

# **Large Scale Stabilized Finite Element Simulation and Modeling for Environmental Flows in Urban Area**

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## **ABSTRACT**

A large-scale finite element simulation and modeling method is presented for environmental flows in urban area. Parallel stabilized finite element method based on domain decomposition method is employed for the numerical simulation. Several GIS and CAD data are used for the preparation of the shape model for landform and urban structures. The present method is applied to the simulation of flood flow and wind flow in urban area. The present method is shown to be a useful planning and design tool for the natural disasters and the change of environments in urban area.

## **INTRODUCTION**

A number of natural disasters occur annually in various parts of the world. Especially, a number of natural disasters in cities increases in accordance with the development of city area, such as flood, wind disaster and air pollution. In order to estimate the extent of a disaster quantitatively, it is necessary to estimate the behavior of natural phenomena which causes the natural disaster. In practical computations of this type of problems, the computational domain is large and the computations need to be carried out over long time durations. Therefore, this type of problem becomes quite large-scale and it is essential to use methods which are as efficient and fast as the available hardware allows. In recent years, parallel finite element computations have been successfully applied to several large-scale simulations for natural phenomena [1-4]. These computations demonstrated the availability of a new level of computational capability to solve practical problems. However, in order to compute natural phenomena in urban area accurately, it is necessary to prepare an accurate shape model for landform, buildings and civil structures.

In this presentation, a large-scale finite element simulation and modeling method is presented for environmental flows in urban area. Several GIS and CAD data are used for the preparation of shape model for landform, buildings and civil structures. In order to express the geographical features accurately, the valley and ridge lines are expressed by an edge of the element. Parallel finite element method based on domain decomposition method and MPI are employed for the numerical simulation. The P1/P1 element (triangular element for 2D problem, tetrahedral element for 3D problem) is employed for the finite element. The stabilized finite element formulation based on the SUPG/PSPG [5] is employed for the discretization in space. The present method is applied to the simulation of flood flow and wind flow in urban area. The shallow water equation and the Navier-Stokes equation are employed for the governing

equations. The present method is shown to be a useful planning and design tool for natural disasters and the change of environments in urban area.

## GOVERNING EQUATIONS AND FINITE ELEMENT METHOD

### Flood Flow Simulation

The governing equations for flood flow can be expressed by the shallow water equations, which are obtained from the conservation of momentum and mass, vertically integrated, assuming a hydrostatic pressure distribution. For the sewer pipe system, the 1D slot model [6] is employed. Figure 1 shows the sketch of the interaction of flood and sewer pipe flow.

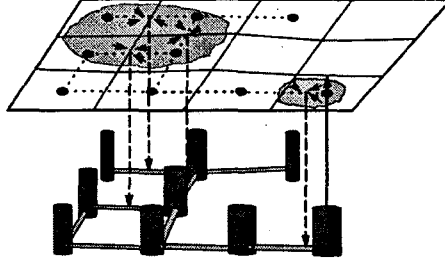


Figure 1 Interaction of flood and sewer pipe flow

The governing equations can be described as follows.  
(for flood area)

$$\dot{u}_i + u_j u_{i,j} + g(\eta + z)_{,i} + \frac{\tau_i^b}{\rho\eta} = 0 \quad (1)$$

$$\dot{\eta} + (u_i \eta)_{,i} = \frac{Q_{up} - Q_{sew}}{A} + r_e \quad (2)$$

where  $u_i$  is the mean velocity,  $\eta$  is the water depth,  $z$  is the elevation from a datum line,  $\rho$  is the density,  $g$  is the gravitational acceleration,  $A$  is the area of element,  $\tau_i^b$  is the bottom shear stress and  $r_e$  is the amount of rainfall, respectively.  
(for sewer pipe system)

$$\dot{v} + v v_{,x} + g h_{,x} - g(S_o - S_f) = 0 \quad (3)$$

$$\dot{h} + v h_{,x} + \frac{A}{B} v_{,x} = \frac{q_{sew}}{B l_e} \quad (4)$$

where  $v$  is the mean velocity,  $h$  is the water depth,  $A$  is the area of section,  $B$  is the width of pipe,  $q_{sew}$  is the inflow from the ground,  $l_e$  is the length of link element,  $S_o$  and  $S_f$  are the gradient of pipe and friction, respectively.

