Optimal Booking Limit Decision in the Presence of Strategic Customer Behavior

Sang-Won Kim
Agency for Defense Development
Taejon, Republic of Korea
studyksw2001@yahoo.com

Abstract

We consider a two-period airline revenue management problem where customers may act strategically. Specifically, we study a two-fare-class airline seat inventory allocation problem which allow for the possibility that a customer may decide to defer to purchase in the hope that a cheaper ticket than those currently on offer (expensive tickets) become available. We also allow for the possibility that some customer will buy a more expensive ticket if the cheaper tickets are not available. We show how to find the optimal booking limits in the presence of such strategic customer behavior and investigate the impact of such strategic customer behavior on the expected revenue. The results are compared with those by the expected marginal seat revenue (EMSR) heuristic approach (Belobaba, 1987, 1989) with strategic customer behavior.

1. Introduction

Airline seat inventory control concerns optimally allocating flight capacity across a set of fare classes. The first published research (Littlewood, 1972) provided a useful analysis of a simple two-fare class model of seat capacity allocation on a single flight leg. He suggested that the point of closing-down the low fare class was when the certain marginal revenue from selling an additional low fare seat was exceeded by the expected marginal revenue of selling the same seat at the higher fare. (Belobaba, 1987) extended Littlewood's work to beyond two fare classes. He first coined expected marginal seat revenue (EMSR) in which Littlewood's rule is applied sequentially in an increasing fare order. The booking limit is a control variable. many flexible travelers who are willing to pay full price would take a discount unit if discount fares were unavailable. This type of customer behavior is referred to as diversion. Belobaba (1987) considered customer diversion in his Ph.D. dissertation and presented a variant of the EMSR heuristic that considered the probability of buying a high-fare when a low-fare was unavailable. Many research papers deal with customer diversion. Brumelle *et al.* (1990) considered two possibly dependent booking classes and derived optimality conditions, assuming that the demands were independent initially. They looked at the effect of diversion by taking into account the probability of an upgrade if a customer was denied a discount seat. Pfeifer (1989) examined a two-fare airline seat allocation problem, which was an extension of the single-period stochastic inventory or the newsvendor problem. He assumed that a customer might buy a more expensive ticket if a less expensive ticket was not available and developed a decision rule.

Bodily and Weatherford (1995) developed a new decision rule for the airline seat allocation problem with customer diversion. Belobaba and Weatherford (1996) developed a new generic decision rule not only for airline seat allocation models but also for general perishable asset revenue management models. The approach was to find the optimal booking limits based on the EMSR approach while considering customer diversions. They used the same decision rules derived by Bodily and Weatherford (1995) and developed a new combined heuristic decision rule for more than three fare classes. The decision rules were compared with EMSR. Weatherford, Bodily and Pfeifer (1993) developed an optimal decision rule for airline revenue management problems with customer diversion using a Bayesian statistics approach based on the EMSR model. They developed heuristic approaches and derived an optimal decision rule with customer diversion for two-fare class models. Sen and Zhang (1999) considered a single-period stochastic inventory problem where the item could be sold to different demand classes at different prices. The objective was to find the optimal replenishment quantity and protection level to maximize the expected profits. In their increasing price model, some of the customers in the lower price demand class might be ready to pay the price of the higher fare class if they were not able to buy the product at the price they had requested. Customer diversion was modeled by assuming that a fixed portion of the unsatisfied lower price demand would join the higher price demand. Anderson and Wilson (2003) studied multi-period seat inventory allocation problems with fixed optimal booking limits by EMSR and strategic customer behavior. Strategic customer behavior was used for situations when customers postpone their purchase decisions in anticipation of opening of the lower price ticket in the next period. Customer might decide to wait for the reopening of a cheaper fare in the future if the cheap fare tickets were sold out in the current period. Buy-up refers to the customer buying a higher fare ticket when lower fare tickets are closed whereas buy-down refers to the substitution of a lower fare for a higher fare customer when lower fares are still open. Anderson and Wilson (2003) considered both buy- down and strategic customer behavior in their model and investigated the impact of these customers' behavior on the total expected profit.

