Assisted GNSS Positioning for Urban Navigation Based on Receiver Clock Bias Estimation and Prediction Using Improved ARMA Model

*LinyuanXia[1], Esmond Mok[2]

[1]129#Luoyu Rd., Wuhan 430079, State Key Laboratory for Information Engineering on Surveying, Mapping and Remote Sensing (LIESMARS) , Wuhan University, China.(E-mail: lyxia@lmars.whu.edu.cn)

[2] Department of Land Surveying and Geo-Informatics, The Hong Kong Polytechnics University, Hung Hom, Kowloon, Hong Kong. (Email: lsemok@polyu.edu.hk)

Abstract

Among the various error sources in positioning and navigation, the paper focuses on the modeling and prediction of receiver clock bias and then tries to achieve positioning based on simulated and predicted clock bias. With the SA off, it is possible to model receiver clock bias more accurately. We selected several types of GNSS receivers for test using ARMA model. To facilitate prediction with short and limited sample pseudorange observations, AR and ARMA are compared, and the improved AR model is presented to model and predict receiver clock bias based on previous solutions. Our work extends to clock bias prediction and positioning based on predicted clock bias using only 3 satellites that is usually the case under urban canyon situation. In contrast to previous experiences, we find that a receiver clock bias can be well modeled using adopted ARMA model. Test has been done on various types of GNSS receivers to show the validation of developed model.

To further develop this work, we compare solution conditions in terms of DOP values when point positioning is conducted using 3 satellites to simulate urban positioning environment. When condition allows, height component is derived from other ways and can be set as known values. Given this condition, location is possible using less than 2 GNSS satellites with fixed height. Solution condition is also discussed for this background using mode of constrained positioning. We finally suggest an effective predictive time span based on our test exploration under varied conditions.

Keywords: receiver clock bias, ARMA model, model and prediction, constrained positioning

1. Introduction

With the advent of Galileo and other satellite navigation systems, we will share services provided by multiple satellite navigation systems in the near future. So far, people are seeking for a seamless system function from positioning and navigation system. It is well known that the limited availability and visibility is caused by terrain features and skyscrapers under urban conditions. So it is still a challenging issue to achieve positioning under this blind area even if satellite source will be enough abounding Current strategies to deal with limited GNSS visibility includes map matching(MM) that needs digital map to be used with GNSS unit and it must be updated timely to ensure its utilization.

For over years, People have also tried GNSS and INS integration, which has been proved to be somewhat costly for high level accuracy purpose and its size is not acceptable for most users. Digital odometer, barometer and other low cost devices are applied to tackle the problem; different algorithms such as Kalman filtering and etc. are also created to accommodate location under urban situations. Adjustment and calibration for these units often give rise to some problems in practical applications.

It will no doubt benefit our application purpose if further potential in positioning model can be explored to exert its usage, such kinds of efforts have been tried before to assist navigation in difficult situation, especially in urban condition[1,2]. Clock bias has been addressed by many discussions[7] and its solution is eaplained in positioning model[5]. In this contribution, we explored a method to further use the role of GNSS receiver clock bias to assist our navigation positioning. We firstly arrive at reasonable clock bias solution from navigation calculation in which ways to deal with clock bias estimation and height determination is discussed. Secondly feasibility for receiver clock bias simulation by ARMA models is explored in which two modified aspects relating ARMA models are addressed. Based on this point, further effort to predict clock bias using AR model is expended and positioning results based on prediction are exemplified.

2. Brief on ARMA models

A time series obtained from deformation observation can also be viewed as a stochastic process through some pre-processing. We assume that it satisfies ergodicity and can be modeled using related approach from theory of time and system. It is already well known that a steady time series x_t (t=1,…, N) can be represented by ARMA (Auto Regressive and Moving Average) model as[3]

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \cdots + \varphi_p x_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \quad (1)$$

Where p and q are positive integers for model ARMA (p,q), indicating the order for auto regressive and moving averaging respectively. $\varphi_1, \varphi_2, \ldots, \varphi_p$ are auto regressive coefficients and $\theta_1, \theta_2, \ldots, \theta_q$ are moving averaging coefficients. α_t is Gauss noise sequence.

