Numerical Analysis of Three Dimensional Supersonic Flow around Cavities

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ABSTRACT

The supersonic flow around tandem cavities was investigated by three- dimensional numerical simulations using the Reynolds-Averaged Navier-Stokes(RANS) equation with the $\kappa-\omega$ turbulence model. The flow around a cavity is characterized as unsteady flow because of the formation and dissipation of vortices due to the interaction between the freestream shear layer and cavity internal flow, the generation of shock and expansion waves, and the acoustic effect transmitted from wake flow to upstream. The upwind TVD scheme based on the flux vector split using van Leer's limiter was used as the numerical method. Numerical calculations were performed by the parallel processing with time discretizations carried out by the 4th-order Runge-Kutta method. The aspect ratio of cavities are 3 for the first cavity and 1 for the second cavity. The ratio of cavity interval to depth is 1. The ratio of cavity width to depth is 1 in the case of three dimensional flow. The Mach number and the Reynolds number were 1.5 and 4.5×10^5, respectively. The characteristics of the dominant frequency between two-dimensional and three-dimensional flows were compared, and the characteristics of the second cavity flow due to the first cavity flow was analyzed. Both two dimensional and three dimensional flow oscillations were in the 'shear layer mode', which is based on the feedback mechanism of Rossiter's formula. However, three dimensional flow was much less turbulent than two dimensional flow, depending on whether it could inflow and outflow laterally. The dominant frequencies of the two dimensional flow and three dimensional flows coincided with Rossiter's 2nd mode frequency. The another dominant frequency of the three dimensional flow corresponded to Rossiter's 1st mode frequency.

INTRODUCTION

The high-speed flight vehicles have cavities such as wheel wells and bomb bays. The supersonic cavity flow is complicated due to vortices, flow separation and reattachment, and shock and expansion waves. The main characteristic of cavity flow is the vortices inside cavity generated from the collision of the unstable shear layer formed at the cavity leading edge to the cavity trailing edge or to the cavity floor as the flow passes the cavity. Such a collision transmits sound waves upstream and affects the shear layer at the leading edge to create a flow circulation that induces oscillation. This type of circulation process was suggested by Rossiter[1],

In the present study, the numerical analysis on tandem cavities flow was performed by the unsteady, compressible, two dimensional and three dimensional Reynolds-Averaged Navier-Stokes(RANS) equations with the $\kappa-\omega$ turbulence model. The Strouhal numbers of the first and second cavities were compared and verified by Rossiter's equation[1] and experimental values of Xin Zhang and Edwards[2,3]. The cavity model used for numerical calculation had a depth(D) of $15mm$, first cavity L/D of 3, second cavity L/D of 1, and cavity interval of S/D of 1. The supersonic flow had a Reynolds number of 4.5×10^5 based on cavity depth, and Mach number of 1.5. The same basic flow conditions were applied for two dimensional and three dimensional analyses. The width to depth ratio(W/D) was 1.0 for three dimensional analysis. The parallel numerical analysis was carried out, which used the 4th order Runge-Kutta method for time discretization and the second order upwind TVD scheme using the flux vector splits for spatial discretization.

Based on the PSD(Power Spectral Density) analysis of the pressure variation at the first cavity floor, the dominant frequency was reasonable in comparison with the results of Rossiter[1] and Xin Zhang and Edwards[3]. The dominant frequency of the second cavity followed that of the first cavity. This result was in agreement with the results of Xin Zhang and Edwards[3]. For two dimensional flow, the dominant frequency is almost same to the 2nd mode dominant frequency of Rossiter's equation[1]. The 1st mode dominant frequency as well as the 2nd mode frequency appears for the three dimensional flow. These frequencies are supposed due to the lateral inflow and outflow, which makes that the three dimensional flow inside the cavity was less violent than the two dimensional flow.

GOVERNING EQUATIONS AND NUMERICAL METHOD

Three dimensiona Navier-Stokes equation non-dimensionalized by the characteristic values is as follows:

$$\frac{\partial\overline{Q}}{\partial t}+\frac{\partial\overline{E}}{\partial\xi}+\frac{\partial\overline{F}}{\partial\eta}+\frac{\partial\overline{G}}{\partial\zeta}=\frac{\partial\overline{E}_V}{\partial\xi}+\frac{\partial\overline{F}_V}{\partial\eta}+\frac{\partial\overline{G}_V}{\partial\zeta}+\overline{S} \tag{1}$$

The $\kappa-\omega$ model was adapted as the turbulence model. The turbulence kinetic energy k and specific dissipation rate ω were non-dimensionalized by the characteristic velocity and length. The turbulence formula vectors corresponding to each flux vector of the Navier-Stokes equation and constants are as follows[4].

