt}^{D} - V_i\lambda_{jt}) y_{ijt}$$

subject to

(6), (7) and (10)

2.1.1 Solving Subproblem $L_1(\lambda)$ with λ given

Relaxing an integrality constraint (8) makes the problem to become a simple linear programming problem to solve optimally via the simplex method. To obtain a good lower bound for the original problem, the LP relaxed problem of Subproblem $L_1(\lambda)$ has to get tight by employing some valid inequalities. In the proposed problem, backlogging is not allowed so that the maximum ordering quantity at the warehouse, q_t, can be determined up to the sum, M_t, of the overall net demands over the periods from t through T for all the retailers such that $M_t = \sum_{k=t}^{T}\sum_j d_{jk}$.

Proposition 1.

The following inequalities are valid for Subproblem $L_1(\lambda)$.

$$\sum_j I_{j,k-1} \geq \sum_{t=k}^{l}(\sum_j d_{jt})(1-z_k-\cdots-z_t) \quad \text{for } 1 \leq k \leq l \leq T.$$

Moreover, the relations $I_{j0}=0$ and $z_1=1$ can be included as constraints, because the initial inventory at the warehouse and retailers is assumed to be zero. Therewith, an LP relaxed problem of Subproblem $L_1(\lambda)$, $LPRL_1(\lambda)$, can be expressed as follows.

Problem $LPRL_1(\lambda)$:

$$Z_{LPRL_1}(\lambda) = Min\sum_j\sum_t(c_{jt}^Q+\lambda_{jt})x_{jt} + \sum_j\sum_t h_{jt}I_{jt} + \sum_t o_t z_t$$

subject to

$$I_{0t} = I_{0t-1} + q_t - \sum_j x_{jt} \qquad \forall t$$

$$I_{jt} = I_{jt-1} + x_{jt} - d_{jt} \qquad \forall j,t$$

$$q_t \leq (\sum_{k=t}^{T}\sum_j d_{jk})z_t \qquad \forall t$$

$$\sum_j I_{j,k-1} \geq \sum_{t=k}^{l}(\sum_j d_{jt})(1-z_k-\cdots-z_t) \qquad \text{for } 1 \leq k \leq l \leq T$$

$$0 \leq z_t \leq 1 \qquad \forall t$$

$$x_{jt}, I_{jt}, q_t \geq 0 \qquad \forall j,t$$

2.1.2 Solving Subproblem $L_2(\lambda)$

For any given λ, Subproblem $L_2(\lambda)$ can be further decomposed into several single-period, single-vehicle scheduling problems $L_2^{it}(\lambda)$ for $\forall i,t$ as follows.

Subproblem $L_2^{it}(\lambda)$:

$$Z_{L^{it}_2}(\lambda) = Min\sum_j(c_{ijt}^D - V_i\lambda_{jt})y_{ijt}$$

subject to

$$\sum_j \tau_j y_{ijt} \leq \tau$$

$$y_{ijt} \in \{0,1,2,\cdots\} \qquad \forall j$$

In this paper, the integer knapsack problem is solved by a dynamic programming algorithm, referring to Nemhauser and Wolsey [27]. The algorithm solves the problem optimally in the complexity order *O(NR)*, where N is the number of retailers and R is the length of a period.

3.3 Converting of Infeasible Solutions

Procedure-FS 1 is executed to find a feasible solution which is an upper bound for the original problem P. The Procedure-FS 2 checks if any further improvement can be made in vehicle schedules so as to lead to reduction in the distribution and inventory cost while maintaining the feasibility of the solution.

Procedure-FS 1

Phase 1. (Increasing the number of trips)

Let $\overline{x_{jt}}$ *be the solution values of Subproblem* $LPRL_1(\lambda)$, *and* $y_{ijt}=0$ *for all i, j and t.*

For t=1 to T

  Sort all the retailers in nonincreasing order of their round trip times (τ_j).

    For j=1 to N, do;

      Repeat if $\overline{x_{jt}} > \sum_i V_i y_{ijt}$:

        Find $i^* = \mathrm{argmin}_i(c_{ijt}^D - V_i\lambda_{jt})$ *such that* $\sum_j \tau_j y_{ijt} + \tau_j \leq \tau$ for i^*

        Let $y_{i^*jt} \leftarrow y_{i^*jt} + 1$.

