ECONOMIC DESIGN OF SCREENING PROCEDURES CONSIDERING INSPECTION ERRORS

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ABTRACT

The implementation of a screening procedure for removing non-conforming products has become a common practice especially in high-tech manufacturing industries. Screening procedures involve a measurement on the quality characteristic of interest since decisions regarding the conformance to specifications are usually made on the basis of the realization of measurement. A significant variability in measurement procedures may result in the misclassification of an outgoing product (that is, falsely accepting defectives or falsely rejecting conforming items), which may lead to wrong interpretation on product quality. It may thus be necessary to consider the impacts of misclassification errors due to measurement variability when designing screening procedures. Along this line, this article investigates the design of screening procedures based on the assessment of misclassification errors. The main objective is to determine the screening limits on measured values so that two types of misclassification errors may properly be compromised.

1. INTRODUCTION

Screening procedures are decision- making processes to improve product quality by determining the most economical set of screening limits under a100% inspection scheme. As advanced types of automated inspection equipment become an integral part of modern manufacturing systems, the implementation of the screening procedures in manufacturing processes has experienced an increased support in recent years and become an attractive means for quality improvement. Screening procedures involve a measurement of the quality characteristic of interest since decisions regarding the conformance to specifications are usually made on the basis of the realization of measurement for the quality characteristic. As more companies strive to improve product quality, enhancement of measurement procedures associated with the complete inspection scheme has become an integral part of quality improvement. However, measurement errors are commonly incurred due to the variations in imprecise devices and/or unskilled operators. Since a significant variability in measurement procedures may lead to a wrong interpretation of the product quality, understanding the notion of measurement variability may be crucial for quality improvement. From this perspective, the study of measurement variability has recently drawn a particular attention from researchers in the context of the so-called gauge study. See, for example, Lin *et al.* (1997), Mader *et al.* (1999), McCarville and Montgomery (1996), Montgomery and Runger (1993a, b), Tsai (1988), and Vardeman and VanValkenburg (1999).

A significant variability in measurement procedures may lead to economic penalties associated with the misclassification of outgoing items. For example, suppose that a defective is falsely accepted and shipped to the customer. A monetary loss may then be incurred to replace the defective items. On the other hand, rejection costs, such as scrap and rework costs, may also be incurred by the manufacturer for falsely rejected conforming products. In this regard, Mader *et al.* (1999) recently evaluated economic impacts of measurement errors for the complete inspection plan. There have also been several studies to reduce the impacts of measurement errors when designing screening procedures. The most immediate approach to control measurement errors may be the selection of measurement precision level since economic penalties associated with measurement errors may be avoided by improving the level of measurement precision. As more precise measurement devices and/or better-trained operators are required, inspection cost may be increasingly incurred to reduce economic penalties. Thus, there is a need for a tradeoff among cost factors associated with measurement errors and the selection of measurement precision level. Readers are referred to Chandra and Schall(1988), Chen and Chung (1996), and Tang and Schneider (1988).

Regardless of how precise the measurement procedures are, however, measurement errors are commonly incurred. Given the level of measurement precision, it will thus be beneficial to design screening procedures based on the assessment of misclassification errors. In this regard, this article proposes optimization models for determining screening limits on measured values so that the probabilities of two types of misclassification errors (i.e., falsely accepting defectives and falsely rejecting conforming items) may properly be compromised. The assessment of misclassification errors is first investigated in section 2, based on which the screening limits on measured values may be determined. A numerical example is then provided to demonstrate the proposed models in section 3, and conclusions are drawn in the last section.

2. ASSESSMENT OF INSPECTION ERRORS

Measurement errors are commonly incurred which may lead to the misclassification of outgoing products such as a false acceptance of defective items and a false rejection of conforming ones. The false rejection of conforming products is often referred to as a type I error (or producer's risk) whereas the false acceptance of defectives as a type II error (or consumer's risk). Since these misclassification errors may result in economic penalties, there is a need for incorporating the impacts of misclassification errors when designing screening procedures. The effects of misclassification errors are depicted in Figure 1, where L and U represent the lower and upper specification limits, respectively, and x denotes the true value of the quality characteristic. The big curve represents the density function of the actual value of the quality characteristic while the small curve represents the density function of the realization of measurement given the actual value. It is worth noting that the probability of misclassification errors may be adjusted by setting the screening limits on measured values different from the specification limits. When the screening limits are located inside the specification limits, for instance, the probability of a false acceptance error may be reduced with a higher risk of false rejection. On the other hand, a lower probability of a false rejection error may be achieved at a higher risk of false acceptance by setting the screening limits outside the specification limits.