With introduction of rearward shift operator

$$Bx_t = x_{t-1}, B^i x_t = x_{t-i}$$

and symbols

$$\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p$$

and

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$$

(1) becomes：

$$(1-\varphi_1 B-\varphi_2 B^2-\cdots-\varphi_p B^p)x_t = (1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q)a_t \qquad (2)$$

or

$$\varphi(B)x_t = \theta(B)a_t \qquad (3)$$

We have three choices among models AR, MA and ARMA to facilitate specific application. Basically auto correlation function for every model can be employed to distinguish MA and other two, while partial correlation function can be used to select from AR and ARMA. Among them, AR model is expressed as

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \cdots + \varphi_p x_{t-p} + a_t$$

$$\varphi(B)x_t = a_t \qquad (4)$$

It is denoted as AR(p,0), where P is the order of AR model and can be determined from partial correlation function values using least variance estimation between actual series and estimated AR series. Coefficients for AR(p,0) is estimated from auto correlation function values using Yule-Walker equation.

In case of MA model, it is simplified as：

$$x_t = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q)a_t$$

or

$$x_t = \theta(B)a_t \qquad (5)$$

It is denoted as MA(0,q), where q is the order of MA model.

For AR model, we have explicit linear solution. When its order is determined through model identification, Yule-Walker equation by Box-Jenkins gives its solution as(Gareth Janacek,1993):

$$\begin{pmatrix} \hat{\varphi}_1 \\ \hat{\varphi}_2 \\ \vdots \\ \hat{\varphi}_p \end{pmatrix} = \begin{pmatrix} 1 & \rho_1 & \cdots & \rho_{p-1} \\ \rho_1 & 1 & \cdots & \rho_{p-2} \\ & \cdots & \cdots & \\ \rho_{p-1} & \rho_{p-2} & \cdots & 1 \end{pmatrix}^{-1} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_p \end{pmatrix} \qquad (6)$$

Where ρ_k represents autocorrelation values of observation series x_t, solution stability for above expression is guaranteed by positive definition of autocorrelation matrix. Solution quality is indicated by model variance that is calculated by ：

$$\hat{\sigma}_\alpha^2 = \hat{\gamma}_0 (1 - \sum_{j=1}^{p} \hat{\varphi}_j \rho_j) \qquad (7)$$

Where $\hat{\gamma}_0$ is autocorrelation value with zero delay. It is clear that equation solution is determined by a matrix that is composed of autocorrelation values ρ_k.

3. Receiver Clock and Its Modeling

3.1 Receiver Clock Solution

In normal application, receiver clock bias in navigation positioning is calculated with position solution. With signals from *k* GPS satellites for instance, observation equation and solution can be expressed respectively as:

$$\begin{bmatrix} \gamma_{i\,(t)}^1 \\ \gamma_{i\,(t)}^2 \\ \gamma_{i\,(t)}^3 \\ . \\ \gamma_{i\,(t)}^k \end{bmatrix} = \begin{bmatrix} P_{i0(t)}^1 \\ P_{i0(t)}^2 \\ P_{i0(t)}^3 \\ . \\ P_{i0(t)}^k \end{bmatrix} - \begin{bmatrix} l_{i(t)}^1 & m_{i(t)}^1 & n_{i(t)}^1 & -1 \\ l_{i(t)}^2 & m_{i(t)}^2 & n_{i(t)}^2 & -1 \\ l_{i(t)}^3 & m_{i(t)}^3 & n_{i(t)}^3 & -1 \\ . & . & . & . \\ l_{i(t)}^k & m_{i(t)}^k & n_{i(t)}^k & -1 \end{bmatrix} \begin{bmatrix} \delta X_i \\ \delta Y_i \\ \delta Z_i \\ \delta \rho_i \end{bmatrix}$$