In this e-commerce age, customers are becoming familiar with the existence of pricing and revenue management structures employed by the airline companies, and their buying behavior is becoming more complex. Due to the flexible customers who are willing to pay a higher fare or wait for reopening a lower fare in the subsequent period, the realized demand for a fare class is not statistically independent.

Therefore, customer diversion and strategic customer behavior are important phenomena that have a profit implication in airline seat inventory allocation. We study the way in which an airline can change its strategy for setting up optimal booking limits if customers behave strategically. The purpose of this study is to investigate these impacts on the optimal booking limits and total expected revenues in the presence of strategic customer behavior.

2. Two-Period Model

We will analyze the situation where there are two periods and two fare classes. Assume that in period i, a fraction d_i, for $i \in$ 1, 2, of customers will purchase the more expensive ticket if the cheap one is not available. We allow for strategic customer behavior by assuming that a fraction, w, will wait until period 2 if a cheap fare is not available in period 1. The capacity at the beginning of period 1 will be denoted C, and the capacity at the beginning of period 2 will be denoted c with $C - c$ (>0) seats sold in period 1. There are two fare classes with revenues of r_1 and r_2 where $r_1 < r_2$. For $i, j \in$ 1, 2, the demand for fare class j in period i will be denoted by D_{ij}. The corresponding density function and distribution function will be denoted by $f_{ij}(D_{ij})$ and $F_{ij}(D_{ij})$ each.

2.1 First Period

The objective is to set the booking limit l_1 to maximize revenues. For the first period, the contribution maximization problem is

$$\underset{l_1}{Max}\, E(\pi_1) = r_1 E(D_{11}^*) + r_2 E(D_{12}^*) \tag{1}$$

Expected demand is

$$\begin{aligned} E(D_{11}^*) &= \int_0^{l_1} D_{11} f_{11}(D_{11}) dD_{11} + \int_{l_1}^{\infty} l_1 f_{11}(D_{11}) dD_{11} \\ E(D_{12}^*) &= \int_0^{l_1}\int_0^{C-D_{11}} D_{12} f_{12}(D_{12}) dD_{12} f_{11}(D_{11}) dD_{11} \\ &+ \int_0^{l_1}\int_{C-D_{11}}^{\infty} (C-D_{11}) f_{12}(D_{12}) dD_{12} f_{11}(D_{11}) dD_{11} \\ &+ \int_0^{C-l_1}\int_{l_1}^{\frac{(C-l_1+d_1 l_1-D_{12})}{d_1}} (D_{12}+d_1(D_{11}-l_1)) f_{11}(D_{11}) dD_{11} f_{12}(D_{12}) dD_{12} \\ &+ \int_0^{C-l_1}\int_{\frac{(C-l_1+d_1 l_1-D_{12})}{d_1}}^{\infty} (C-l_1) f_{11}(D_{11}) dD_{11} f_{12}(D_{12}) dD_{12} \\ &+ \int_{C-l_1}^{\infty}\int_{l_1}^{\infty} (C-l_1) f_{11}(D_{11}) dD_{11} f_{12}(D_{12}) dD_{12} \end{aligned} \tag{2}$$

The first-order condition for optimal booking limit with respect to l_1 is

$$\begin{aligned} \frac{\partial E(\pi_1)}{\partial l_1} &= (r_1 - d_1 r_2)(1-F_{11}(l_1)) - r_2(1-d_1)(1-F_{11}(l_1))(1-F_{12}(C-l_1)) \\ &- r_2(1-d_1)\int_0^{C-l_1}\left\{1-F_{11}\left\{\frac{(C-l_1+d_1 l_1-D_{12})}{d_1}\right\}\right\} f_{12}(D_{12}) dD_{12} = 0 \end{aligned} \tag{3}$$

2.2 Expected Revenues and Booking Limits for Period Two

Consider the case where the booking limit for the low fare is l_1 in period 1, D_{11} is less than l_1 and there is capacity c going into the second period. Then, the optimal booking limit, $l_2(c)$, for the low fare in period 2 is either 0, c or the value of l_2 that satisfies the following:

$$\begin{aligned} \frac{\partial E(\pi_2)}{\partial l_2} &= (r_1 - r_2) + (r_2 - r_1)P[D_{21} < l_2] - r_2(1-d_2)P(D_{22} \le c - l_2) \\ &+ r_2(1-d_2)\int_0^{c-l_2} P\left[D_{21} < \frac{c - l_2(1-d_2) - x}{d_2}\right] f_{22}(x) dx = 0 \end{aligned} \tag{4}$$