$$Q=\begin{bmatrix}\rho\kappa\\ \rho\omega\end{bmatrix},\quad E=\begin{bmatrix}\rho u\kappa\\ \rho u\omega\end{bmatrix},\quad F=\begin{bmatrix}\rho v\kappa\\ \rho v\omega\end{bmatrix},\quad G=\begin{bmatrix}\rho wk\\ \rho w\omega\end{bmatrix} \tag{2}$$

$$E_v=\begin{bmatrix}\mu\kappa\dfrac{\partial\kappa}{\partial x}\\ \mu\omega\dfrac{\partial\omega}{\partial x}\end{bmatrix},F_v=\begin{bmatrix}\mu\kappa\dfrac{\partial\kappa}{\partial y}\\ \mu\omega\dfrac{\partial\omega}{\partial y}\end{bmatrix},\quad G_v=\begin{bmatrix}\mu\kappa\dfrac{\partial\kappa}{\partial z}\\ \mu\omega\dfrac{\partial\omega}{\partial z}\end{bmatrix}\quad S=\begin{bmatrix}P-\beta^{*}\rho\omega\kappa\\ (\alpha P-\beta\rho\omega\kappa)\dfrac{\omega}{\kappa}\end{bmatrix}$$

The 4th order Runge-Kutta method was used for the time discretization. The 2nd order upwind TVD scheme using the flux vector split with van Leer's limiter was used for the spacial discretization. The parallel process by PC-Cluster was used for calculations.

RESULT AND DISCUSSION

Fig. 1 shows a part of the three dimensional grid system around the cavity. The grid was concentrated near the wall. For two dimensional analysis, the grid number for upper zone was 350×100, for the first cavity 100×70, and for the second cavity 50×70. For three dimensional analysis, the grid numbers were 140×40×40, 50×30×20, and 30×30×20, respectively.

Fig. 2 and Fig. 3 show the frequency characteristics of the SPL, which are adapted to analyze the dominant frequencies of the two dimensional and three dimensional flow, respectively. The frequency characteristics of second cavity are the same as those of first cavity. This result shows that the first cavity flow is transferred to the second cavity with only the phase shift. As shown in Fig.3, the dominant oscillation frequency of three dimensional flow is $5.5kHz$, which is the same as that of two dimensional flow, but due to the two kinds of pressure amplitudes, which appear alternately in three dimensional flow, there is another dominant frequency of $2.59kHz$. This low frequency component clearly appears in the first cavity more than the second cavity, because the amplitude of the first cavity is larger than that of the second cavity.

Table 1 shows the comparison of the frequency characteristics of this study with those obtained from Rossiter's equation[1] and with those of Xin Zhang and Edwards[3], from which it is thought that the present results are very resonable.

Fig. 6 shows the comparison of the non-dimensional Strouhal numbers[1] obtained by Eq.(3) with the experimental values of Heller[5] and present results.

$$St = \frac{fL}{U} = \frac{n-\gamma}{1/k_n - M} \qquad (3)$$

St is the non-dimensional Strouhal number, n is the n-th oscillation mode, k_v (=0.57) is the constant representing the vortex convection speed as a fraction of the freestream flow speed at the cavity entrance, and M is the freestream Mach number. γ (=0.25) is a constant obtained from experiments[6]. The Strouhal number of Heller[5] is 0.6. The 2nd mode($n = 2$) Strouhal number calculated in the present research for two dimensional and three dimensional flows is 0.57, which is almost the same as Rossiter's formula and the experimental value of Heller[5]. In three dimensional flow, the 1st mode($n = 1$) Strouhal number is observed as 0.269, which is similar to the value of the 1st mode by Rossiter's formula[1].

Fig. 5 shows the streamlines of two dimensional flow for one cycle of non-dimensional time step of about 0.65. Fig. 6 shows the streamlines of three dimensional flow at z/D =0.5 for one cycle of non-dimensional time step of 1.3

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Table 1 Comparison of dominant frequency (mode number n=2)