Where *l*, *m* and *n* are direction numbers from observer point to tracked satellite. P_i indicates receiver position vector This equation can be symbolized in matrix form as:

$$A\delta X + L = 0 \qquad (8)$$

Least square solution is obtained as:

$$\delta X = -(A^T PA)^{-1} A^T PL \qquad (9)$$

Positioning performance is usually given by DOP values, which is derived from normal equation formed from above observation

$$Q_{X'} = (A^T PA)^{-1} = \begin{bmatrix} q_{XX} & q_{XY} & q_{XZ} & q_{Xt} \\ & q_{YY} & q_{YZ} & q_{Yt} \\ & & q_{ZZ} & q_{Zt} \\ & & & q_{tt} \end{bmatrix}$$

We have PDOP= $\sqrt{(q_{XX} + q_{YY} + q_{ZZ})}$ and TDOP= $\sqrt{q_{tt}}$, through transform into local level coordinate system, we can have DOP values regarding plane and height precision index.

In urban condition, poor DOP usually comes with highly correlation of height and clock bias solution. At some situation, height information is not important or can be roughly obtained from other ways. It can be set as known or a loose constraint in location calculation. When DOP value is poor, we can easily see that satellite is within a restricted area, even Galileo and other satellite systems are added, this will not be improved obviously for a restricted urban region because this is caused by urban building environment. And more signals within a restricted area will result in strong relation between direction vectors, which means near ill matrix structure when solution is resolved.

This effect is reflected by unsteady solution output by location calculation when the number of satellite is not ideal, for example, 4 or 5 satellites with poor DOP values used for location computation.

Different kinds of GNSS receivers can exhibit different clock bias features. Following is a list for some receiver clock bias solutions.

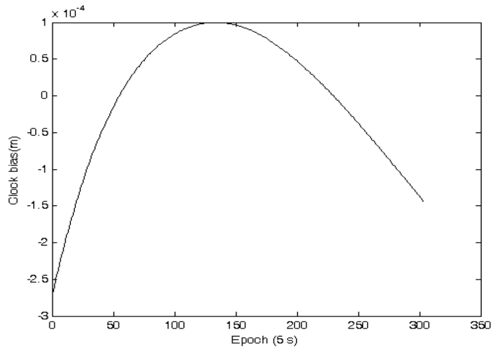

Figure 1. LOCUS single frequency GPS receiver clock bias.

In case of this trend value, it is usually modeled by some polynomial firstly and the rest part of clock bias is treated in the discussion.

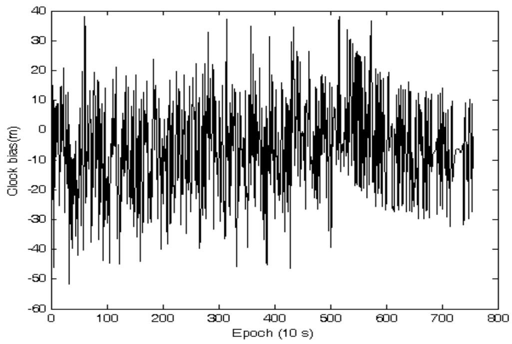

Figure 2. SR299 clock bias estimate

It is noted that estimate is based on observations with SA on, and SA dithering is absorbed into clock normal solution, height and plane positioning solutions.

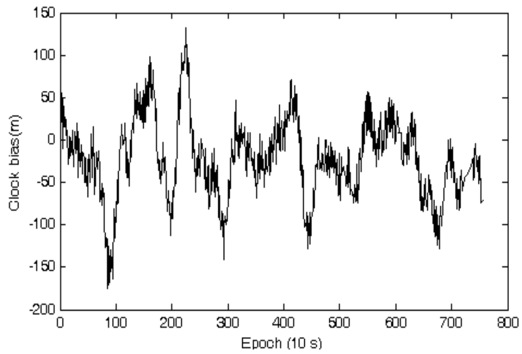

Figure 3. SR299 clock bias estimate with height constrained

From contrast, we can see the differences of the same clock estimatesfor SR299 within this time span. With SA on and height constrained solution shown in figure 3, the solution partly contains SA dithering imposing on satellite signals,other part of SA is contained in plane positioning solutions.

