A Maximum Likelihood Estimator Based Tracking Algorithm for GNSS Signals

*Jong-Hoon Won[1], Thomas Pany[2], Bernd Eissfeller[3]

[1]Institute of Geodesy and Navigation, University FAF Munich, Germany (jonghoon.won@unibw-muenchen.de)
[2]Institute of Geodesy and Navigation, University FAF Munich, Germany (Thomas.pany@unibw-muenchen.de)
[3]Institute of Geodesy and Navigation, University FAF Munich, Germany (bernd.eissfeller@unibw-muenchen.de)

Abstract

This paper presents a novel signal tracking algorithm for GNSS receivers using a MLE technique. In order to perform a robust signal tracking in severe signal environments, e.g., high dynamics for navigation vehicles or weak signals for indoor positioning, the MLE based signal tracking approach is adopted in the paper.

With assuming white Gaussian additive noise, the cost function of MLE is expanded to the cost function of NLSE. Efficient and practical approach for Doppler frequency tracking by the MLE is derived based on the assumption of code-free signals, i.e., the cost function of the MLE for carrier Doppler tracking is used to derive a discriminator function to create error signals from incoming and reference signals. The use of the MLE method for carrier tracking makes it possible to generalize the MLE equation for arbitrary codes and modulation schemes. This is ideally suited for various GNSS signals with same structure of tracking module. This paper proposes two different types of MLE based tracking method, i.e., an iterative batch processing method and a non-iterative feed-forward processing method. The first method is derived without any limitation on time consumption, while the second method is proposed for a time limited case by using a 1st derivative of cost function, which is proportional to error signal from discriminators of conventional tracking methods. The second method can be implemented by a block diagram approach for tracking carrier phase, Doppler frequency and code phase with assuming no correlation of signal parameters. Finally, a state space form of FLL/PLL/DLL is adopted to the designed MLE based tracking algorithm for reducing noise on the estimated signal parameters.

Keywords: GNSS, GPS, Galileo, MLE, PLL, FLL, DLL

1. Introduction

Conventional tracking methods widely used in hardware-based GNSS receivers such as Phase Lock Loop (PLL), Frequency Lock Loop (FLL) and Delay Lock Loop (DLL) use a feed forward filtering approach and have a limited performance in high dynamic or weak signal environments [1,2,3].

Recent signal processing technology makes it possible to use a Software Defined Radio (SDR) approach in designing a GNSS receiver with a commercial purpose microprocessor. This provides a flexible and a very easy design method to use a sophisticated mathematical algorithm for the improved signal tracking in a GNSS receiver.

The Maximum Likelihood Estimation (MLE) approach, which is based on the premise that over a sufficient short observation interval the signal parameters may be viewed as unknown constant quantities, is known to yield the best performance for the GNSS signal tracking [3,4,5]. That means for time-invariant systems with stationary noise processes and constant parameter vector, the MLE approach provides the following desirable properties for the GNSS signal tracking as time goes to infinity: consistency, unbiasedness, normality, and efficiency, i.e., achieving Cramer-Rao bound of estimation error variance [6]. Therefore, in order to track the GNSS signal in such a severe signal environments, the use of MLE based signal tracking method is preferable.

This paper presents two types of MLE based GNSS signal method, i.e., an iterative batch processing method and a non-iterative feed-forward processing method. A log-likelihood cost function proposed in Ref. 4 and 5 is reviewed and also used to derive solution algorithms for GNSS signal parameters with a very well understandable mathematical manner. Also this paper shows a quadratic cost function of Nonlinear Least-Squares Estimation (NLSE) for the GNSS signal parameters is equivalent to the cost function of MLE with a white noise assumption on the received signals. Using an assumption of code-free signal, a solution method which uses the Doppler frequency as a state variable is provided for efficient computational processing. For more improvement of computational performance, this paper expands the MLE method to a non-iterative form of solution by a feed-forward approach. This method can be ideally suited for the hardware-based correlator implementation. In order to mitigate noise effect on the estimated signal parameters, a four state fixed rate optimal filter which uses the code phase, the carrier phase, the Doppler frequency and the rate of Doppler frequency as state variables is designed.

2. MLE for GNSS Signal Tracking

The MLE algorithm is derived based on the premise that over a sufficient short observation interval, e.g., a coherent integration time interval, the GNSS signal parameters may be viewed as unknown constant quantities and the MLE of those parameters yields the best performance in terms of minimum noise and no loss-of-lock.