For the discretization in space, the stabilized finite element method based on the SUPG (streamline-upwind/Petrov-Galerkin) method [5] with shock capturing is employed. The linear triangular element is employed for the spatial discretization for the flood area and the linear line element is used for the sewer pipe system. The explicit method based on the multi-pass algorithm is employed for the discretization in time. The Eulerian approach using fixed mesh is applied to the treatment method of moving boundaries.

### Wind Flow Simulation

The Navier-Stokes equation based on Boussinesq approximation is employed for the wind flow.

$$\dot{u}_i + u_j u_{i,j} + \frac{1}{\rho_0} p_{,i} - \nu(u_{i,jj} + u_{j,ij}) + \frac{\rho}{\rho_0} g \delta_{i3} = 0 \quad (5)$$

$$u_{i,i} = 0 \quad (6)$$

$$\dot{\rho} + u_i \rho_{,i} = -u_i \rho_{B,z} \delta_{i3} \quad (7)$$

where  $u_i$  is the velocity,  $p$  is the pressure,  $\rho_o$  is the reference density,  $\rho$  is the gap to the reference density,  $\rho_z$  is the vertical gradient of the density,  $\nu$  is the viscosity of fluid and  $\delta_{ij}$  is the Kronecker's delta, respectively.

For the discretization in space, the stabilized finite element method based on the SUPG/PSPG (pressure-stabilizing/Petrov-Galerkin) method [5] with shock capturing is employed for the Navier-Stokes equation. For the discretization of the density equation, the SUPG method is employed. The P1/P1 finite element (linear interpolation both for velocity and pressure) is employed for the finite element. On the other hand, the Crank-Nicolson scheme is employed for the discretization in time. The element-by-element Bi-CGSTAB2 method is applied to solve the simultaneous equation.

## PARALLEL IMPLEMENTATION

A parallel computational scheme based on the domain decomposition method [1-4] is developed in order to reduce the CPU time and computer storage required. A parallel implementation using the MPI suitable for unstructured grid is designed for the use on PC cluster parallel computer. The PC cluster consists of 20 nodes, each with a Intel-Xeon 3.06 GHz processor, 2GB memory and 1MB cache size. The giga-bit Ethernet (1000Base-T network) and a switching hub are employed for the network environment. The MPICH-SCore is used for MPI and PGI Fortran is used for a compiler. The automatic mesh decomposer METIS [7] based on the multilevel k-way partitioning scheme [8] is employed to minimize the amount of interprocessor communication. For each sub-domain, the processor associated with that sub-domain carries out computations independently.

The element-by-element Bi-CGSTAB2 method is employed to solve the finite element equations. The parallel implementation results in a quick and efficient solution to the resulting equation system. The parallel implementation requires two types of communications. The communication between neighbor processors is needed for the assembly of element level vector and that among all of processors is needed for the calculation of residual norm.

## MODELING SYSTEM OF URBAN AREA

The modeling system for urban area is expressed in this section. In case of the modeling of urban area, the building data is needed in addition with the elevation data. For the data for buildings, the 2D and 3D GIS data obtained by the aerial-photo and -laser surveying are employed. The ArcView software is used for the GIS system.

### Modeling System for Flood Flow Simulation

The 2D finite element model for flood flow simulation is prepared by the 2D GIS data (Mapple 2500) which is obtained by the aerial-photo. For the data of land elevation, the digital elevation map issued by the geographical survey institute of Japan is employed. In case of the modeling of flood flow in urban area, the shape model for sewer pipe system is also needed. Figure 3 shows the shape model for both buildings and sewer pipe system for the studied area. In this figure, the node denotes the manhole and the link denotes the sewer pipe, respectively. Figure 4 shows the finite element mesh based on the linear triangular element. The modified Delaunay method [2] is employed for generating the finite element mesh. The linear line element is employed for the sewer pipe system. The mesh size is assumed to be 2m for both flood area and sewer pipe system. The total number of nodes and elements for flood area are 33,223 and 53,272, respectively.