Now suppose that l_1 is the booking limit in period 1, that all of these seats are sold out and that there are c unsold seats going into the second period. In this situation, all that is known about the demand for the cheap seats in period 1 is that it was at least l_1. The conditional distribution function for the number of customers, Y, who will wait until the second period is

$$\begin{aligned} P(Y \le y) &= P(w(D_{11} - l_1) \le y \mid D_{11} \ge l_1) \\ &= P(l_1 \le D_{11} \le \frac{y}{w} + l_1)\frac{1}{P(D_{11} \ge l_1)}. \end{aligned} \tag{5}$$

Differentiate the above to see that the density function for Y is given by

$$f_Y(y) = \frac{f_{11}(l_1 + y/w)}{wP(D_{11} \ge l_1)} \quad \text{for } y \ge 0. \tag{6}$$

The demand for low fares in period 2 is D_{21} plus the demand from those customers who wait from period 1. Obtaining the optimal booking limit for the second period now becomes a one-period problem and can be found from (4) by replacing D_{21} with $D_{21} + Y$. Denote this random variable by $D_{21}(l_1)$. Replace $P[D_{21} < l_2]$ in (4) with

$$\begin{aligned} P(D_{21}(l_1) < l_2) &= P(D_{21} + Y < l_2) \\ &= \int_0^{l_2} P(D_{21} < l_2 - y) f_Y(y) dy \\ &= \frac{\int_0^{l_2} P(D_{21} < l_2 - y) f_{11}(l_1 + \frac{y}{w}) dy}{wP(D_{11} \ge l_1)} \end{aligned} \tag{7}$$

Similarly, replace $P\left[D_{21} < \frac{c - l_2(1-d_2) - x}{d_2}\right]$ in (4) with

$$P\left[D_{21}(l_1) < \frac{c - l_2(1-d_2) - x}{d_2}\right] \tag{8}$$

Now the optimal booking limit in period 2, $l_2(c,l_1)$, is a function of both l_1 and the number of available seats. Use (7) and (8) in (4) to see that $l_2(c,l_1)$ is either 0, c or the value of l_2 that satisfies the following equation:

$$\begin{aligned} & wP(D_{11} \ge l_1)(r_1 - r_2) + (r_2 - r_1)\int_0^{l_2} P(D_{21} < l_2 - y) f_{11}(l_1 + \frac{y}{w}) dy \\ & - wP(D_{11} \ge l_1) r_2(1-d_2) P(D_{22} \le c - l_2) \\ & + r_2(1-d_2)\int_0^{c-l_2}\int_0^{\frac{c-l_2(1-d_2)-x}{d_2}} P\left[D_{21} < \frac{c - l_2(1-d_2 y)}{d_2}\right] f_{11}(l_1 + \frac{y}{w}) dy f_{22}(x) dx = 0 \end{aligned} \tag{9}$$

If the capacity going into the second period is c and the booking limit is not reached at l_1 in period 1, then the expected revenue for the second period can be written as:

$$\begin{aligned} & r_1 E\left[min\{D_{21}, l_2(c)\}\right] + \\ & r_2 E\left[min\{max\{c - l_2(c), c - D_{21}\}, D_{22} + d_2(D_{21} - l_2(c))^+\}\right] \end{aligned} \tag{10}$$

If the capacity is c going into the second period and the booking limit l_1 is reached in period 1, then the expected return in the second period is given by (10) with D_{21} and l_2(c) replaced by $D_{21}(l_1)$ and $l_2(c,l_1)$ respectively.