2.1 Signal Models

For a visible GNSS satellite, the received IF signal at the end of GNSS receiver's RF frontend is modeled as [1]

$$r(t) = A(t) \cdot C(t-\tau) \cdot D(t-\tau) \cdot \cos\left(2\pi (f_{IF} + f_d)t + \phi\right) + n(t) \quad (1)$$

where $r(t)$ is the received IF GNSS signal at the end of the RF frontend at time t, $A(t)$ is the signal amplitude, $C(t)$ is the PRN (or BOC) code sequence for GNSS, $D(t)$ is the navigation data bit, $n(t)$ is the thermal noise, τ is the code delay time in seconds, f_d is the carrier Doppler frequency shift in Hz, and ϕ is the carrier phase in radian.

With assuming the navigation data bit does not change in the coherent integration time interval, we use the received signal model in Eq. (1) in order to apply estimation technique for tracking GPS signal given by

$$r(k) = A \cdot C(k-\tau)\cos\left(2\pi T(f_{IF} + f_d)k + \phi\right) + n(k)\,, \tag{2}$$

where T represents the sampling time interval in second and the signal parameters (A, τ, f_d, ϕ) are assumed to change slowly enough to be considered unknown constants over any observation interval.

In order to estimate signal parameters in Eq. (2), the conventional tracking method widely uses a 2nd order FLL for Doppler frequency tracking, a 3rd order PLL for carrier phase tracking and a carrier aided 1st order DLL with narrow bandwidth for code phase tracking [1,2,3].

2.2 MLE Cost Function [4,5]

The joint probability density of the 1st N complex samples conditioned on the signal parameters can be expressed as

$$p(r_N \mid A, \tau, f_d, \phi) \equiv \frac{1}{(\pi N_0)^N}\exp\left(-\frac{1}{N_0}(r_N - \hat{r}_N)^T W (r_N - \hat{r}_N)\right) \tag{3}$$

where $r_N = \{r(0), r(1), \ldots\ldots, r(N-1)\}^T$ is a given N-dimensional observation vector, W is the weighting matrix for r_N and $N_0/2$ represents the two-sided power spectral density of the noise $n(k)$ in Eq. (2).

The best estimator of the signal parameters based on the observation vector r_N, i.e., the limit of an infinite number of samples, with a Gaussian noise assumption are obtained by simultaneously maximizing the joint conditional probability density function in Eq. (3). This is the MLE where a Minimum Variance Unbiased Estimate (MVUE) with the Cramer-Rao lower bound is achievable. The computation of the joint probability density function in the above equation is very difficult, because the measurements are correlated, i.e., the joint probability density function cannot be expressed with the product of individual probability density function. Fortunately, because the measurements and the innovations are causally invertible and the innovations are all uncorrelated, the log-likelihood cost function of the joint probability density function in Eq. (3) is defined using an assumption on the scalar weighting factor for each measurement as

$$L(A, \tau, f_d, \phi \mid r_N) \equiv -\frac{1}{N_0}\sum_{k=0}^{N-1} w_k \left|r(k) - \hat{r}(k)\right|^2 \cdot \tag{4}$$

Thus, the MLE of the signal parameters are obtained by maximizing the log-likelihood cost function in Eq. (4) based on the observation vector r_N satisfying

$$\frac{\partial L(\theta \mid r_N)}{\partial \theta} = 0 \tag{5}$$

where θ represents the signal parameter vector (=$[A, \tau, f_d, \phi]^T$).

Substituting Eq. (2) to Eq. (4) and simple manipulating based on the assumption on the unity scalar weight factor ($w_k = 1$) yield

$$\begin{aligned} N_0 L(A, \tau, f_d, \phi \mid r_N) \equiv &-\sum_{k=0}^{N-1} r^2(k) - \hat{A}^2\sum_{k=0}^{N-1}\cos^2\left(2\pi T(f_{IF} + \hat{f}_d)k + \hat{\phi}\right) \\ &+ 2\hat{A}\sum_{k=0}^{N-1} r(k)\cdot C(k-\hat{\tau})\cos\left(2\pi T(f_{IF} + \hat{f}_d)k + \hat{\phi}\right) \end{aligned} \tag{6}$$