Let X be the actual value of the quality characteristic of interest, which is normally distributed with mean μ and variance σ_x^2. Denoting the measured value of X by Y, further assume that the conditional distribution of Y, given that $X=x$, is normally distributed with mean x and variance $\sigma_{y|x}^2$. Suppose that the conformance of a product is determined on the basis of measurement in spite of measurement errors. Then, a product passes the inspection and is shipped to the customer if $Y \in [v, w]$, where v and w represent the lower and upper screening limits on measured values, respectively. Let β represent the probability of a false acceptance error, that is, $\beta = P(Y \in [v,w] \mid X \notin [L,U])$, which is then given by

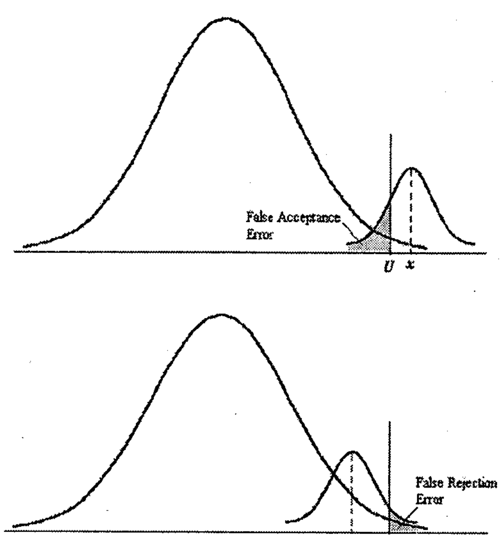
$$\beta = \left[\int_U^{\infty} \int_v^{w} h(x,y)\,dxdy + \int_v^{w} \int_{-\infty}^{L} h(x,y)\,dxdy \right] \times \left[1 - \int_L^U f(x)dx \right]^{-1} \qquad (1)$$

where $f(x)$ and $h(x,y)$ represent the marginal density function of X and the joint density function of X and Y, respectively. It can easily be shown that X and Y jointly follow a bivariate normal distribution with a mean vector of (μ, μ) and a variance-covariance matrix of Σ given by

$$\Sigma = \begin{bmatrix} Var(X) & Cov(X,Y) \\ Cov(X,Y) & Var(Y) \end{bmatrix} = \begin{bmatrix} \sigma_x^2 & \sigma_x^2 \\ \sigma_x^2 & \sigma_y^2 \end{bmatrix},$$

where σ_y^2 represents the variance of marginal distribution of Y, and $\sigma_y^2 = \sigma_x^2 + \sigma_{y|x}^2$. Note that the correlation coefficient of X and Y, denoted by γ, is defined as

$$\gamma = \frac{Cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}} = \frac{\sigma_x}{\sigma_y}.$$



[Fig. 1] False Acceptance and False Rejection Errors.

It can easily be shown that equation (1) is expressed as follows:

$$\beta = \begin{bmatrix} \Phi\left(\dfrac{w-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}\right) - \Phi\left(\dfrac{v-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}\right) \\ + BVN\left(\dfrac{w-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{U-\mu}{\sigma_x}; -\gamma\right) \\ - BVN\left(\dfrac{w-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{L-\mu}{\sigma_x}; -\gamma\right) \\ - BVN\left(\dfrac{v-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{U-\mu}{\sigma_x}; -\gamma\right) \\ + BVN\left(\dfrac{v-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{L-\mu}{\sigma_x}; -\gamma\right) \end{bmatrix} \times \left[1 - \Phi\left(\dfrac{U-\mu}{\sigma_x}\right) + \Phi\left(\dfrac{L-\mu}{\sigma_x}\right)\right]^{-1} \quad (2)$$

On the other hand, the probability of a false rejection error, denoted by α, can similarly be derived as follows:

$$\alpha = \begin{bmatrix} \Phi\left(\dfrac{w-\mu}{\sigma_x}\right) - \Phi\left(\dfrac{v-\mu}{\sigma_x}\right) \\ + BVN\left(\dfrac{w-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{U-\mu}{\sigma_x}; -\gamma\right) \\ - BVN\left(\dfrac{w-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{L-\mu}{\sigma_x}; -\gamma\right) \\ - BVN\left(\dfrac{v-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{U-\mu}{\sigma_x}; -\gamma\right) \\ + BVN\left(\dfrac{v-\mu}{\sqrt{\sigma_x^2+\sigma_{y|x}^2}}, -\dfrac{L-\mu}{\sigma_x}; -\gamma\right) \end{bmatrix} \times \left[\Phi\left(\dfrac{U-\mu}{\sigma_x}\right) - \Phi\left(\dfrac{L-\mu}{\sigma_x}\right)\right]^{-1} \quad (3)$$

To evaluate α and β, we first need to calculate the bivariate normal probabilities. This article uses the well-known numerical method developed by Drezner and Wesolowsky (1990) for evaluating the bivariate normal integrals. Readers are also referred to Drezner (1976) and Mee and Owen (1983).

