Reduced Complexity Schnorr-Euchner Sphere Decoders in MIMO Applications

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ABSTRACT

We present two techniques based on lookup tables to reduce complexity of the well-known Schnorr-Euchner (SE) sphere decoder (SD) without introducing performance degradation. By the aid of lookup tables, the computational loads caused by the SE enumeration and decision feedback are reduced at the cost of higher storage capacity. Simulation results are provided to verify performance and complexity of the proposed decoders.

Keywords

Multiple-input multiple-output, maximum likelihood detection, sphere decoding, wireless communication.

Ⅰ. Introduction

The application of multiple transmit and receive antennas to wireless communication systems, i.e., multiple input multiple output (MIMO) systems, has drawn a lot of attention from researchers because MIMO systems are theoretically shown to remarkably increase spectral efficiencies [1]. However, for signal detection in MIMO system, the optimal brute-force maximum-likelihood (BF-ML) decoder has very high or even infeasible complexity since its complexity grows exponentially in the number of transmit antennas. In order to achieve ML performance at reduced complexity, a class of detection algorithms, referred to as sphere decoders, has been developed [2]-[5]. Among them, the sphere decoder based on Schnorr-Euchner enumeration (SE-SD) [4]-[5] is of more interest due to its low complexity.

In this paper, we present two approaches for lowering the complexity of the SE-SD based on the lookup tables. The first method, called LT1 (Lookup Table 1), eliminates the SE enumeration, thereby making the SE-SD more compact at the cost of a small increase in memory capacity. Based on the finite property the transmission constellation, the second one, called LT2, generates a number of lookup tables, thanks to which the computational load of the decision feedback can be reduced. The amount of storage capacity required by the LT2 increases proportionally to the number of transmit antennas and the constellation size. Simulation results are provided to demonstrate performance and complexity of the proposed methods.

Ⅱ. System model

Consider an uncoded MIMO system with n_T transmit and n_R receive antennas, denoted as (n_T, n_R) system. At the transmitter, the input data sequence is partitioned into n_T sub-streams (layers), each of which is then modulated by an M-level QAM modulation scheme and transmitted from a different transmit antenna. The transmission isperformed in a burst by burst basis over a quasi-static Rayleigh fading channel changing randomly from one burst of length L (symbol durations) to the other. The power launched by each transmit antenna is in proportion to $1/n_T$ so that the total transmitted power is a constant and independent of n_T.

The complex baseband signal model is given by:

$$\mathbf{r}_c = \mathbf{H}_c\mathbf{s}_c + \mathbf{w}_c \quad (1)$$

where $\mathbf{r}_c$ and $\mathbf{s}_c$ are respectively the $n_R \times 1$ received signal, $n_T \times 1$ transmitted signal vectors, the $n_R \times 1$ vector $\mathbf{w}_c$ is an i.i.d zero-mean complex Gaussian noise vector with variance σ^2 per complex dimension, $\mathbf{H}_c$ is the $n_R \times n_T$ channel matrix, whose entries are the path gains between transmit and receive antennas modelled as the samples of a zero-mean complex Gaussian random variable with equal variance of 0.5 per real dimension. In the sequel, we assume $n_T = n_R$.

One can equivalently express the system (1) as [3]:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \quad (2)$$

where $\mathbf{r} = \begin{bmatrix} \Re\{\mathbf{r}_c\} \\ \Im\{\mathbf{r}_c\} \end{bmatrix}$, $\mathbf{s} = \begin{bmatrix} \Re\{\mathbf{s}_c\} \\ \Im\{\mathbf{s}_c\} \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} \Re\{\mathbf{w}_c\} \\ \Im\{\mathbf{w}_c\} \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} \Re\{\mathbf{H}_c\} & \Im\{\mathbf{H}_c\} \\ -\Im\{\mathbf{H}_c\} & \Re\{\mathbf{H}_c\} \end{bmatrix}$, $\Re\{\mathbf{a}\}$ and $\Im\{\mathbf{a}\}$ respectively denote the real and imaginary part of $\mathbf{a}$. $\mathbf{H}$ has a full rank of $m = 2n_T$.

Under the assumption that $\mathbf{H}$ is perfectly known at the receiver, the transmitted vector is ML decoded according to:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\Omega} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2 \quad (3)$$

where Ω is the set of $N = M^{1/2}$ integers, from which a QAM constellation is carved, e.g., $\Omega = \{-3, -1, 1, 3\}$ for 16-QAM.