With assuming $N >> (f_{IF} + f_d)^{-1}$, the 2nd component of the right side can be as $\sum_{k=0}^{N-1}\cos^2\left(2\pi T(f_{IF} + \hat{f}_d)k + \hat{\phi}\right) \approx N/2$. Thus, the 1st and the 2nd components on the right side have a constant value (no information on the signal parameters) and only the 3rd term on the right side has signal parameter components $\hat{\theta}$, i.e., $\hat{A}$, $\hat{\tau}$, $\hat{f}_d$ and $\hat{\phi}$ as variables explicitly. As a result a MLE of $\hat{\theta}$ in Eq. (6) can be obtained when the 3rd component on the right side in Eq. (6) is maximized. Note that the 3rd component on the right side is a product of incoming and replica IF signals. This is why we can use a mixer operation (multiplication) to track GNSS signals [7].

The 3rd component of Eq. (6) can be rewritten as

$$\begin{aligned} &2\hat{A}\sum_{k=0}^{N-1} r(k)\cdot C(k-\hat{\tau})\cos\left(2\pi T(f_{IF} + \hat{f}_d)k + \hat{\phi}\right) = \\ &2\hat{A}\,\mathrm{Re}\left\{\exp(j\hat{\phi})\sum_{k=0}^{N-1} r(k)\cdot C(k-\hat{\tau})\exp\left(2\pi T(f_{IF} + \hat{f}_d)k\right)\right\}. \end{aligned} \tag{7}$$

Without any doubt, for any complex z, $\mathrm{Re}\{\exp(j\phi)z\}$ is maximum with respect to ϕ when $\phi = -\arg(z)$. Thus, the MLE of $\hat{\phi}$ is therefore

$$\hat{\phi} = -\arg\left\{2\hat{A}\sum_{k=0}^{N-1} r(k)\cdot C(k-\tau)\exp\left(j2\pi T(f_{IF} + f_d)k\right)\right\}\cdot \tag{8}$$

Substituting $\hat{\phi}$ in Eg. (8) into Eq. (6), differentiating with respect to $\hat{A}$ and equaling to zero yield a MLE of A given by

$$\hat{A} = \frac{2}{N}\left|\sum_{k=0}^{N-1} r(k)\cdot C(k-\tau)\exp\left(j2\pi T(f_{IF} + f_d)k\right)\right|\cdot \tag{9}$$

Substituting the results in Eqs. (8) and (9) into Eq. (4) yields a new two-dimensional MLE cost function for τ and f_d given by

$$L(\tau, f_d \mid r_N) = \frac{1}{N_0}\left|\sum_{k=0}^{N-1} r(k)C(k-\tau)\exp(j2\pi T(f_{IF} + f_d)k)\right|^2\cdot \tag{10}$$

The above equation is similar to the square of the amplitude of the discrete Fourier transform on $r(k)C(k-1)$ scaled by $1/N_0$ and can be also expanded to the sin-cos form using the relationship of $\exp(j\theta) = \cos\theta + j\sin\theta$ as

$$\begin{aligned} L(\tau, f_d \mid r_N) = &\frac{1}{N_0}\left\{\sum_{k=0}^{N-1} r(k)C(k-\tau)\cos(2\pi T(f_{IF} + f_d)k)\right\}^2 \\ &+ \frac{1}{N_0}\left\{\sum_{k=0}^{N-1} r(k)C(k-\tau)\sin(2\pi T(f_{IF} + f_d)k)\right\}^2. \end{aligned} \tag{11}$$

The above equation simply means that the two-dimensional MLE cost function for τ and f_d can be expressed with the sum of the In-phase (I) and the quad-phase (Q) components which is similar with the conventional correlation function except for the initial carrier phase. In Eqs. (8), (9) and (11), the unknown signal parameters ϕ and A are nuisance parameters that must be estimated simultaneously with the desired parameters τ and f_d. As a result, the primary measurements of GNSS receiver can be regarded as the carrier phase, the Doppler frequency, code phase and signal-to-noise ratio.

