# 다중반응표면 최적화 분야의 최근 연구 동향
# Recent Advances in Multiresponse Systems

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**ABSTRACT**

A common problem encountered in product or process design is the selection of optimal parameter levels which involve simultaneous consideration of multiresponse variables. To date, various methods have been proposed for multiresponse optimization. In this paper, we briefly review the existing methods and then discuss some recent advances in this field.

## 1. INTRODUCTION

Most of the work in response surface methodology has focused on the case where there is only one response of interest. However, a common problem in product or process design is to determine the optimal parameter levels when there are multiple responses which should be considered simultaneously. Such a problem is called a multiresponse problem [13].

To date, various methods have been proposed for multiresponse optimization. In this paper, we briefly review the existing methods and then discuss some recent advances in this field.

## 2. EXISTING APPROACHES IN MULTIRESPONSE OPTIMIZATION

### 2.1. Priority-based approach

The priority-based approach selects the most important response among a number of responses and uses it as the objective function. Myers and Carter [23] proposed an optimization formulation that maximizes (or minimizes) the primary response with an equality constraint on the other response. Biles [2] extended this idea by allowing not only more than two responses, but also inequality constraints on the secondary responses.

### 2.2. Desirability function approach

The desirability function approach transforms an estimated response (e.g., the $i$th estimated response $\hat{y}_i$) into a scale-free value, called a desirability (denoted as $d_i$ for $\hat{y}_i$). The overall desirability $D$ is defined by combining the individual desirability values (i.e., $d_i$'s). Then, the optimal setting is determined by optimizing $D$.

Harrington [8] first proposed a simple form of a desirability function. Derringer and Suich [6] extended Harrington's approach by suggesting a more systematic transformation scheme from $\hat{y}_i$ to $d_i$.

### 2.3. Loss function approach

The loss function approach aims to find the optimal parameter setting by minimizing the expected loss function. Pignatiello [27] first proposed the use of a squared error loss function in multiresponse optimization.

Vining [30] proposed a modification to the Pignatiello's model by employing a vector of the estimated responses $\hat{\mathbf{y}}(\mathbf{x})$ in loss function, instead of $\mathbf{y}(\mathbf{x})$. Ko *et al.* [15] proposed an improvement over the Pignatiello's and Vining's models. They employ $\hat{\mathbf{y}}_{new}(\mathbf{x})$ in the loss function, as opposed to $\mathbf{y}(\mathbf{x})$ in the Pignatiello's or $\hat{\mathbf{y}}(\mathbf{x})$ in Vining's model.

### 2.4. Process capability approach

The process capability approach derives a process capability index using the estimated mean and standard deviation of a response. The overall capability index is obtained by combining the individual process capability indices. Then, the optimal setting is determined by maximizing the overall capability index.

Barton and Tsui [1] proposed a performance centering as a process capability index. Plante [28] extended the Barton and Tsui's approach by developing several multicriteria models based on the performance centering. Plante [29] proposed the use of two typical process capability indices, *Cpk* and *Cpm*.

### 2.5. Probability-based approach

The probability-based approach assumes a multivariate probability distribution of a multivariate response **Y**. It first models the distributional parameters in terms of input variables and then finds the optimal setting which maximizes the probability that all responses simultaneously meet their specifications.

Chiao and Hamada [4] first proposed this approach with a multivariate normal distribution assumption. Peterson [26] and Miró-Quesada *et al.* [20] estimated the distributional parameters in the multivariate $t$ distribution using a Bayesian approach.

## 3. RECENT RESEARCH ISSUES

### 3.1. Interactive approach to multiresponse optimization

Most of the existing work in multiresponse optimization is categorized into prior preference articulation methods [24, 25]. Recently, Jeong and Kim [9, 10] and Köksalan and Plante [16] proposed an interactive method. Although not included in the major approaches, Montgomery and Bettencourt [22], Mollaghasemi and Evans [21] and Boyle and Shin [3] also proposed interactive methods.

Interactive methods are desirable in that it is easy and effective to extract the DM's preference since he/she has only to provide the information by a local level in an interactive manner.

### 3.2. Consideration of both mean and variability

The major focus of the existing approaches to multiresponse optimization is on the location effect only, ignoring the dispersion effect of the responses. Kim and Lin [14] proposed an integrated modeling approach to simultaneously optimize both the location and dispersion effects of multiple responses. Lee and Kim [17] also proposed an expected desirability concept to consider both the location and dispersion effects in the well-known desirability function framework.

### 3.3. Determination of weights for bias and variance in dual response optimization

Lin and Tu [19] proposed a simple, yet effective approach based on mean squared error (MSE) minimization. A natural extension of MSE, called a weighted MSE (WMSE), is formed by imposing the relative weights on the bias and variance terms. Jeong et al. [11] proposed a systematic method to determine the relative weights of bias and variance in accordance with a decision maker's preference structure. As an extension of the aforementioned work, Jeong et al. [12] proposed a scheme to construct a probability distribution of the relative weight using the Bayesian approach.

### 3.4. Data mining approach to multiresponse optimization

The patient rule induction method (PRIM) [7] is an alternative method to the response surface methods. PRIM aims to find optimal input variables directly from historical data without constructing an explicit functional model. PRIM has been applied successfully to various areas such as geology, finance, bioinformatics, and process optimization [5]. The conventional PRIM has been developed and applied for the single response case. Lee and Kim [18] proposed an extended version of PRIM, called MR-PRIM, to accommodate multiple responses.

## 4. CONCLUDING REMARKS

The existing work in multiresponse optimization has been reviewed. Some recent advances have also been discussed. The research in this field has been quite active in the literature in recent years. Considering its applicability in real-world problems, more research endeavors are warranted in the future.

