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선박용 디지털 컴퍼스에 적용하기 위한 지구편차 모형 개발

람파드하사하* · 임정빈**
*목포해양대학교 대학원 석사과정, **목포해양대학교 해상운송시스템학부 교수

A Study on the Earth's Variation Model to Adopt Ship's Digital Compass

Rampadha Saha · Jeong-Bin Yim***

**Graduate School of Mokpo Maritime University, Mokpo 530-729, Korea*

***Division of Maritime Transportation System, Mokpo Maritime University, Mokpo 530-729, Korea*

ABSTRACT : *The Earth's spherical harmonic model of the main field and of the secular variation, of the geomagnetic field gives the intensity and geomagnetic structure at any location around the earth, assuming an undistorted, steady state field that no external sources or localized earth anomalies. To consider the practical use of a ship's digital compass in earth's magnetic field, Earth's spherical harmonic model is searched for the related practical methods and procedures as a basic study in this work.*

KEY WORDS : geomagnetic field, secular variation, geomagnetic variation, spherical harmonic model, digital compass,

요 약 : 지구자장의 주자장과 경년변화에 대한 원통형 지구 조화 모델은, 지구가 지엽적으로 불균형 또는 외부 소스가 없는 정상상태라고 가정하는 경우, 어느 지역의 지구자장 구조와 자장의 세기를 나타낼 수 있다. 이 연구에서는 기초 연구로서 선박용 디지털 컴퍼스를 실제 지구자장에 적용하는 경우의 원통형 조화 모델에 대한 관련 방법과 절차 등을 조사하였다.

핵심용어 : 지구자기장, 경년변화, 자차, 원통형 조화 모델, 디지털 컴퍼스

1. Introduction

The Earth's magnetic field is neither uniform, stationary, nor perfectly aligned with the planet, is approximately a magnetic dipole, with one pole near the North Pole and the other near the geographic South Pole. The Earth's magnetic field is generated in the fluid outer core by a self-exiting dynamo process. Electrical currents flowing in the slowly moving molten iron generated the magnetic field. In addition to sources in the Earth's core the magnetic field observable at the Earth's surface has sources in the crust, in the ionosphere and magnetosphere. The intensity and structure of the earth's magnetic field are always changing, slowly but erratically reflecting the influence of the flow of thermal currents within the iron core. It varies on a range of scales and a description of these variations is now made, in the order low frequency to high frequency variations, in both the space and time domains. The field, as measured by magnetic sensor on or above Earth's surface.

* 학생회원 : jbyim@mmu.ac.kr 061)240-7051
**종신회원 : jbyim@mmu.ac.kr 061)240-7051

2. The Earth's Variation

2.1 Observation

The Earth's magnetic field (B) is a vector quantity varying in space (r) and time (t). The field, as measured by a magnetic sensor on or above Earth's surface, is actually a composite of several magnetic fields, generated by a variety of sources. These fields are superimposed on each other and through inductive processes interact with each other. The most important of these geomagnetic sources are (a) the main field generated in earth's conducting fluid outer core (B_m), (b) the crustral field from earth's crust/upper mantle (B_c), (c) the combined disturbance field from electrical currents flowing in the upper atmosphere and magnetosphere, which also induce electrical currents in the sea and the ground (B_d).

Thus, the observed magnetic field is a sum of combination as in Eq.(1),

$$B(r,t)=B_m(r,t)+B_c(r)+B_d(r,t). \qquad (1)$$

Where B_m is the dominating part of the field, accounting for over 95% of the field strength at the earth's surface. Secular variation is the slow change in time of B_m. B_c. the field arising from magnetized crustral rocks, varies spatially, but is considered here. B_c is usually much smaller in magnitude than B_m. The crustal field is constant over the time scales considered here. The field arising from currents flowing in the ionosphere and magnetosphere and their resultant induced currents in the earth's mantle and crust, B_d, varies both with location and time.

To create an accurate main field model, it is necessary to have data with good global coverage and as low a noise level as possible. The Denis Ø ersted and German CHAMP satellites data sets satisfy these requirements. Both satellites provide high quality vector and scalar data at all latitudes and longitudes, but not during all latitudes needed for modeling.

The observatory data therefore provide valuable constraints on the time variations of the geomagnetic field. Used together, satellite and observatory data provide an exceptional quality data set for modeling the behavior of the main field magnetic field in space and time.

2.2 Theoretical Modeling

In Eq.(1) B_c has spatial variations on the order of meters to thousands of kilometers and can't be fully modeled with low degree spherical harmonic models. B_c is usually smaller at sea than on land and decreases with increasing altitude. The rock magnetization resulting in B_c may be either induced (by the main magnetic field) or remnant or combination of both.

The field arising from currents flowing in the ionosphere and magnetosphere and their associated induced currents in the earth, B_d varies both with location and time.