2.3 NLSE Cost Function

A direct approach for obtaining signal parameters is obtainable to choose them so as to minimize the weighted sum of squared errors for 1st N complex samples expressed by

$$J(A,\tau,f_d,\phi)=(r_N-\hat{r}_N)^T W(r_N-\hat{r}_N)\,. \tag{12}$$

This is a quadratic cost function of the NLSE for errors and the NLSE of the signal parameters are obtained by minimizing this quadratic cost function based on the observation vector r_N. With assuming a unity scalar weight factor such that $w_k=1/\sigma^2_{r(k)}=1$, the quadratic cost function of NLSE in Eq. (12) can be rewritten as

$$J(A,\tau,f_d,\phi)=\sum_{k=0}^{N-1}\left|r(k)-\hat{r}(k)\right|^2\Big/\sigma^2_{r(k)}\,. \tag{13}$$

Obviously the assumption of a unity scalar weight factor for the received IF GNSS signal is reasonable in case of additive white Gaussian noise. Therefore, the minimization problem of the quadratic cost function of NLSE in Eq. (12) is equivalent to the maximization problem of the log-likelihood cost function of MLE in Eq. (4). As a result, the above equation can also be used to estimate signal parameters as similar way to the MLE except for the minimization criteria satisfying

$$\frac{\partial J(\theta)}{\partial\theta}=0\,. \tag{14}$$

The use of Jaccobian matrix based on this equation makes it possible to take a faster NLSE solution algorithm compared to the MLE (we describe it in a later section).

3. Practical Approach to MLE with Code-free Assumption

The MLE cost function in Eq. (11) contains the discrete nature of code signal and does not satisfy the convexity property of the continuous time function for all range of code delay values. Assuming the estimate of τ available from a conventional DLL such that $C(k-\tau)C(k-\hat{\tau})\approx 1$ for a small code delay estimate error, we can obtain the code free signal model given by

$$\begin{aligned}\bar{r}(k)&=r(k)\cdot C(k-\hat{\tau})\\&=\cos(2\pi T(f_{IF}+f_d)k+\phi)+\bar{n}(k)\end{aligned} \tag{15}$$

where $\bar{n}(k)=n(k)C(k-\hat{\tau})$ and $\bar{r}(k)$ denotes the code-free signal.

Assuming the initial carrier phase ϕ known, a new MLE cost function of the code-free signal, i.e., known τ, is given by

$$\begin{aligned}L(f_d\mid r_N)=&\frac{1}{N_0}\left\{\sum_{k=0}^{N-1}\bar{r}(k)\cos(2\pi T(f_{IF}+f_d)k)\right\}^2\\&+\frac{1}{N_0}\left\{\sum_{k=0}^{N-1}\bar{r}(k)\sin(2\pi T(f_{IF}+f_d)k)\right\}^2.\end{aligned} \tag{16}$$

Assuming that code synchronization and dispreading are performed prior to carrier phase tracking since sufficient signal-to-noise ratio is necessary for the DLL to operate successfully, we can use the above cost function to derive the MLE algorithm for tracking the carrier Doppler frequency.

The reason why we can use the code-free assumption on the MLE is as follows: For the code tracking, the carrier aided 1st order DLL is widely used successfully since the DLL measurement is only used to remove long term bias. Also, a quite small bandwidth can be used for the DLL because the receiver motion is captured by the measured change in carrier phase, i.e., the Doppler frequency. A well-designed carrier tracking loop contains a PLL which is assisted by a FLL because the FLL is less sensitive than the PLL and provides us with more robust tracking performance. As a result, the code phase can be successfully estimated by using a carrier-aided DLL and also can be assumed to be known, i.e., if the Doppler frequency is well tracked, the code phase is considered as a known value because it is directly obtainable by integrating the Doppler frequency.

4. Solution Methods for MLE

Because the signal parameters are located in the cost functions in Eq. (4) and (13) with a complicated nonlinear manner, no simple close form solution is possible. The one way to obtain the MLE is the estimation technique by means of mathematical programming with iterative approach, e.g., Levenberg-Marquardt method or NLSE method. The other way is given from a tracking technique with non-iterative approach by feed-forward manner directly using a gradient assuming no coupling between signal parameters.