III. Schnorr-Euchner Sphere Decoder

Instead of ML decoding $\mathbf{s}$ using (3), a sphere decoder try to search for the ML solution based on the following equation [2]-[5]:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\Omega} \|\mathbf{v} - \mathbf{R}\mathbf{s}\|^2$$
$$= \arg\min_{\mathbf{s}\in\Omega} \sum_{k=1}^{m} |v_k - R_{k,k}s_k - \xi_k|^2 \quad (4)$$

where $\mathbf{v} = \mathbf{Q}^{\mathrm{T}}\mathbf{r}$, $\mathbf{Q}$ is a $m \times m$ unitary matrix, i.e., $\mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{I}_m$, $\mathbf{I}_m$ is a $m \times m$ identity matrix, $\mathbf{R}$ is a $m \times m$ upper triangular matrix, $\mathbf{Q}$ and $\mathbf{R}$ are obtained from the QR decomposition of $\mathbf{H}$, i.e., $\mathbf{H} = \mathbf{QR}$, $\xi_k = \sum_{i=k+1}^{m} R_{k,i}s_i$.

To avoid an exhaustive search, a sphere decoder examines only signal points that satisfy:

$$\|\mathbf{v} - \mathbf{R}\mathbf{s}\|^2 \le C \quad (5)$$

where the initial sphere radius $\sqrt{C}$ large enough to contain the ML solution.

For the sake of completeness, the SE-SD that solves (5) is summarized below. Readers are referred to [5] for more details of the SE-SD.

SE-SD Algorithm- Input: $\mathbf{v}, \mathbf{R}, C$. Output: $\hat{\mathbf{s}}$

Step 1: (Initialization) Set $k := m$, $T_m := 0$, $\xi_m := 0$, and $D_{\min} := C$.

Step 2: Set $s_k := round\left(\frac{v_k - \xi_k}{R_{k,k}}\right)$, $\Delta_k := sign(v_k - \xi_k - R_{k,k}s_k)$, and $l_k := 1$.

Step 3: (Main step) If ($D_{\min} \ge |v_k - \xi_k - R_{k,k}s_k|^2 + T_k$) or ($l_k > N$), then if $k = m$, terminate, else go to Step 5.

Step 4: If $k > 1$, then {let $\xi_{k-1} := \sum_{i=k}^{m} R_{k,i}s_i$, $T_{k-1} := |v_k - \xi_k - R_{k,k}s_k|^2 + T_k$, $k := k-1$, and go to Step 2}, else {set $D_{\min} := |v_k - \xi_k - R_{k,k}s_k|^2 + T_k$, save new solution $\hat{\mathbf{s}} := \mathbf{s}$, let $k := k+1$}.

Step 5: (Schnorr-Euchner enumeration) If ($l_k \le N$), then {let $s_k := s_k + 2\Delta_k$, $\Delta_k := -\Delta_k - sign(\Delta_k)$}.

Step 6: If $|s_k| \ge M$, then go to Step 5, else {set $l_k := l_k + 1$ and go to Step 3}.

Here $round(.)$ denotes rounding to the closest integer $\in \Omega$, and $sign(.)$ denotes the sign of the term inside the bracket.

IV. Proposed Decoders

1. Proposed LT1-SD Decoder

In the SE-SD presented above, s_k is drawn from a sequence of values generated by the SE enumeration. Specifically, at level k, the decoder first determines the midpoint $\tilde{s}_k := round\left(\frac{v_k - \xi_k}{R_{k,k}}\right)$ end computes $v_k - \xi_k - R_{k,k}\tilde{s}_k$. The sequence of values at level k is generated as follows.

- If $v_k - \xi_k - R_{k,k}\tilde{s}_k \geq 0$:
  $s_k \in \{\tilde{s}_k, \tilde{s}_k + 2, \tilde{s}_k - 2, \tilde{s}_k + 4, \tilde{s}_k - 4, \ldots\} \cap \Omega$
- If $v_k - \xi_k - R_{k,k}\tilde{s}_k < 0$:
  $s_k \in \{\tilde{s}_k, \tilde{s}_k - 2, \tilde{s}_k + 2, \tilde{s}_k - 4, \tilde{s}_k + 4, \ldots\} \cap \Omega$

The proposed LT1-SD decoder will eliminate the SE enumeration by the observation that when s_k is selected from the above sequences, it creates a corresponding sequence of Euclidean distances, $|v_k - \xi_k - R_{k,k}s_k|^2$, that obeys an ascending order. This order is referred to as "optimal testing" order.Unlike the SE-SD, the proposed LT1-SD determines the optimal testing order without having to utilize the SE enumeration.