Fig. 1 shows the variations currents systems. The disturbance field can vary both regularly, with fundamental periods of one day and one year, as well as irregularly on time scales of seconds to days. The regular variations are both diurnal and annul and they are essentially generated by the daylight atmosphere.

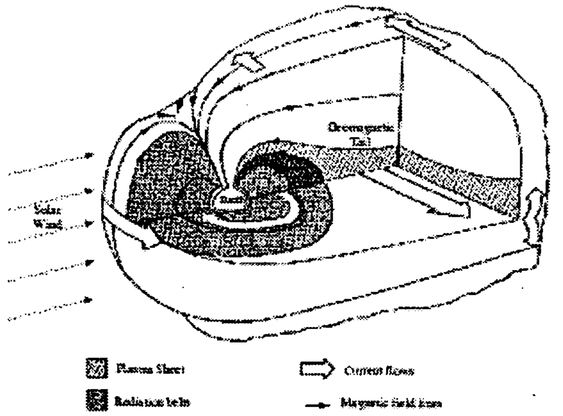

Fig. 1. schematic of the magnetosphere showing the current flows and magnetic field lines(Nils Olsen, 2002).

3. Model Parameterization & Estimation

3.1 Model Parameterization

The geomagnetic field measured at the Earth's surface or at satellite altitude is the sum of the field generated by sources internal or external to the solid Earth. Away from its sources, the internal magnetic field B is a potential field and therefore can be written as the negative gradient of a scalar potential as in Eq.(2) (Nils Olsen, 2002),

$$\begin{aligned} V = a\Bigg(& \sum_{n=1}^{N_{MF}} \sum_{m=0}^{n} (g_n^m \cos m\varnothing + h_n^m \sin m\varnothing)\left(\frac{a}{r}\right)^{n+1} \\ & \times P_n^m(\cos\Theta) + \sum_{n=1}^{N_{SV}} \sum_{m=0}^{n} (g_n^m \cos m\varnothing + h_n^m \sin m\varnothing)(t-t_o)\left(\frac{a}{r}\right)^{n+1} \\ & \times P_n^m(\cos\Theta) + \sum_{n=1}^{2} \sum_{m=0}^{n} (q_n^m \cos m\phi + s_n^m \sin m\phi) \\ & \times \left(\frac{r}{a}\right)^n P_n^m(\cos\Theta) + RC[\left(\frac{r}{a}\right) + Q_1\left(\frac{a}{r}\right)^2] \\ & \times [\tilde{q}_1^0 p_1^0(\cos\Theta) + \tilde{q}_1^1 \cos\varnothing + \tilde{s}_1^1 \sin\varnothing) + p_1^1(\cos\Theta)]\Bigg) \end{aligned} \qquad (2)$$

Where a (=6371.2km) is the mean radius of the earth, (r, Θ, $\varnothing$) are geocentric spherical coordinates, where r, Θ and $\varnothing$ are the geocentric distance, co elevation and east longitude from Greenwich, $p_n^m(\cos\Theta)$ are the associated Schmidt semi-normalized Legendre functions of degree n and order m, (g_n^m, h_n^m) and (q_n^m, h_n^m) are the

Gauss coefficients describing sources internal and external to the Earth, respectively, (g_n^m, h_n^m) describe the (linear) secular variation around model epoch t_0. The expansion of the above series is an infinite series, but it in practice it is usually limited to $n = 10$ or $n = 12$.

In addition, the $n = 1$, $n = 2$; $m = 0$, terms incorporated an annual and semi-annual variation. The last part of the above equation (coefficients $\tilde{q}_1^0$, $\tilde{q}_1^1$ and $\tilde{s}_1^1$) accounts for the variability of contributions from the magneto spheric ring current (as measured by RC (Ring Current)) plus their internal, induced counterpart.

3.2 Estimation Procedures

The B field in tangential coordinates is calculated as

$$V(r,\Theta,\phi) = a\sum_{n=1}^{k}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}(g_n^m\cos m\phi + h_n^m\sin m\phi)P_n^m(\Theta) \tag{3}$$

$$B_r = \frac{-\partial V}{\partial r} = \sum_{n=1}^{k}\left(\frac{a}{r}\right)^{n+2}(n+1)\sum_{m=0}^{n}(g^{n,m}\cos m\phi + h^{n,m}\sin m\phi)P^{n,m}(\Theta) \tag{4}$$

$$B_\Theta = \frac{-1}{r}\frac{\partial V}{\partial \Theta} = \sum_{n=1}^{k}\left(\frac{a}{r}\right)^{n+2}(n+1)\sum_{m=0}^{n}(g^{n,m}\cos m\phi + h^{n,m}\sin m\phi)\frac{\partial P^{n,m}(\Theta)}{\partial\Theta} \tag{5}$$

$$B_\phi = \frac{-1}{r\sin\Theta}\frac{\partial V}{\partial\phi} = \frac{-1}{\sin\Theta}\sum_{n=1}^{k}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}m(-g^{n,m}\sin m\phi + h^{n,m}\cos m\phi)P^{n,m}(\Theta) \tag{6}$$

