Parallel Deferred Correction for CFD Problems

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The development of large computational resources, as the grid computing, leads to search for parallel implementation not only based on space decomposition as domain decomposition or operator splitting which are classical parallel methods in CFD. Indeed these methods have to face the lack of computational granularity when the computing resources are very large. One promising direction of parallelism can be the time domain decomposition, where the solution of problem is computed in parallel on different time slices.

Nevertheless the difficulty of this approach comes from the matching conditions for the solution at the boundaries of the time slices. Notably, when CFD problems have nonlinear behavior. The initial guesses at the begin of time slices have to be enhanced iteratively until to reach a given tolerance. In this presentation, we will continue the investigation on time domain decomposition initiated in the Pita algorithm [1], the parareal algorithm [4] and to the multiple shooting method [2] (see [3] for details).

In order to actually investigate the benefit of the present implementation we focus on large ODEs systems coming from the discretisation of CFD problems, as the lead driven cavity problem and also combustion problem as the Bratu problem. More precisely, we will introduce parallelism in the Deferred Correction Method (DC) [5] that can be summarized as follows:

Considering an ODE systems of the form

$$\{ \phi'(t) = f(t, \phi(t)), \qquad \phi(0) = \phi_0. \tag{1}$$

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where $\phi_0, \phi(t) \in \mathbb{C}^N$ and $F : \mathbb{R} \times \mathbb{C}^N \to \mathbb{C}^N$.

The single time step on a DC begins by first dividing the time step $[t_0, t_{end}]$ into a set of intermediate sub-steps defined by the points $t_0 < t_1 < \cdots < t_p = t_{end}$. Next a provisional approximation $[\phi^{[0]}(t_0), \phi^{[0]}(t_1), \cdots, \phi^{[0]}(t_p)]$ is computed. Using standard approximation or interpolation theory, an equation for the error $\delta(t) = \phi(t) - \phi^{[0]}(t)$ is built. This correction equation can be approximated using a similar low order method, and an improved numerical solution is constructed.

Let δ_m be an approximation to $\delta(t_m)$, discretisation ([5]) yields to

$$\delta_{m+1} = \delta_m + \Delta_m \Big(f(t_{m+1}, \phi^{[0]}_{m+1} + \delta_{m+1}) - f(t_{m+1}, \phi^{[0]}_{m+1}) \Big) + \epsilon_{m+1} - \epsilon_m \quad (2)$$

where ϵ_m is the value of the residual function at time t_m.

The same parallelism difficulty as in the Parareal scheme ([3]), occurs with the correction step in the DC which is sequential in nature. More precisely for stiff problem the Jacobian Matrix of the problem have to be constructed by automatic differentiation and a linearized problem must be solved. This two features should be parallelized to obtain efficient solver. We investigate to solve this sequential correction step by using a Schur Decomposition method providing a full parallel solver.

References

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