Let $\mathbf{z} = (z_1, z_2, \ldots, z_N)$ be a vector containing N integers $\in \Omega$ arranged in an increasing order. For example, for 16-QAM, $\mathbf{z} = (-3, -1, 1, 3)$ with $N = 4$. Illustrated in Fig. 1 is the approach to determine the optimal testing order at layer k for 16-QAM simply by comparing $\eta_k = v_k - \xi_k$ with appropriate boundaries (b_1, b_2, b_3 and c_1, c_2), called b-boundaries and c-boundaries.

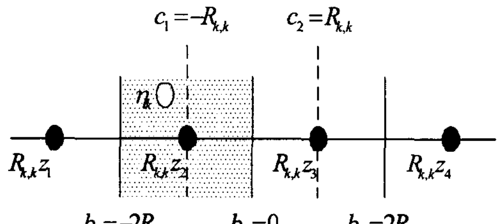

Fig. 1: Determining optimal testing order at level k for 16-QAM.

For example, since $b_1 < \eta_k < b_2$, i.e., η_k is in the dotted region, $s_k = z_2$ is the first value to be tested. In addition, since $\eta_k < c_1$, the optimal testing order for all four values in $\mathbf{z}$ at level k is (z_2, z_1, z_3, z_4). In case $\eta_k > c_1$, the optimal testing order becomes (z_2, z_3, z_1, z_4). Note that each QAM constellation has its own b-boundaries and c-boundaries, which are known in advance.

In the LT1-SD, the optimal testing order is stored in lookup table $\mathbf{i}_k = (i_1^k, i_2^k, \ldots, i_N^k)$ in terms of indexes of integer values in $\mathbf{z}$. The LT1-SD is summarized as follows.

LT1-SD Algorithm- Input: $\mathbf{v}, \mathbf{R}, N, C$. Output: $\hat{\mathbf{s}}$

Step 1: (Initialization) Set $k := m$, $T_m := 0$, $\eta_m := v_m$, and $D_{\min} := C$.

Step 2: Use η_k to generate lookup table $\mathbf{i}_k$ at level k, set $l_k := 1$.

Step 3: If ($l_k \leq N$), then {get s_k by setting $s_k := z_{i_{l_k}^k}$ and let $D := |\eta_k - R_{k,k}s_k|^2 + T_k$}.

Step 4: If $(D \geq D_{min})$ or $(l_k > N)$, then if $(k = m)$, terminate, else {set $k := k+1$, $l_k := l_k + 1$ and go to Step 3}.

Step 5: If $k > 1$, then {let $\eta_{k-1} := v_{k-1} - \sum_{i=k}^{m} R_{k,i}s_i$, $T_{k-1} := D$, $k := k-1$, and go to Step 2}, else {set $D_{\min} := D$, save new solution $\hat{\mathbf{s}} := \mathbf{s}$, let $k := k+1$, $l_k := l_k + 1$, and go to Step 3}.

One can see from the LT1-SD that the SE enumeration no longer exists.

2. Proposed LT2-SD Decoder

In the LT1-SD, the computation of the feedback term $\xi_k = \sum_{i=k}^{m} R_{k,i}s_i$ is also a possible factor that results in high complexity, particularly at low signal-to-noise ratio, because during the search ξ_k may be repeatedly computed. Since s_i are selected from a finite set Ω, in addition to the quasi-static of the channel, the complexity of the LT1-SD can be further reduced by pre-computing the products $R_{k,i}s_i$ and storing them in lookup tables.

For example, consider a MIMO system with 16-QAM. After performing the QR decomposition of $\mathbf{H}$, the LT2-SD computes the product of the matrix $\mathbf{R}$ with the four values $(-3, -1, 1, 3) \in \Omega$ and stores the results in 4 matrices: $\mathbf{R}_1 = -3\mathbf{R}$, $\mathbf{R}_2 = -\mathbf{R}$, $\mathbf{R}_3 = \mathbf{R}$, and $\mathbf{R}_4 = 3\mathbf{R}$. Clearly, this approach allows the decoder to have low complexity at the cost of higher capacity storage. Thus, it is suitable for MIMO applications with small numbers of transmit antennas.

Now, the LT2-SD is summary as follows.

LT2-SD Algorithm- Input: $\mathbf{v}, \mathbf{R}, \mathbf{R}_1, \ldots, \mathbf{R}_N, N, C$. Output: $\hat{\mathbf{s}}$

Step 1: (Initialization) Set $k := m$, $T_m := 0$, $\eta_m := v_m$, and $D_{\min} := C$.