Where

$$g^{n,m} \equiv s_{n,m}\, g_n^m \tag{7}$$

$$h^{n,m} \equiv s_{n,m}\, h_n^m \tag{8}$$

And

$$s_{n,m} = \left[\frac{(2-\delta_m^0)(n-m)!}{(n+m)!}\right]\frac{(2n-1)!!}{(n-m)!} \tag{9}$$

The kronecker delta, $\delta_j^i = 1$ if $i = j$ and 0 otherwise. In addition, $P^{n,m}$ is defined by

$$P^{0,0} = 1 \tag{10}$$

$$P^{n,n} = \sin\Theta\, P^{n-1,n-1} \tag{11}$$

$$P^{n,m} = \cos\Theta\, P^{n-1,m} - k^{n,m}\, P^{n-2m} \tag{12}$$

Where

$$k^{n,m} = \left.\begin{matrix}\dfrac{(n-1)^2 - m^2}{(2n-1)(2n-3)}, & n>1 \\ 0, & n=1\end{matrix}\right\} \tag{13}$$

4. Spherical Harmonic Model

For analytic purposes, a tilted dipole model of the geomagnetic field can be obtained by calculating the spherical harmonic model to the first degree ($n = 1$) and orders ($m = 0, 1$). The scalar potential, V, becomes (K. L. Makovec, 2001; White Paper, 2005)

$$\begin{aligned} V(r,\Theta,\varnothing) &= \left(\frac{a}{r^2}\right)[g_1^0 p_1^0(\Theta) + (g_1^1\cos\varnothing + h_1^1\sin\varnothing)p_1^1(\Theta)] \\ &= \left(\frac{1}{r^2}\right)(g_1^0 a^3\cos\Theta + g_1^1 a^3\cos\varnothing\sin\Theta + h_1^1 a^3\sin\varnothing\sin\Theta) \end{aligned} \tag{14}$$

The total dipole strength is given by

$$a^3 H_0 = a^3(g_1^{0^2} + g_1^{1^2} + h_1^{1^2})^{1/2} \tag{15}$$

Which leads to a value of $H_o = 30,115nT$.

The co elevation of the dipole is

$$\Theta'_m = \cos^{-1}\left(\frac{g_1^0}{H_0}\right) \tag{16}$$

And the east longitude of the dipole is

$$\varnothing'_m = \tan^{-1}\left(\frac{h_1^1}{g_1^1}\right) \tag{17}$$

The magnetic field in local tangential coordinates with a tilted dipole model is

$$B_r = 2\left(\frac{a}{r}\right)^3[g_1^0\cos\Theta + (g_1^1\cos\varnothing + h_1^1\sin\varnothing)\sin\Theta] \tag{18}$$

$$B_\Theta = \left(\frac{a}{r}\right)^3[g_1^0\sin\Theta - (g_1^1\cos\varnothing + h_1^1\sin\varnothing)\cos\Theta] \tag{19}$$

$$B_\varnothing = \left(\frac{a}{r}\right)^3[g_1^1\sin\varnothing - h_1^1\cos\varnothing] \tag{20}$$

By assuming that the magnetic of the earth is due to a vector dipole with strength and pole direction given above, the magnetic field can be calculated in vector form:

$$B(R) = \left(\frac{a^3 H_0}{R^3}\right)[3(\tilde{m}\tilde{R})\tilde{R} - \tilde{m}] \tag{21}$$

Where R is the position vector of the desired point in the magnetic field, and $\dot{m}$ is the dipole direction. R. And $\tilde{m}$ designate unit vectors. This vector can be calculated in any coordinate system, and this value in the geocentric inertial frame is shown below,

$$\tilde{m}=\begin{bmatrix} \sin\Theta'_m \cos\alpha_m \\ \sin\Theta'_m \sin\alpha_m \\ \cos\Theta'_m \end{bmatrix} \qquad (21)$$

Where

$$\alpha_m=\Theta_{g0}+\omega_{\oplus}t+\varnothing'_m \qquad (22)$$

Where Θ_{g0} is the Greenwich sidereal time at some reference time, $\omega_{\oplus}$ is the average rotation rate of the earth equal to $7.2921152\times10^{-5}\,rad/\sec$, t is the time since reference, Θ'_m and $\varnothing'_m$ are defined in Eq. (16) and Eq. (17).

5. Conclusions

When using the spherical harmonic expansions to describe the geomagnetic field, it usually follows that the more coefficients there are, the more accurate results. However, using too many coefficients can be unfavorable because of an increase in computation time that does not correspond with an increase in accuracy.

As further works, the improvement and modification of the Earth's variation model using different sources of present magnetic data are remained.

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