4.1 Iterative Batch Solution Methods

Levenberg-Marquardts method

The Levenberg-Marquardt method is known as the most effective optimization method to determine the MLE [6]. This algorithm requires the computation of the gradient as well as the Hessian matrix of the log-likelihood function given by

$$\hat{\theta}^{i+1}_{ML}=\hat{\theta}^{i}_{ML}-(H_i+D_i)^{-1}G_i,\quad \text{for } i=0,1,... \tag{17}$$

where θ_{ML} is the 2-by-1 MLE state vector ($=[\tau,f_d]^T$), G_i and H_i are the 2-by-1 gradient vector and the 2-by-2 pseudo-Hessian matrix, respectively:

$$G_i=\left[\frac{\partial L(\theta\mid Z)}{\partial\theta}\right]_{\theta=\hat{\theta}^i} \tag{18}$$

$$H_i=\left[\frac{\partial^2 L(\theta\mid Z)}{\partial\theta^2}\right]_{\theta=\hat{\theta}^i} \tag{19}$$

$$\text{with}\quad \frac{\partial L}{\partial\theta}=\begin{bmatrix}\dfrac{\partial L}{\partial\tau}\\[2mm]\dfrac{\partial L}{\partial f_d}\end{bmatrix}\quad\text{and}\quad \frac{\partial^2 L}{\partial\theta^2}=\begin{bmatrix}\dfrac{\partial^2 L}{\partial\tau^2} & \dfrac{\partial^2 L}{\partial\tau\partial f_d}\\[2mm]\dfrac{\partial^2 L}{\partial f_d\partial\tau} & \dfrac{\partial^2 L}{\partial f_d^2}\end{bmatrix} \tag{20}$$

and D_i is a diagonal matrix chosen to force $H_i + D_i$ to be positive definite so that $(H_i + D_i)^{-1}$ will always be computable, and i represents the iteration index. Detailed expressions for Eq. (20) are given as follows:

$$\frac{\partial L}{\partial \tau} = -2\left\{\sum_k r\cdot C\cdot \cos\right\}\times\left\{\sum_k r\cdot C'\cdot \cos\right\} -2\left\{\sum_k r\cdot C\cdot \sin\right\}\times\left\{\sum_k rC'\cdot \sin\right\} \tag{21}$$

$$\frac{\partial L}{\partial f_d} = -4\pi T\left\{\sum_k r\cdot C\cdot \cos\right\}\times\left\{\sum_k r\cdot C\cdot k\cdot \sin\right\} +4\pi T\left\{\sum_k r\cdot C\cdot \sin\right\}\times\left\{\sum_k r\cdot C\cdot k\cdot \cos\right\} \tag{22}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial \tau^2} = {} & 2\left\{\sum_k r\cdot C'\cdot \cos\right\}^2 + 2\left\{\sum_k rC'\cdot \sin\right\}^2 \\ & +2\left\{\sum_k r\cdot C\cdot \cos\right\}\times\left\{\sum_k r\cdot C''\cos\right\} \\ & +2\left\{\sum_k r\cdot C\cdot \sin\right\}\times\left\{\sum_k r\cdot C''\sin\right\}\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial f_d^2} = {} & 8\pi^2T^2\left\{\sum_k r\cdot C\cdot k\cdot \sin\right\}^2 + 8\pi^2T^2\left\{\sum_k r\cdot C\cdot k\cdot \cos\right\}^2 \\ & +8\pi^2T^2\left\{\sum_k r\cdot C\cdot \cos\right\}\times\left\{\sum_k r\cdot C\cdot k^2\cdot \cos\right\} \\ & -8\pi^2T^2\left\{\sum_k r\cdot C\cdot \sin\right\}\times\left\{\sum_k r\cdot C\cdot k^2\cdot \sin\right\}\end{aligned} \tag{24}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial \tau \partial f_d} = {} & 4\pi T\left\{\sum_k r\cdot C\cdot k\cdot \sin\right\}\times\left\{\sum_k r\cdot C'\cdot \cos\right\} \\ & +4\pi T\left\{\sum_k r\cdot C'\cdot k\cdot \sin)\right\}\times\left\{\sum_k r\cdot C\cdot \cos\right\} \\ & -4\pi T\left\{\sum_k r\cdot C\cdot k\cdot \cos\right\}\times\left\{\sum_k r\cdot C'\cdot \sin\right\} \\ & -4\pi T\left\{\sum_k r\cdot C'\cdot k\cdot \cos\right\}\times\left\{\sum_k r\cdot C\cdot \sin\right\} \\ = {} & \frac{\partial^2 L}{\partial f_d \partial \tau}\end{aligned} \tag{25}$$

where $\sin = \sin(2\pi T(f_{IF} + f_d)k)$,
$\cos = \cos(2\pi T(f_{IF} + f_d)k)$,
$C = C(k-\tau)$,
$C' = C'(k-\tau)$,
$C'' = C''(k-\tau)$,
$r = r(k)$,
$\sum_k(\bullet) = \sum_{k=0}^{N-1}(\bullet)$.