Step 2: Use η_k to generate lookup table $\mathbf{i}_k$ at level k, set $l_k := 1$.

Step 3: If $(l_k \le N)$, then {get s_k by setting $s_k := z_{i_{l_k}}^{k}$ and let $D := \left|\eta_k - R_{k,k} s_k\right|^2 + T_k$}.

Step 4: If $(D \ge D_{\min})$ or $(l_k > N)$, then if $(k = m)$, terminate, else {set $k := k+1$, $l_k := l_k + 1$ and go to Step 3}.

Step 5: If $k > 1$, then {let $\eta_{k-1} := v_{k-1} - \sum_{j=k}^{m} R_{k,j}^{i_{l_k}^k}$, $T_{k-1} := D$, $k := k-1$, and go to Step 2}, else {set $D_{\min} := D$, save new solution $\hat{\mathbf{s}} := \mathbf{s}$, let $k := k+1$, $l_k := l_k + 1$, and go to Step 3}.

V. Simulation Results

We investigate performances and complexities of the proposed decoders by applying to a (4, 4) system with 16-QAM modulation. In the simulations, signals are transmitted in a burst by burst basis with burst length of 100 symbol durations. In addition, the channel matrix $\mathbf{H}$ is assumed to remain fixed within one burst and changes randomly from this burst to the next. Thus, QR decomposition of $\mathbf{H}$ is performed once per burst. The initial square sphere radius C is set equal to 100. If no signal point exists inside the sphere, the radius will be increased by a step of $0.2C$ until a point is found. The algorithms are all implemented in floating-point C, then converted into mex file and used in MATLAB 6. We use the number of floating point operations (flops), i.e., addition, subtraction, multiplication, division, as a measure for complexity.

Illustrated in Fig. 1 are the bit error rate (BER) curves of the SE-SD, LT1-SD, and LT2-SD in a (4, 4) system employing 16-QAM modulation. One can see from Fig. 1 that performances of the three decoders are almost identical. Clearly, the proposed LT1-SD and LT2-SD are able to provide the system with ML performance. The important point is that, when utilizing the proposed decoders, the system achieves ML performance at lower complexity than when using the conventional SE-SD as shown in Fig. 2. For example, at SNR = 24 dB, the LT1-SD and LT2-SD allows the system to reduce the complexity by factors of around 1.38 and 1.68, respectively.

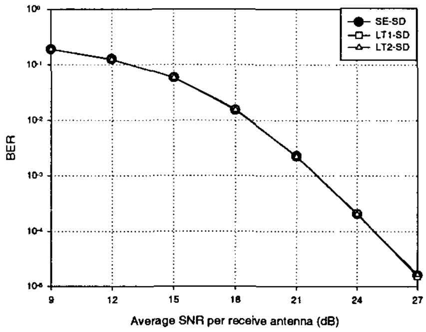

Fig. 1: Performance of the SE-SD, LT1-SD, and LT2-SD in a (4, 4) system with 16-QAM symbols.

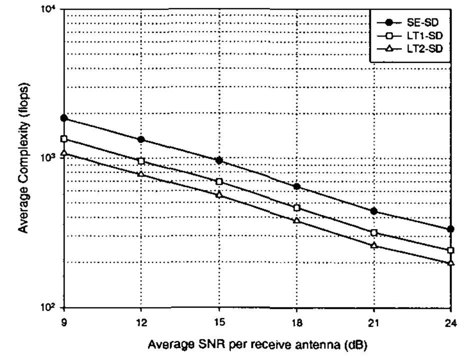

Fig. 2: Complexity of the SE-SD, LT1-SD, and LT2-SD in a (4, 4) system with 16-QAM symbols.

VI. CONCLUSION

Two reduced complexity SE sphere decoders are proposed for signal detection in MIMO systems. The proposed LT1-SD eliminates the SE enumeration in the conventional SE-SD by storing the optimal testing order in lookup tables. The proposed LT2-SD is modified from the LT1-SD by pre-computing the decision feedback terms and saving them in the memory. Simulation results show that the proposed decoders enable the system to have low detection complexity without altering their optimality. Yet, low complexity is achieved as the cost of higher storage capacity.

ACKNOWLEDGMENT

This work is supported by the Korea Science and Engineering Foundation (KOSEF) under ERC program through the Intelligent Radio Engineering Center (IREC) at Information and Communications University (ICU), Republic of Korea.

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