2.3 Density Functions for Capacity

In order to calculate the expected revenues and optimal booking limits for a two-period problem, we used the same probability density function for capacity at the end of period 1. The capacity remaining at the beginning of the second period, $C(l_1)$, is given by

$$C(l_1) = C - \min\{D_{11}, l_1\} - \min\{\max\{C - l_1, C - D_{11}\}, D_{12} + d_1(D_{11} - l_1)^+\} \quad (11)$$

Let $g_1(c,l_1)$ denote the density function for $C(l_1)$ conditioned on the event that $D_{11} < l_1$ and let $g_2(c,l_1)$ denote the density function conditioned on $D_{11} \geq l_1$ and $C(l_1) > 0$, i.e.

$$g_1(c,l_1) = \frac{\frac{\partial}{\partial c} P[0 < C(l_1) \leq c, D_{11} < l_1]}{P[C(l_1) > 0, D_{11} < l_1]} \quad (12)$$

and,

$$g_2(c,l_1) = \frac{\frac{\partial}{\partial c} P[0 < C(l_1) \leq c, D_{11} \geq l_1]}{P[C(l_1) > 0, D_{11} \geq l_1]} \quad (13)$$

2.4 Total Expected Revenues and Booking Limits

Suppose the booking limit for the first period is l_1 and that optimal booking limits are $l_2(c)$ and $l_2(c,l_1)$ for the second period are found using the procedure of section 2.2. We will show how to write the expected return for both periods, assuming l_1 is the first-period booking limit. The expected revenue for the first period can be written as

$$r_1 E[\min\{D_{11}, C, l_1\}] + r_2 E[\min\{\max\{C - l_1, C - D_{11}\}, D_{12} + d_1(D_{11} - l_1)^+\}] \quad (14)$$

If demand for the low fares in the first period does not reach the booking limit l_1, then no customer needs to wait until period 2 to obtain a low fare. The contribution to the total expected revenue is given by

$$P[D_{11} < l_1, C(l_1) > 0] \left\{ \begin{array}{l} \int_0^C r_1 E[\min\{D_{21}, c, l_1\}] g_1(c,l_1) dc + \\ \int_0^C r_2 E\left[\min\left\{ \begin{array}{l} \max\{c - l_2(c), c - D_{21}\}, \\ D_{12} + d_2(D_{21} - l_2(c))^+ \end{array} \right\}\right] g_1(c,l_1) dc \end{array} \right\} \quad (15)$$

Otherwise, a fraction of those who could not get a low fare w will wait until the second period. The contribution to the total expected revenue is provided by

$$P[D_{11} \geq l_1, C(l_1) > 0]\left\{ \int_0^C r_1 E[\min\{D_{21}(l_1), c, l_2(c,l_1)\}] g_2(c,l_1) dc \right\} + P[D_{11} \geq l_1, C(l_1) > 0]\left\{ \int_0^C r_2 E\left[\min\left\{ \begin{array}{l} \max\{c - l_2(c,l_1), c - D_{21}(l_1)\}, \\ D_{12} + d_2(D_{21}(l_1) - l_2(c,l_1))^+ \end{array} \right\}\right] g_2(c,l_1) dc \right\} \quad (16)$$

The expected return for the two periods is given by adding (14), (15) and (16). In the following section, we provide numerical examples and investigate the impact of strategic customer behavior under various conditions and parameter settings.

2.5 Example

The following are a series of numerical examples to illustrate the impacts of model parameters upon total expected revenue.

Let C = 35, r_1 = 1 and r_2 = 2. Basically, 30% of customers who cannot get a saver fare ticket are willing to wait from period 1 to period 2 for saver fares and 30% of customers who cannot get a saver fare ticket are also willing to buy more expensive fares. The demand is assumed to follow a normal distribution with a mean of 10 and a standard deviation of 3 for each fare class in each period. We assume that the fraction of buying-up in each period has the same value: $d_1 = d_2 = s$. The fraction of buying-up s has a more significant effect than does the fraction of waiting w. In Figure 1, we see that the optimal booking limit increases if we decrease the fraction of buying-up or waiting. The optimal booking limit is quite sensitive to the fraction of buying-up or waiting.

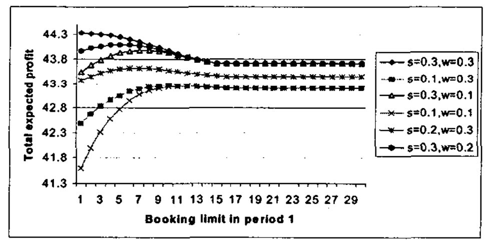

Figure 1. Total expected revenue as a function of the booking limit in period 1 with different combinations of s and w

Figure 2 and Figure 3 present the expected profit curves as a function of the fraction of buying-up at a given w = 0.3 for the case of r_2 = 1.5 and 2, at a given r_1 = 1.5. In both cases, increasing s will change the optimal booking in period 1 and will significantly change the total expected revenue.