Phase 2. (Find a binary value of variable zt)

Let $(\overline{z_t}, \overline{q_t})$ *be the optimal solution value of Subproblem* $LPRL_1(\lambda)$, *and* $\overline{y_{ijt}}$ *is the current solution after Phase 1 is executed.*

For t=1 to T,

  For $\overline{z_t}=1$, *find the smallest* t^* *such that*

$$\sum_{k=t}^{t^*}(1-\overline{z_k})o_k \leq \sum_{k=t+1}^{t^*}\left(\sum_{l=t}^{k-1}h_{0t}\right)\overline{q_k}, \quad t^* > t.$$

  Let $z_{t^*}=1$ *and* $z_k=0$ *for* $t<k<t^*$.

  Solve Problem (LP1).

  Problem (LP1):

$$\min\sum_i\sum_j\sum_t c_{ijt}^D\overline{y_{ijt}} + \sum_j\sum_t c_{jt}^Q x_{jt} + \sum_j\sum_t h_{jt}I_{jt} + \sum_t o_t\overline{z_t}$$

subject to

$$q_t \le \left(\sum_{k=t}^{T}\sum_{j} d_{jk}\right)\overline{z_t} \qquad \forall t$$

$$x_{jt} \le \sum_i V_i \overline{y_{ijt}} \qquad \forall j,t$$

$$I_{0t} = I_{0t-1} + q_t - \sum_j x_{jt} \qquad \forall t$$

$$I_{jt} = I_{jt-1} + x_{jt} - d_{jt} \qquad \forall j,t$$

$$x_{jt}, I_{jt}, q_t \ge 0 \qquad \forall j,t$$

Procedure-FS 2

Let $\Omega = \{(i,j,t) | \overline{y_{ijt}} > 0, i \in I, j \in J, t \in T\}$ *where* $\overline{y_{ijt}}$ *is the current solution obtained from Phase 1.*

Repeat if $\Omega \ne \emptyset$

Find $(i,j,t)^*$ *such that*

$$(i,j,t)^* = \operatorname{argmax}_{(i,j,t) \in \Omega}\left(c_{ijt}^{D} - V_i \lambda_{jt}\right).$$

Let $\Omega = \Omega \setminus (i,j,t)^*$.

Solve Problem (LP2) with $\overline{z_t}$ *(the current solution from Phase 2).*

If the objective value of Problem (LP2) is smaller than the objective value of the current solution, then let $\overline{y_{(i,j,t)^*}} \leftarrow \overline{y_{(i,j,t)^*}} - 1$ *, where Problem (LP2) is expressed as*

Problem (LP2):

$$\min \sum_i \sum_j \sum_t c_{ijt}^{D} \overline{y_{ijt}} + \sum_j \sum_t c_{jt}^{Q} x_{jt} + \sum_j \sum_t h_{jt} I_{jt} + \sum_t o_t \overline{z_t}$$

subject to

$$x_{jt} \le \sum_i V_i \overline{y_{ijt}} - V_{i^*} \qquad \forall j,t$$

$$I_{0t} = I_{0t-1} + q_t - \sum_j x_{jt} \qquad \forall t$$

$$I_{jt} = I_{jt-1} + x_{jt} - d_{jt} \qquad \forall j,t$$

$$q_t \le \left(\sum_{k=t}^{T}\sum_{j} d_{jk}\right)\overline{z_t} \qquad \forall t$$

$$x_{jt}, I_{jt}, q_t \ge 0 \qquad \forall j,t$$

3.4 Lagrangean Heuristic Procedure

Procedure-LH

Step 1: Initialize the Lagrangean multipliers and parameters as follows;

1.1: Set the improvement counter at *k*=0, the iteration counter at *l*=0.

1.2: Set the Lagrangean multiplier $\lambda^k = 1$.

1.3: Set the current best lower bound, LB, to negative infinity and the current best upper bound, UB, to positive infinity.

Step 2: Generate a lower bound and use it to update the current best lower bound.

2.1: Given λ^k solve the Lagrangean Problem $L(\lambda^k)$ using the simplex method and the DP algorithm (referring to Sections 3.1.1 and 3.1.2), and obtain the value for $Z_L(\lambda^k)$ which is a lower bound for the original problem P. If the solutions are feasible to the original problem, terminate and give $Z_L(\lambda^k)$ as the final solution.

2.2: If $Z_L(\lambda^k) > LB$, then let $LB = Z_L(\lambda^k)$ and $l = 0$. Otherwise, let $l = l + 1$.