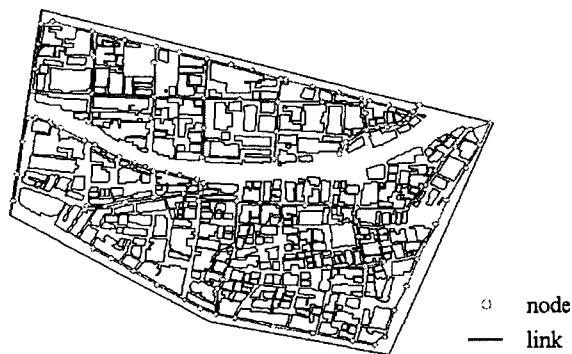


Figure 3 2D shape model

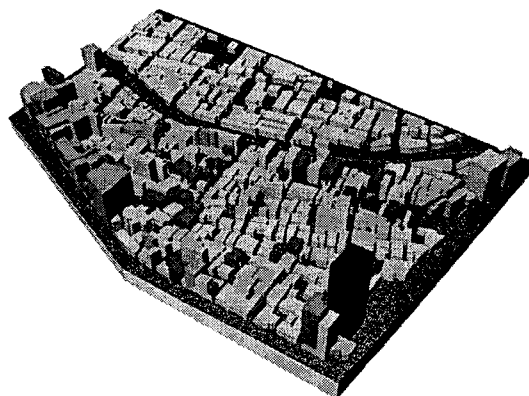


Figure 4 Finite element mesh

### Modeling System for Wind Flow Simulation

The 3D finite element model for wind flow simulation is also prepared by using the 2D GIS data. The vertical shape for low-storied buildings is assumed to be straight and that for the high-storied buildings with complicated shape are prepared by the CAD system (Auto CAD). The civil structures such as highway are also prepared by the CAD system. Figure 5 shows the 3D shape model for Shinjuku-area of Tokyo and the computed results of shade simulation. Figure 6 show the finite element mesh for the surface of the shape model. The linear tetrahedral element is employed for the finite element mesh. The minimum mesh size is assumed to be 1m. The total number of nodes and elements are 2,868,335 and 15,871,126, respectively.

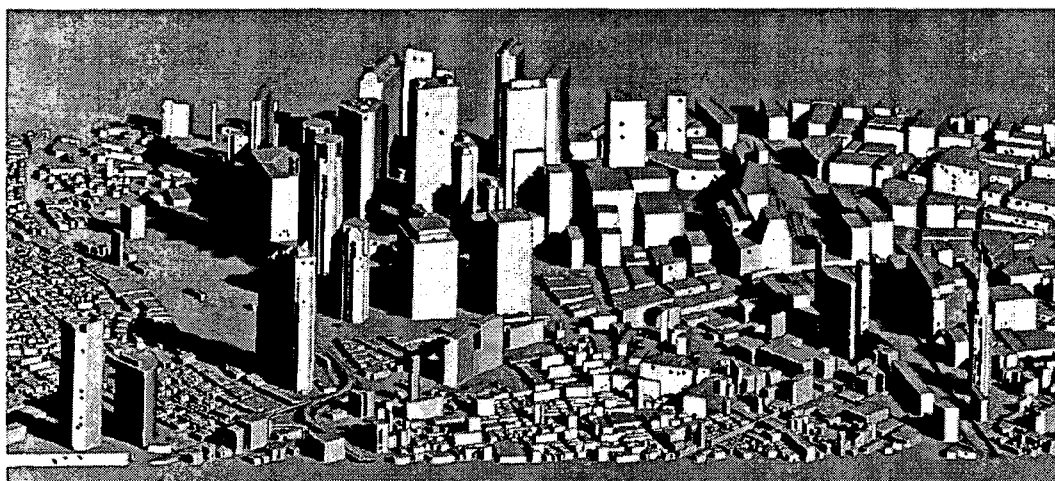


Figure 5 3D shape model for Shinjuku, Tokyo

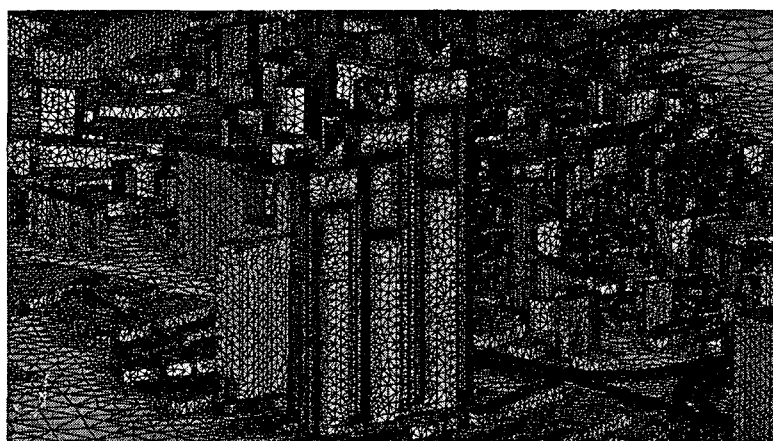


Figure 6 Surface mesh for 3D shape model

## NUMERICAL EXAMPLES

The stabilized finite element method is employed for the simulation of environmental flows in urban area. The finite element mesh obtained by the present method are applied to the simulation of flood flow and wind flow in urban area.