When satellite geometry is not good, we know that there will be more obvious correlation between height and clock bias solution, so it will result in a more objective clock bias solution if height can be roughly obtained or set as known value.

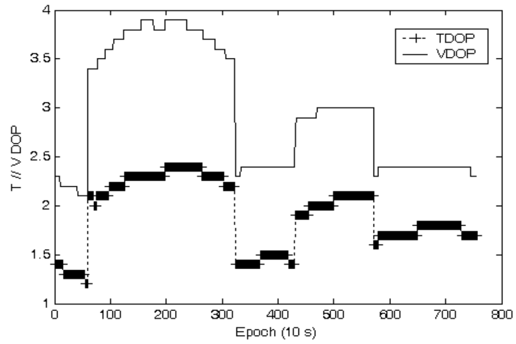

Figure 4. DOP values for SR299 clock bias estimate

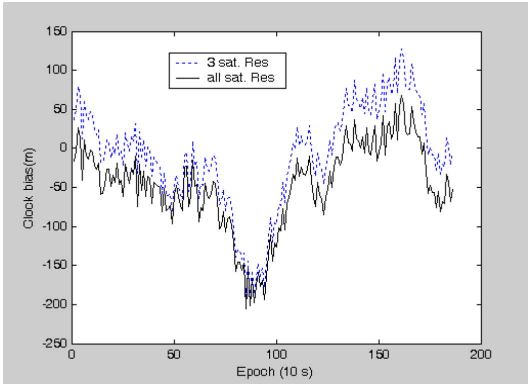

Figure 5. clock biases from 3 satellites and normal estimate

This result shows that clock bias results from 3 satellites with height fixed and normal solution from all visible satellites. (We used 30 minutes observation), difference may reach as large as 20 meters although the general trend is almost identical.

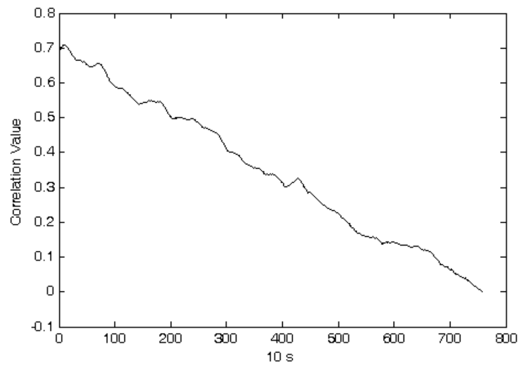

Figure 6. correlation of clock bias and height estimates

It is the usual case that we will have poor DOP values in urban canyon positioning, correlation for height and clo ck bias series is as large as 0.7 by figure 6, so it is worth while to firstly assess impact from this correlation from T DOP and HDOP analysis. If they are strongly correlated to each other, we can set height as known values as a test, perhaps value with some weight from error knowledge in el ectronic maps if condition allows. Given this reasonably der ived clock estimation series, we can go ahead to model clo ck behavior using ARMA models.

3.2 Receiver Clock Modeling

Formula (6) and (7) gives commonly used Box-Jinkens solution for a random series. Depending on nature of series residuals, BJ solution is not ensured with the most optimal and unbiased estimate although the method is simple and has less computation. Its accuracy can not match with following Least Square method stated below.

LS method ensures an unbiased estimation of model parameters with higher accuracy, but computation speed is relatively slow, however, LS based on autocorrelation efficients is more reasonable in terms of reliable normal equation (no ill-conditioned equation) and direct use of information derived from series.