The 1st and 2nd derivative of the delayed code $C(k-\tau)$ with respect to τ is obtained from the Early and Late arm given by

$$C'(k-\tau) = \frac{\partial C(k-\tau)}{\partial \tau} \approx \frac{C(k-\tau-d/2) - C(k-\tau+d/2)}{d} \tag{26}$$

$$C''(k-\tau) = \frac{\partial^2 C(k-\tau)}{\partial \tau^2} \tag{27}$$

where d represents correlator spacing of Early-minus-Late codes in chips and also in practice can be set go to zero.

The procedure of the Levenberg-Marquardt algorithm is as follows: Initial values for $\hat{\theta}^0$ must be specified and matrix D_0 should be initialized to a diagonal matrix. The Levenberg-Marquardt algorithm corrects its state using Eq. (17) and the MLE cost function $L(\hat{\theta}^{i+1} \mid Z)$ is compared with the previous cost function to increase D_i and try again or to check convergence. By accepting $\hat{\theta}^i$ only if $L(\hat{\theta}^1 \mid Z) \ge L(\hat{\theta}^{1-1} \mid Z)$, we guarantee that each iteration will improve the likelihood of our estimates. This iteration is terminated when the change in $L(\hat{\theta}^i \mid Z)$ falls below some predefined threshold.

The carrier phase is obtained by the MLE solution equation for $\hat{\phi}$ in Eq. (8) where the estimate of carrier frequency is integrated by iterative manner as

$$\hat{\phi}^{i+1} = \hat{\phi}^i + \delta\hat{\phi} \tag{28}$$

where $\delta\hat{\phi}$ denotes the carrier phase difference between the incoming and the replica IF signals at each iteration which is given by arctangent of Q over I.

For an example of the carrier Doppler frequency error in GPS L1 CA code signals, the cost function in Eq. (16), the gradient function in Eq. (18) and the pseudo-Hessian function in Eq. (19), can be normalized as depicted in Figure 1. The cost function has a bell shape between approximately ±900 Hz and has a maximum value at the zero Doppler frequency error. For the smaller Doppler frequency error, the gradient value becomes smaller while the pseudo-Hessian value becomes larger. It means that the correction to the next step, i.e., $(H_i + D_i)^{-1}G_i$, in Eq. (17) becomes smaller and results in a fine convergence. In contrast, for the larger Doppler frequency error, the correction becomes having larger value. This makes it possible for the estimator to converge fast to the true value. By choosing the D_i to force $H_i + D_i$ to be positive definite at every iteration, we can control the convergence speed of the estimator.

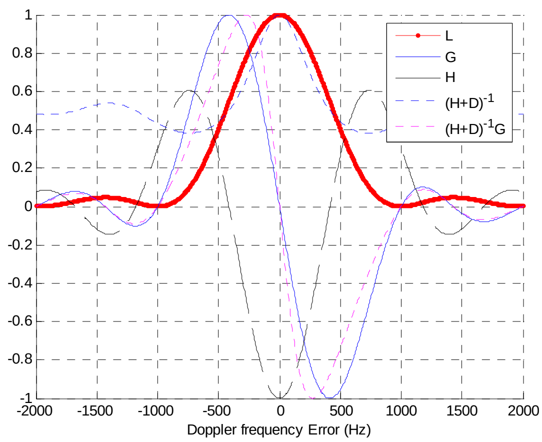

Figure 1 Graphical description on Levenberg-Marquardt method for Doppler frequency estimation

Assuming the code phase τ is available from a conventional DLL, i.e., code-free signal model, substituting the efficient cost function for code-free signal in Eq. (16) into Eqs. (22) and (25) yields a scalar (1-by-1) gradient and Hessian only in terms of Doppler frequency f_d given by

$$\frac{\partial L}{\partial f_d} = -4\pi T\left\{\sum_k \bar{r}\cdot\cos\right\}\times\left\{\sum_k \bar{r}\cdot k\cdot\sin\right\} + 4\pi T\left\{\sum_k \bar{r}\cdot\sin\right\}\times\left\{\sum_k \bar{r}\cdot k\cdot\cos\right\} \tag{29}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial f_d^2} = {} & 8\pi^2T^2\left\{\sum_k \bar{r}\cdot k\cdot\sin\right\}^2 + 8\pi^2T^2\left\{\sum_k \bar{r}\cdot k\cdot\cos\right\}^2 \\ & -8\pi^2T^2\left\{\sum_k \bar{r}\cdot\cos\right\}\times\left\{\sum_k \bar{r}\cdot k^2\cdot\cos\right\} \\ & -8\pi^2T^2\left\{\sum_k \bar{r}\cdot\sin\right\}\times\left\{\sum_k \bar{r}\cdot k^2\cdot\sin\right\}\end{aligned} \tag{30}$$

where $\bar{r} = \bar{r}(k)$.