## REFERENCE


[1] Barton, R. S. and Tsui, K. L., "Multivariate Yield Maximization Using CAD/CAE Models: Efficient Approximations Based on Mean and Variance," *Design Theory and Methodology (ASME)*, Vol. 31(1991), pp.31-35.

[2] Biles, W. E., "A Response Surface Methods for Experimental Optimization of Multi-Response Processes," *Industrial and Engineering Chemistry Process Design and Deployment*, Vol.14(1975), pp.152-158.

[3] Boyle, C. R. and Shin, W. S., "An Interactive Multiple-Response Simulation Optimization Method," *IIE Transactions*, Vol.28(1996), pp.453-462.

[4] Chiao, C. and Hamada, M., "Analyzing Experiments with Correlated Multiple Responses," *Journal of Quality Technology*, Vol.33(2001), pp.451-465.

[5] Chong, I., Albin, S., and Jun, C., "A Data Mining Approach to Process Optimization without an Explicit Quality Function," Working Paper, (2006).

[6] Derringer, G. and Suich, R., "Simultaneous Optimization of Several Response Variables," *Journal of Quality Technology*, Vol.12(1980), pp.214-219.

[7] Friedman, J. and Fisher, N., "Bump Hunting in High Dimensional Data," *Statistics and Computing*, Vol.9(1999), pp.123-143.

[8] Harrington, E., Jr., "The Desirability Function," *Industrial Quality Control*, Vol.21(1965), pp.494-498.

[9] Jeong, I. and Kim, K., "Interactive Desirability Function Approach to Multi-Response Surface Optimization," *International Journal of Reliability, Quality and Safety Engineering*, Vol.10(2003), pp.205-217.

[10] Jeong, I. and Kim, K., "D-STEM: A Modified Step Method with Desirability Function Concept," *Computers and Operation Research*, Vol.32, No.12(2005), pp.3175-3190.

[11] Jeong, I., Kim, K., and Chang, S., "Optimal Weighting of Bias and Variance in Dual Response Surface Optimization," *Journal of Quality Technology*, Vol.37, No.3(2005), pp.236-247.

[12] Jeong, I., Kim, K., and Lin, D., "Bayesian Analysis for Weighted Mean Square Error in Dual Response Surface optimization," Working Paper, (2006).

[13] Khuri, A. I., "Multiresponse Surface Methodology". *In Handbook of Statistics: Design and Analysis of Experiment (Vol. 13)* (eds. A. Ghosh and C. R. Rao), pp. 377-406, 1996.

[14] Kim, K. and Lin, D., "Optimization of Multiple Responses Considering Both Location and Dispersion Effects," *European Journal of Operational Research*, Vol.169,No.1(2006), pp.133-145.

[15] Ko, Y., Kim, K., and Jun, C., "A New Loss Function- Based Method for Multiresponse Optimization". *Journal of Quality Technology*, Vol.37(2005), pp.50-59.

[16] Köksalan, M. and Plante, R. D., "Interactive Multicriteria Optimization for Multiple-Response Product and Process Design," *Manufacturing and Service Operations Management*, Vol.5(2003), pp.334-347.

[17] Lee, M. and Kim, K., "Expected Desirability Function: Consideration of Both Location and Dispersion Effects in Desirability Function Approach," *Quality Technology and Quantitative Management*, Accepted for Publication, (2006).

[18] Lee, M. and Kim, K., "Optimization of Multiple Responses in a Steel Manufacturing Process via PRIM," Working Paper, (2006).

[19] Lin, D. and Tu, W., "Dual Response Surface Optimization," *Journal of Quality Technology*, Vol.27(1985), pp.34-39.

[20] Miró-Quesada, G., Del Castillo, E., and Peterson, J. J., "A Bayesian Approach for Multiple Response Surface Optimization in the Presence of Noise Variables," *Journal of Applied Statistics*, Vol.31(2004), pp.251-270.

[21] Mollaghasemi, M. and Evans, G. W., "Multicriteria Design of Manufacturing Systems through Simulation Optimization," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol.24(1994), pp.1407-1411.

[22] Montgomery, D. C. and Bettencourt, V. M., Jr., "Multiple Response Surface methods in Computer Simulation," *Simulation*, Vol.29(1977), pp.113-121.

[23] Myers, R. H. and Carter, W. H., "Response Surface Techniques for Dual Response Systems," *Technometrics*, Vol.15(1973), pp.301-317.

[24] Park, K. and Kim, K., "Optimizing Multi-Response Surface Problems: How to Use Multi-Objective Optimization Techniques," *IIE Transactions*, Vol.37, No.6(2005), pp.523-532.

[25] Park, K., Kim, K., and Moskowitz, H., "Multi-Response Surface Optimization and Multi-Objective Optimization: Relationships and Directions," *New Frontiers of Decision Making for the Information Technology Era* (Eds. Shi and Zeleny). World Scientific Publishing, Singapore, pp. 289-303, 2000.

[26] Peterson, J. J., "A Posterior Predictive Approach to Multiple Response Surface Optimization," *Journal of Quality Technology*, Vol.36(2004), pp.139-153.

[27] Pignatiello, J., "Strategies for Robust Multiresponse Quality Engineering," *IIE Transactions*, Vol.25(1993), pp.5-15.

[28] Plante, R. D., "Multicriteria Models for the Allocation of Design Parameter Targets," *European Journal of Operational Research*, Vol.115(1999), pp.98-112.

[29] Plante, R. D., "Process Capability: A Criterion for Optimizing Multiple Response Product and Process Design". *IIE Transactions*, Vol.33(2001), 497-509.

[30] Vining, G., "A Compromise Approach to Multiresponse Optimization," *Journal of Quality Technology*, Vol.30(1998), pp.309-313.