We can see from expressions when the determined order is p, the number of used autocorrelation values for parameter estimation is also p. This kind of processing limits use of autocorrelation values and random property of sampled observations will cause some impact on parameter estimation when sample number is relatively small. We may overcome this demerit using maximum likelihood estimation if distribution function for random error is known. Unfortunately this is not possible for most of cases. Here we try to use commonly used least square method based on consideration for model residual which can be indicated by expression (4) as[9]

$$\varepsilon_t = x_t - \sum_{j=1}^{p} \varphi_j x_{t-j} \qquad (10)$$

squared sum of residual is:

$$V(\varphi) = \sum_{t=p+1}^{N} (x_t - \sum_{j=1}^{p} \varphi_j x_{t-j})^2 \qquad (11)$$

We mark

$$Y = \begin{pmatrix} x_{p+1} \\ \vdots \\ x_N \end{pmatrix}, \quad X = \begin{pmatrix} x_p & x_{p-1} & \cdots & x_1 \\ x_{p+1} & x_p & \cdots & x_2 \\ \vdots & \vdots & & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-P} \end{pmatrix} \qquad (12)$$

$$\varphi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_P \end{pmatrix},$$ then distance in N-p dimensional Euclidean space is:

$$\left\| Y - X\varphi \right\|^2$$

it is easy to derive parameter LS estimation as：

$$\hat{\varphi} = \left(\hat{\varphi}_1, \hat{\varphi}_2, \ldots, \hat{\varphi}_p\right)^T = \left(X^T X\right)^{-1} X^T Y \qquad (13)$$

white noise variance estimate for model is：

$$\hat{\sigma}_\alpha^2 = \frac{1}{N-p} V(\hat{\varphi}) = \frac{1}{N-p} (\hat{\varphi} - \varphi)^T (\hat{\varphi} - \varphi) \qquad (14)$$

It is also easy to realize from estimate theory that derived estimation by (13) is the most optimal and unbiased result when model residuals are independent white noises with the same distribution.

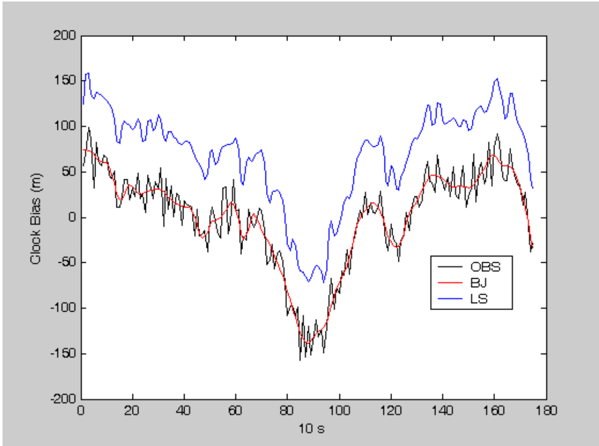

Figure 7. Contrast of BJ LS estimates

Figure 7 lists differences using LS and BJ model. In practical case, this can be selected according to test for different receiver types.

Another improving point is to use AR model as an alternative for MA or ARMA even if they are tested as a choice for modeling receiver clock bias. It is know that solution for AR model has an explicit linear form while non-linear equation is involved in parameter estimation for MA and ARMA. When the identified order is high, seeking for solutions for MA or ARMA becomes inconvenient, this is especially challenging for on line and real time application.

Fortunately we can verify from factorization analysis that any ARMA or MA model can be equivalently replaced by an AR model with a higher order number. This can be explained by introducing Green function G and starting from expression

$$x_t = \frac{\theta(B)}{\varphi(B)} a_t \qquad (15)$$

in case of MA， $\varphi(B) = 1$ and $\theta(B) = 1$ for AR model. This expression can be finally modified by division as

$$x_t = \sum_{i=0}^{\infty} G_i a_{t-i} \qquad (16)$$

The serial is indicated as combination current a_t and past a_t values, in which the role of Green function is similar to weighting function and it specifies weight for every term in expended formula. The model system is steady when $i \to \infty$ and G_i has definite bound, and system model is asymptotically stead when $i \to \infty$ and G_i tends to attenuate. This means we can use AR model to replace ARMA or MA models under some conditions to facilitate our application purpose.