This new solution does not require the redundant calculations on the gradient and Hessian for τ, i.e., the gradient and the Hessian become a scalar. So, the computational load for this case is less than the case before.

NLSE method

From Eq. (2), the nonlinear signal model for N-dimensional observation vector can be written in a vector form given by

$$Z = h_N(\theta) + n_N \tag{31}$$

where Z is the N-by-1 IF signal measurement vector, $[r(0),.....,r(N-1)]^T$, n_N is the N-by-1 measurement error vector, $[n(0),.....,n(N-1)]^T$, and $h_N(\theta)$ is the N-by-1 nonlinear signal model.

Linearizing the Eq. (31) about nominal value $\theta^*(=[\tau^*, f_d^*]^T)$ yields a linear measurement equation given by

$$\delta Z = H\cdot\delta\theta + \bar{n}_N \tag{32}$$

where $\delta Z = [\delta z(0),......,\delta z(N-1)]^T$,

$\delta z(k) = r(k) - \hat{r}(k)$

$\hat{r}(k) = \hat{A}C(k-\hat{\tau})\cos(2\pi T(f_{IF}+\hat{f}_d)k+\hat{\phi})$,

$$H = \frac{\partial h_N(\theta)}{\partial\theta} = \begin{bmatrix} \frac{\partial r(0)}{\partial\tau} & \frac{\partial r(0)}{\partial f_d} \\ \vdots & \vdots \\ \frac{\partial r(N-1)}{\partial\tau} & \frac{\partial r(N-1)}{\partial f_d}\end{bmatrix}, \text{ (Jaccobian)}$$

$\delta\theta = [\delta\tau, \delta f]^T$,

and the elements of Jaccobian matrix are given by

$$\frac{\partial r(k)}{\partial\tau} = -C'\cdot\cos \tag{33}$$

$$\frac{\partial r(k)}{\partial f_d} = -2\pi TC\cdot k\cdot\sin \tag{34}$$

where $\sin = \sin\left(2\pi T(f_{IF}+f_d)k+\phi\right)$

$\cos = \cos\left(2\pi T(f_{IF}+f_d)k+\phi\right)$.

The solution of Eq. (31) can be obtained by the iterative least-squares technique by using Eq. (32) described in the following equations [6]

$$\delta\theta = (H^TH)^{-1}H^T\delta Z \tag{35}$$

$$\hat{\theta}^{i+1} = \hat{\theta}^i + \delta\theta^i \tag{36}$$

Detailed expressions related to the Jaccobian matrix are given as follows:

$$\begin{aligned} H^TH &= \begin{bmatrix} \frac{\partial r(0)}{\partial\tau} & \cdots & \frac{\partial r(N-1)}{\partial\tau} \\ \frac{\partial r(0)}{\partial f_d} & \cdots & \frac{\partial r(N-1)}{\partial f_d}\end{bmatrix}\begin{bmatrix} \frac{\partial r(0)}{\partial\tau} & \frac{\partial r(0)}{\partial f_d} \\ \vdots & \vdots \\ \frac{\partial r(N-1)}{\partial\tau} & \frac{\partial r(N-1)}{\partial f_d}\end{bmatrix} \\ &= \begin{bmatrix} \sum_k\left(\frac{\partial r(k)}{\partial\tau}\right)^2 & \sum_k\frac{\partial r(k)}{\partial\tau}\frac{\partial r(k)}{\partial f_d} \\ \sum_k\frac{\partial r(k)}{\partial f_d}\frac{\partial r(k)}{\partial\tau} & \sum_k\left(\frac{\partial r(k)}{\partial f_d}\right)^2\end{bmatrix}\end{aligned} \tag{37}$$

$$\begin{aligned} H^T\delta Z &= \begin{bmatrix} \frac{\partial r(0)}{\partial\tau} & \cdots & \frac{\partial r(N-1)}{\partial\tau} \\ \frac{\partial r(0)}{\partial f_d} & \cdots & \frac{\partial r(N-1)}{\partial f_d}\end{bmatrix}\begin{bmatrix}\delta z(0) \\ \vdots \\ \delta z(N-1)\end{bmatrix} \\ &= \begin{bmatrix}\sum_k\frac{\partial r(k)}{\partial\tau}\delta z(k) \\ \sum_k\frac{\partial r(k)}{\partial f_d}\delta z(k)\end{bmatrix}.\end{aligned} \tag{38}$$