	Xin Zhang[2,3]	Rossiter's Eq	Two Dimensional flow	Three dimensional flow

$L/D=3$	$5.90\,kHz$	$5.45\,kHz$	$5.40\,kHz$	$5.40\,kHz$

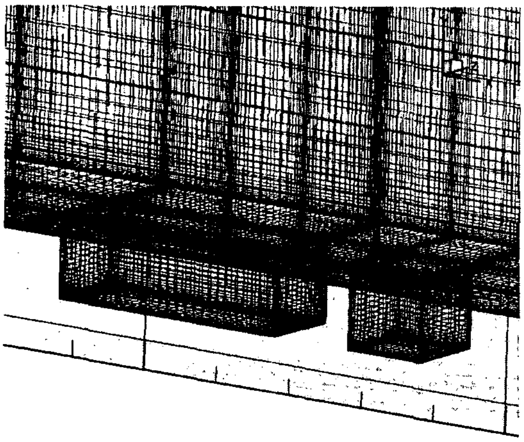

Fig.1 Computational grids for the three dimensional calculation

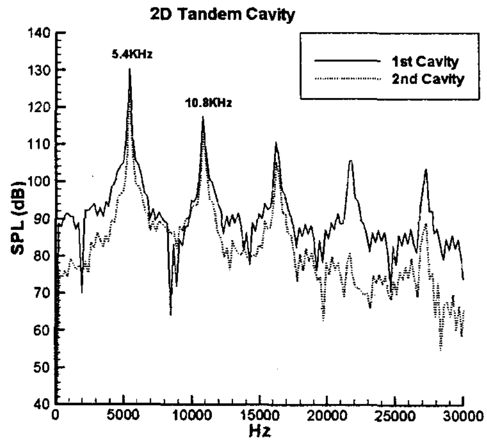

Fig. 2 SPL distribution for two dimensional calculation

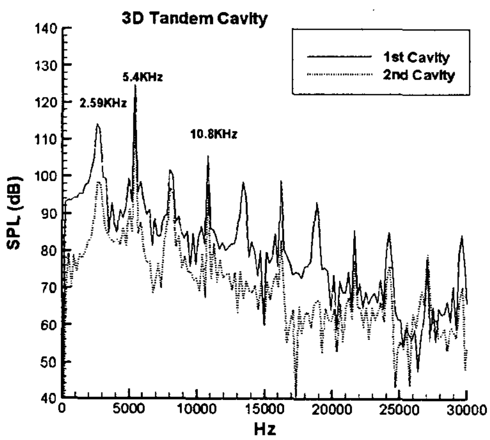

Fig. 3 SPL distribution for three dimensional flow

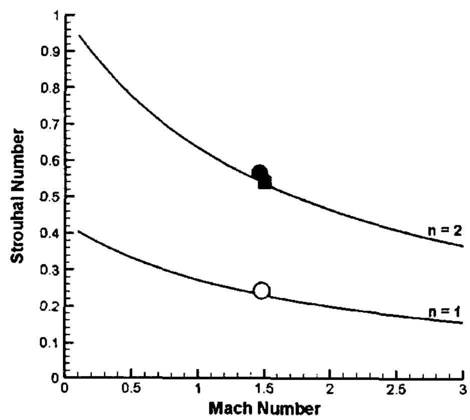

Fig. 4 Non-dimensional resonant frequencies as a function of Mach number, n=mode

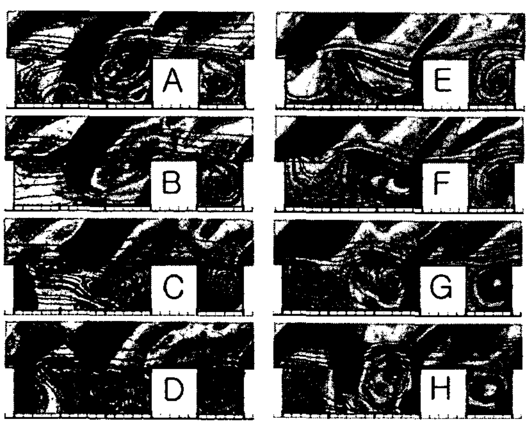

Fig. 5 Streamlines of two dimensional flow

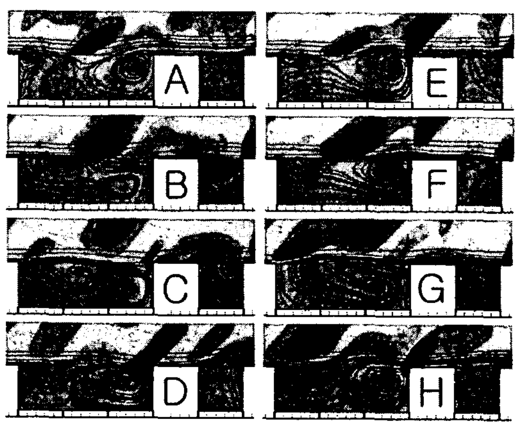

Fig.6 Streamlines of three dimensional flow