### Flood Flow Simulation

For the flood flow simulation, the models based on the shallow water equation system is employed. The stabilized finite element formulation based on the SUPG [5] is employed for the discretization in space. For the discretization in time, the explicit method based on the multi-pass algorithm is employed. In order to treat the moving boundary, the Eulerian approach is applied. For the boundary condition, the discharge is assumed at the inlet of the river. The Manning coefficient is assumed to be 0.014. Figure 7 shows the computed water elevation. It can be seen that the flood occur in accordance with the increase of river discharge.

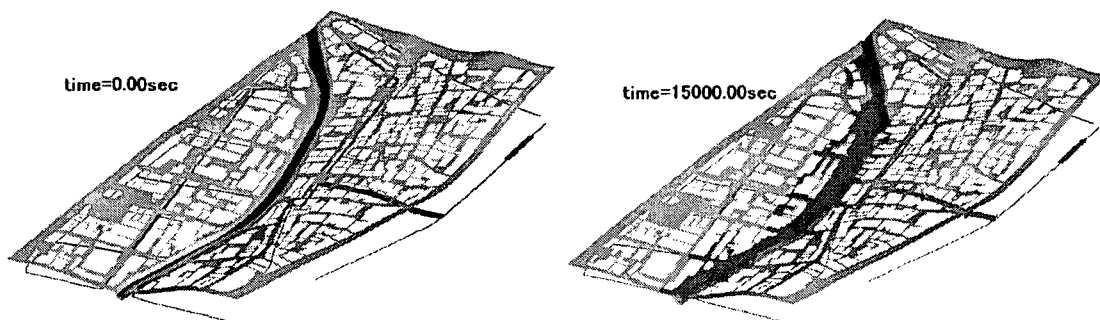


Figure 7 Computed water elevation

### Wind Flow Simulation

For the wind flow simulation, the Navier-Stokes equation based on RANS model is employed for the governing equations. The stabilized finite element method based on the SUPG/PSPG [5] is employed for the discretization in space, and the Crank-Nicolson scheme is employed for the discretization in time. The parallel computational method based on the domain decomposition is employed. Figure 8 shows the domain decomposition for 20 sub-domains.

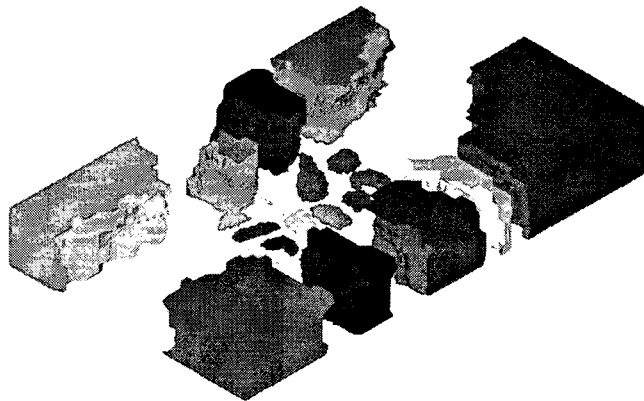


Figure 8 Domain decomposition

The verification of the finite element solver has been done by the comparison with experimental data [9]. The good parallel performance has also obtained using a PC cluster [3]. Figure 9 shows the computed streamlines. The Reynolds number is assumed to be 84,000. From this figure, it can be seen that the wind flow passed large buildings shows the complicated flow field.



Figure 9 Computed streamlines

## CONCLUSIONS

A large-scale finite element simulation and modeling method for environmental flows in urban area has been presented in this paper. The shape model for landform and urban structures can be prepared accurately by using several GIS and CAD data. Parallel stabilized finite element method based on domain decomposition method has been employed for large scale simulations. The present method has been applied to the simulation of flood flow and wind flow in urban area. From the results obtained in this paper, it can be concluded that the present method provides a useful planning and design tool for the natural disasters and the change of environments in urban area.

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