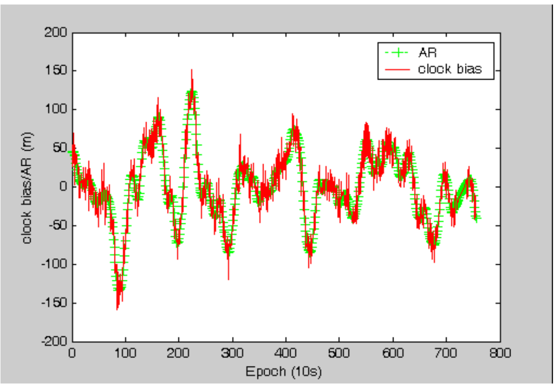

Figure 8. AR model result for SR299 clock bias

Figure 8 is the result using model AR(17, 0) for SR 299 GPS receiver. Standard variance deviation calculated is 1.20. Slightly increased order may reduce variance, but the speed is very slow.

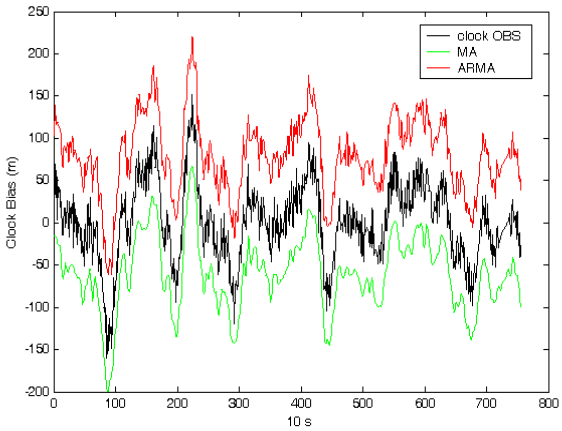

Figure 9. Comparison of AR with MA and ARMA results

MR(0,2) and ARMA(7,2) are used as test for comparison of model selection in contrast to AR with higher order. They show the alternative AR model can approach simulation result as by MA or ARMA. Order selection can be adjusted from test of residuals till an acceptable deviation level is reached.

4. Prediction of Receiver Clock

4.1 Receiver Clock Prediction

Prediction for deformation trend is another important aspect in deformation simulation and monitoring. From ARMA model employed in this research, we see that current observation can be expressed by a certain number of previous observations through established system. We can also predict the next one or two observational trend(s) based on discussed model and limited number of observations. It is essential to achieve prediction using a small number of observations for on line or real time monitoring.

The best prediction based on (16) can be exhibited by:

$$x_{t+l} = \sum_{j=0}^{\infty} G_j a_{t+l-j} \qquad (17)$$

Which can be denoted as:

$$x_{t+l} = (G_0 a_{t+l} + G_1 a_{t+l-1} + \cdots + G_{l-1} a_{t+1}) + (G_l a_t + G_{l+1} a_{t-1} + \ldots) \qquad (18)$$

This means:

$$x_{t+l} = e_{t(l)} + \hat{x}_{t(l)} \qquad (19)$$

The prediction error or residual is:

$$e_{t(l)} = \sum_{j=0}^{l-1} G_j a_{t+l-j} \qquad (20)$$

The prediction can be proved to be the most optimal and unbiased estimate if the serial is normal distributed. The variance for prediction is:

$$Var[e_{t(l)}] = \sigma_a^2 \sum_{j=0}^{l-1} G_j^2 \qquad (21)$$

Prediction efficiency is verified by residual test and model is adjusted until residual is a white noise. Figure 10 gives a practical prediction by one step.

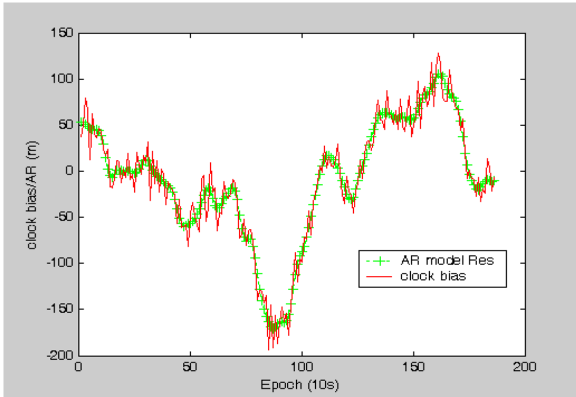

Figure 10. One step prediction using AR model.