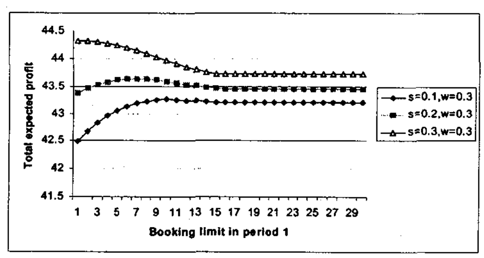

Figure 2. Total expected revenue as a function of the booking limit in period 1 with different combinations of s and w where the full fare is 1.5

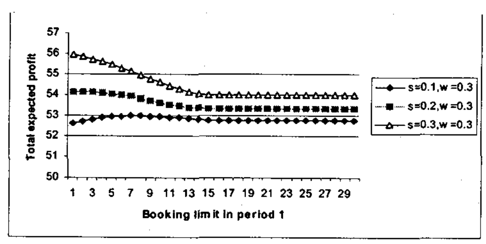

Figure 3. Total expected revenue as a function of the booking limit in period 1with different combinations of s and w where the full fare is 2.0

Consider the case where the fraction of buying up is 0.1, the saver fare is 1 and the full fare is 2. Figure 4 displays optimal booking levels and expected revenues as a function of the fraction of waiting. As the fraction of waiting increases from 0.1 to 0.5, the optimal booking limit in period 1 decreases from 9 to 2. For the case where the fraction willing to wait is 0.1, Figure 5 displays the optimal booking limits and the expected revenue as a function of the fraction of buying-up. When the fraction of willing to buy up is 0.4, the optimal booking limit in period 1 is zero. Figures 4 and 5 illustrate that optimal booking limits decrease significantly in the presence of strategic customer behavior.

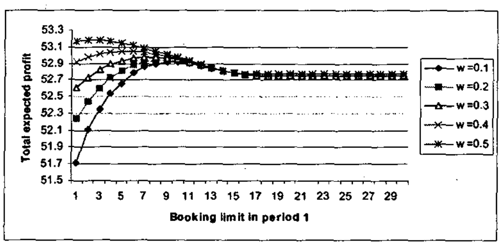

Figure 4. Total expected revenue as a function of period 1 booking limit where the fraction of waiting changes from 0.1 to 0.5

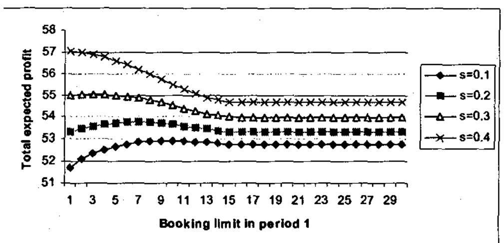

Figure 5. Total expected revenue as a function of period 1 booking limit where the fraction of buying-up changes from 0.1 to 0.4

If no strategic customer behavior is assumed, then the sequential application static single-period models, referred to as advanced static allocation would suggest the optimal booking limit of 15. Table 1 summarizes expected revenues if the airline used EMSR type rules in the presence of strategic customer behavior. As the Table indicates, not accounting for strategic behavior results in revenue losses in excess of 10% under certain parameter settings.

Table 1. Expected revenues using EMSR versus optimal booking limits

% Divert	% Wait	EMSR	Optimal	% Gain
10	10	78.27	78.70	0.54
20	10	78.78	81.01	2.83
30	10	79.09	83.88	6.06
40	10	79.28	87.71	10.63
10	20	78.27	78.79	0.66
10	30	78.27	78.82	0.82
10	40	78.27	79.06	1.01

3. Conclusion

Customer diversion for one-period models has been investigated by a number of researchers. Finding optimal booking limits for multi-period perishable asset management models has proven to be a difficult task. Many of the multi-period models in the literature are heuristic in nature. Allowing customers to behave strategically by either diverting to another product or waiting to see whether or not a cheaper product will become available adds greatly to the modeling complexity. The contribution of this paper is to demonstrate that finding optimal booking limits for a two-period model where customers may wait can ce reduced to solving a one-dimensional problem.

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