CIVA-Lattice Boltzmann Method for Sallow Water Flow Based on Unstructured Grid

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ABSTRACT

This paper presents a CIVA-Lattice Boltzmann Method (CIVA/LBM) for shallow water flow with a complicated flow domain. The LBM is advantageous in computational time and memory required comparing with the conventional numerical methods, because the LBM simply solves the particle distribution explicitly. Recently, the LBM has applied to the shallow water flow [1]. The practical computation of shallow water flow problems such as flow in ocean, river and estuaries, the computational domain is large and the computations need to be carried out over long time durations. Therefore, the LBM is to be an effective method for shallow water flow analysis.

However, it is difficult to apply the LBM to the complicated flow domain, because LBM uses a regular lattice. In order to overcome the problem, some LBM applicable to the irregular lattice have been presented in recent years. These methods can be classified into two types; the interpolation supplemented LBM and the descretized method based LBM such as finite difference, finite volume and finite element methods based LBM. The interpolation supplemented LBM is robustness in applicability, however in order to obtain the accurate solutions, an accurate interpolation scheme is needed to interpolate the particle distribution function in the streaming step.

This paper presents a CIVA-LBM based on unstructured mesh for shallow water flow. The cubic interpolation scheme based on the CIVA method [2] is employed in order to interpolate the particle distribution function in the streaming step accurately. In the collision step of the lattice Boltzmann method, two dimensional nine-speed (2D9V) lattice BGK model is employed as a collision operator. After the streaming and relaxation steps, the velocity and water elevation can be obtained from the computed particle distribution function.

The lattice Boltzmann equation for shallow water flow can be expressed as:

$$f_\alpha(x+e_\alpha\Delta t,t+\Delta t)-f_\alpha(x,\Delta t)=-\frac{1}{\tau}\left[f_\alpha(x,t)-f_\alpha^{eq}(x,t)\right]+\frac{\Delta t}{6e^2}e_{\alpha i}F_i \qquad (1)$$

The left side denotes the streaming term and the first term of right side represents the collision term and the second term represents the force term. Where f_α is the particle distribution function, f_α^{eq} is the equilibrium distribution function, e_α is the velocity vector of a particle, and τ is the single relaxation time which is defined as:

$$\tau = \frac{3\nu_e}{e^2 \Delta t} + \frac{1}{2} \tag{2}$$

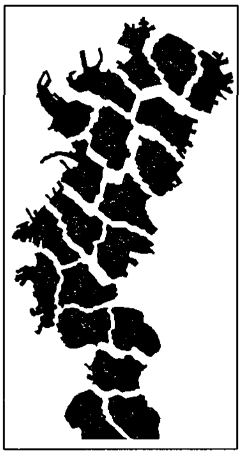
where ν_e is the vertically integrated kinematic viscosity. And F_i is the component of the force which can be given as:

$$F_i = -\rho g \frac{\partial z_b}{\partial x_i} + \frac{\tau_{wi}}{\rho} - \frac{\tau_{bi}}{\rho} \tag{3}$$

where z_b is the bottom elevation from the datum, ρ is the density, τ_{wi} and τ_{bi} are the free surface friction and bottom friction, respectively.

The macroscopic variables, water depth and velocity, can be evaluated by the computed particle distribution function. A parallel computational scheme based on the domain decomposition method is developed in order to reduce the CPU time and computer storage required. A parallel implementation using the MPI suitable for unstructured grid was designed for the use on PC cluster parallel computer. The automatic mesh decomposer METIS based on the multilevel k-way partitioning scheme is employed to minimize the amount of interprocessor communication.

The present method is applied to several numerical examples, such as river and coastal flows. Fig.1 shows the unstructured grid and domain decomposition for 20 sub-domains used in the compuation of tidal flow in Tokyo Bay. Fig.2 shows the computed velocity at high tide and low tide. The computed results are compared with the obserbed data and the computed results by FEM. The present method is shown to be an useful tool for the shallow water flow with with a complicated flow domain.



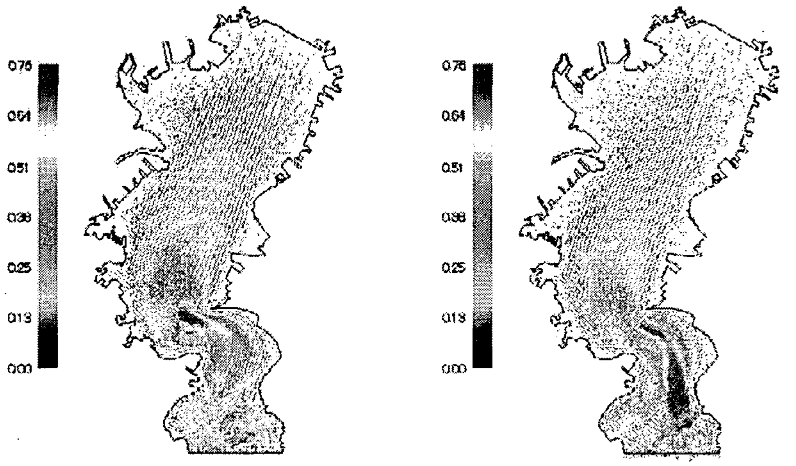


Fig.1 Domain decomposition   Fig. 2 Computed velocity at high-tide (left) and low-tide (right)

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