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Clock bias prediction based on short time span clock behavior is useful for practical navigation. We use 30m time span clock bias that is got from normal solution with height fixed. For multi-step prediction, effective forward prediction span can be determined by residual test based on suggested AR model.

4.2 Positioning Based on Clock Prediction

To simulate navigation positioning under urban canyon condition, predicted clock bias is employed to bridge positioning gap in case of satellite source scarcity.

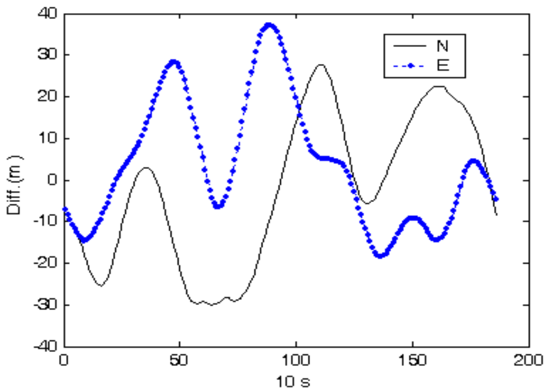

Figure 11. Positioning differences based on predicted clock bias

Figure 11 shows differences of positioning results for N and E components from predicted clock bias with normal solution. This is result of one step prediction, test has shown that positioning based on multi-step prediction is possible, however, effective prediction depends on sample rate and number of known clock bias values used for forward calculation. The interval can be controlled by prediction variance and positioning root mean squares test.

5. Conclusion and Summary

The paper focuses on positioning method on urban canyon condition supported by predicted receiver clock bias. The suggested use of modified ARMA models in which the modified aspects includes LS based AR parameter estimation and replacement of MA and ARMA using equivalent AR model. Analysis using practical observation data shows that the suggested plan can work well based on well processed clock bias.

Future plan include issues how to further relate this work with urban navigation application.

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Reference

1. MOK Esmond, Lawrence LAU (2001) "GPS vehicle location tracking with the Minimum Range ERror Algorithm (MRERA)". Proceedings, ION GPS-2001, Salt Lake City, Utah, 11-14 September,2001, pp. 2564-2570.
2. MOK Esmond, Guenther Retscher and Yong-Qi Chen (2000), "Conceptual Design of an Intelligent GPS Vehicle Navigation System Suitable for Hong Kong". The Hong Kong Surveyors.
3. Gareth Janacek and Louise Swift, (1993) Time Series, Forecasting, Simulation, Applications, Ellis Horwood Limitted, New York.
4. Kai Borre, Christian Tiberius,(2000), Time Series Analysis of GPS Observables, Proceedings of the 13th International Technical Meeting of the Satellite Division of the Institute of Navigation GPS 2000, September 19-22,Salt Lake City, Utah, USA,P245-248.
5. Young C. Lee, Receiver Autonomous Integrity Monitoring Availability for GPS Augmented with Barometeric Altimeter Aiding and Clock Coasting, B.W.Parkinson, J.J.Spilker Jr, Global Positioning System: Theory and Applications, American Institute of Aeronautics and Astronautics, Inc., 1996.
6. Leon Cohen, (1995) Time–Frequency Analysis: Theory and Application. Prentice Hall.
7. Pratap N. Misra. The Role of the Clock in a GPS Receiver [J]. GPS World, April 1996, Volume 7 Number 4:60-66
8. L. Xia, J. Liu, L.Lu, “Research on Matching and Constrained GPS Positioning in ITS”. ITS_NAV 2002, October 11-13, 2002, Nanjing, China.
9. Zhang X.D. (1995)， Modern Signal Processing, Qinghua University Press.