Screening procedures may be considered a class of hypothesis testing to determine whether an outgoing item is conforming or not. The general procedure in hypothesis testing is to specify a value of the probability of type I error α, and then to design a test procedure so that a small value of the probability of type II error β is obtained. This may formally be expressed as

$$\begin{aligned} &\text{Minimize} \quad \beta \\ &\text{subject to} \quad \alpha \le \alpha_0 \end{aligned}$$

where α and β are given in equations (2) and (3), respectively, and α_0 denotes the maximum allowable value of α.

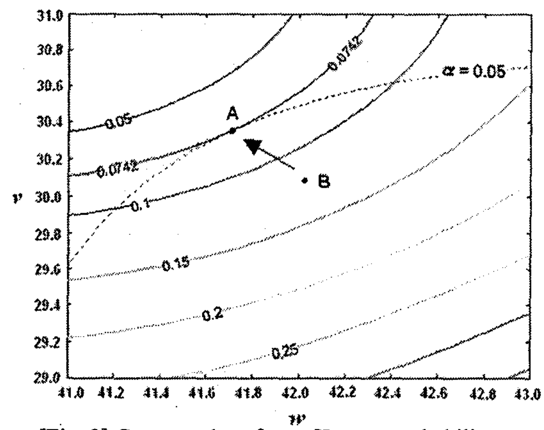
3. A NUMERICAL EXAMPLE

To demonstrate the proposed model, consider the example of an optical scanning device illustrated in Mader *et al.* (1999). The device uses a lamp to illuminate the target and the important quality characteristic of the lamp is the luminance measured in candelas per square meter (cd/m^2). The luminance for the lamps is normally distributed with a mean of 35,200 cd/m^2 and a standard deviation of 4,100 cd/m^2 (i.e., $\mu =$ 35,200 and $\sigma_x = 4{,}100$). The upper and lower specification limits for the luminance are 30,000 and 42,000 cd/m^2, respectively, and thus L=30,000 and U=42,000. Further, the variability of a single measurement given the actual value of the quality characteristic is $\sigma_{y|x} = 774.6$ cd/m^2. It is a common practice to design a hypothesis testing procedure to minimize β while limiting the probability of type I error at 5%, i.e., $\alpha_0 = 0.05$. The screening limits on measured values can then be determined by solving the optimization model in equation (4), which involves a great deal of computational resources mainly due to the evaluation of bivariate normal probabilities. In this respect, an approximation method using the Gaussian quadrature formulas based on Legendre polynomials (Drezner and Wesolosky, 1990) is implemented to evaluate the bivariate normal integrals. A popular mathematical software Matlab is used to find the optimal solution by implementing a well-known numerical optimization technique by Hooke and Jeeves (1966). The optimal solution to the example problem is found to be v^*=30351.3 and w^*=41701.5 with β^*=0.0742. The contour plot of type II error probability β with respect to v and w is depicted in Figure 2, where the probability of type I error is to be 0.05 along the dotted line. If outgoing items are screened against the specification limits, the type I and type II misclassification errors are 0.0310 and 0.1284, respectively, which is represented as point B in Figure 2. Referring to point A in Figure 2, the probability of a false acceptance error is significantly reduced by 0.0542 (from 0.1284 to 0.0742) while the probability of a false rejection error increases by 0.0190 (from 0.0310 to 0.0500).

4. CONCLUSIONS

One of the most important aspects in designing screening procedures is implementing adequate screening limits to ensure the outgoing product quality. Regardless of how precise the measurement procedures are, however, measurement errors are commonly incurred which may lead to a wrong interpretation on product quality.

Nonconforming items may falsely be accepted, and conforming items may also be misclassified as defectives. It may thus be necessary to consider the impacts of misclassification errors when designing screening procedures. Along this line, this article investigates the design of screening procedures based on the assessment of inspection errors. Optimization models are proposed to determine the screening limits on measured values so that the impacts of inspection errors may be reduced.



[Fig. 2] Contour plot of type II error probability β with respect to v and w.

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