Step 3: Generate a Lagrangean heuristic solution and use it to update the current best upper bound.

3.1: Generate a Lagrangean heuristic solution by using Procedure-FS 1 and compute the feasible solution value of $\overline{Z_{opt}}$ which is an upper bound for the original problem P.

3.2: If k is a multiple of *KH*, execute Procedure-FS 2 and update the feasible solution value of $\overline{Z_{opt}}$. Otherwise, go to Step 3.3.

3.3: If $\overline{Z_{opt}} < UB$, then let $UB = \overline{Z_{opt}}$.

3.4: If $\left(\frac{UB - LB}{LB}\right) \times 100 \le 1$, terminate and give UB as the final solution. Otherwise, go to Step 4.

Step 4: If $k > K$ or $l > L$, terminate and give UB as the final solution. Otherwise, go to Step 5.

Step 5: Update the Lagrangean multipliers using the subgradient optimization method (referring to Section 3.2). Set $k = k + 1$ and go to Step 2.

4. Computational results

All the other parameters are generated by referring to Kim and Kim [7] [8].

1. Demands in each time period (d_{jt}) are generated from $DU(100,500)$.
2. Loading capacities (V_i) of small, medium and large size vehicles are set at the largest integers not greater than $\left(\sum_j \sum_t d_{jt}\tau_j\right)/(\tau N)$ times 0.7, 0.85 and 1.0, respectively.
3. Transportation time between the warehouse and each retailer (τ_j) is generated from $DU(0.2\mu_1, \mu_1)$, where $\mu_1 = \tau M/N$.

4. Distance-based transportation cost $\left(c_{ijt}^{D}\right)$ is generated from $U(5\mu_2, 20\mu_2)$, where $\mu_2 = V_i\tau_j/\tau$.
5. Quantity-based transportation cost $\left(c_{jt}^{Q}\right)$ is generated from $U(5,10)$.
6. Per unit inventory holding costs at each retailer $\left(h_{jt}\right)$ and the warehouse $\left(h_{0t}\right)$ are generated from $U(5,10)$ and $U(1,5)$, respectively.
7. Ordering cost at the warehouse is generated from $U(\mu_3, 10\mu_3)$, where $\mu_3 = 3\left(\sum_j \sum_t d_{jt}\right)/T$.

Table 1 gives the average, minimum and maximum percentage gaps of the Lagrangean heuristic solutions and CPU times for each problem set.

Table 1. Results of the computational experiment

		Percentage gap (%)[b]			CPU time (s)		
I^a	J	Min	Mean	Max	Min	Mean	Max
20	50	3.33	3.76	4.59	91.3	114.1	129.0
20	60	3.24	3.76	4.18	162.1	206.9	228.6
20	70	3.10	3.41	3.86	216.4	250.5	265.0
30	70	3.14	4.02	5.68	202.5	227.1	294.7
30	80	3.05	3.45	3.77	265.9	313.2	370.5
30	90	3.07	3.58	4.93	482.2	547.3	618.1
40	90	3.43	4.17	5.47	328.9	361.7	382.6
40	110	2.93	3.40	4.51	724.1	1138.7	1548.1
40	130	3.24	3.61	4.24	2276.2	2502.7	2848.4
50	110	3.11	4.07	5.73	497.7	568.0	784.7
50	130	2.85	3.23	3.62	1029.7	1731.9	2212.6
50	150	3.12	3.43	4.50	2842.6	3324.4	3661.3

[a] *I and J denote the number of vehicles and the number of retailers, respectively.*

[b] *Percentage gap = 100×(upper bound lower bound)/lower bound.*

5. Conclusion

This paper deals with an integrated inventory-distribution problem with a fleet of heterogeneous vehicles employed to distribute a single type of product from a single warehouse to spatially distributed retailers to satisfy their dynamic deterministic demands.

The problem is formulated as an Mixed Integer Programming where the objective function consists of inventory holding cost, transportation cost and ordering cost at the warehouse, subject to inventory-balancing constraints, ordering related constraints, vehicle capacity constraints and transportation time constraints.

As a solution approach, the Lagrangean relaxation method is adapted to derive a Lagrangean heuristic procedure. To solve the proposed problem more efficiently, some valid inequalities are proposed. In order to evaluate the effectiveness and efficiency of the proposed algorithm, computational experiments are performed with some numerical instances that are randomly generated. The experiment results show that the proposed algorithm solves the problem within a reasonable time and give good solutions at 3.66% gap in average.