Substituting Eqs. (33) and (34) into Eqs. (37) and (38) yields final results as:

$$H^TH = \begin{bmatrix}\sum_k C'^2\cdot\cos^2 & 2\pi T\sum_k k\cdot C\cdot C'\cdot\cos\cdot\sin \\ 2\pi T\sum_k k\cdot C\cdot C'\cdot\cos\cdot\sin & 4\pi T^2\sum_k k^2\cdot\sin^2\end{bmatrix} \tag{39}$$

$$H^T\delta Z = \begin{bmatrix}-\sum_k C'\cdot\cos\cdot\delta z(k) \\ -2\pi T\sum_k k\cdot C\cdot\sin\cdot\delta z(k)\end{bmatrix}. \tag{40}$$

Since the matrix size of H and δZ are N-by-2 and N-by-1, respectively, those of H^TH and $H^T\delta Z$ becomes 2-by-2 and 2-by-1, respectively.

As in the case before, applying the efficient cost function for code-free signal in Eq. (16) to the NLSE yields the more fast solution algorithm since it does not require the redundant computation of Jaccobian for the code phase τ. Thus, a matrix computation in Eq. (35) becomes a simple scalar division and multiplication process:

$$(H^TH)^{-1} = 1\Big/4\pi T^2\sum_k k^2\cdot\sin^2 \tag{41}$$

$$H^T\delta\bar{Z} = -2\pi T\sum_k k\cdot\sin\cdot\delta\bar{z}(k) \tag{42}$$

where $\delta\bar{z}(k) = \bar{r}(k) - \hat{\bar{r}}(k)$.

Note that the solution of NLSE is more sensitive to the accuracy of the signal amplitude estimate than the MLE. This is because the NLSE uses the difference of received and locally generated signals, i.e., $\delta z(k) = r(k) - \hat{r}(k)$ in the solution algorithm as in

Eq. (40) while the MLE uses the product of received and locally generated signals as in Eqs. (21) to (25).

4.2 Non-iterative Feed-Forward Solution Methods

Non-iterative solution for MLE is obtainable directly from the log-likelihood cost function of the MLE in Eq. (4). As mentioned before, with same assumption on N as in Eq. (6), the maximization or the minimization problem of the cost function becomes same to obtaining θ satisfying [7]

$$\frac{\partial L(\theta \mid r_N)}{\partial \theta} = 0 \quad \text{or} \quad \frac{\partial J(\theta)}{\partial \theta} = 0 \cdot \tag{43}$$

Normally, since the signal amplitude A is easily obtainable by using a sum of square of I and Q as shown in Eq. (9), A is assumed to be a known parameter.

<u>Carrier Phase Tracking Loop</u>

Assuming the parameters of τ and f_d are known, the gradient of log-likelihood cost function for ϕ is given by

$$\left(\frac{\partial L}{\partial \phi}\right)_{\phi=\hat{\phi}} = -2A\sum_{k=0}^{N-1} r(k)C(k-\tau)\sin\left(2\pi T(f_{IF}+f_d)k+\hat{\phi}\right) = 0 \tag{44}$$

The above equation is obtained based on Single-Input-Single-Output (SISO) system assumption, i.e., no coupling between signal parameters, and equals to the Q value in the prompt arm of a normal PLL.

For an example Figure 2 depicts the normalized cost function and gradient function for GPS L1 CA signal with 1 ms coherent integration time with respect to carrier phase error in Eqs. (7) and (44) with assuming no error ($\tilde{\tau}=0$ and $\tilde{f}_d=0$ where $\tilde{\bullet}$ represents the estimate error) for noise free environments. It is easily shown in the figure that the gradient function provides a linearly proportional value to carrier phase error which is similar to discriminator output of PLL.

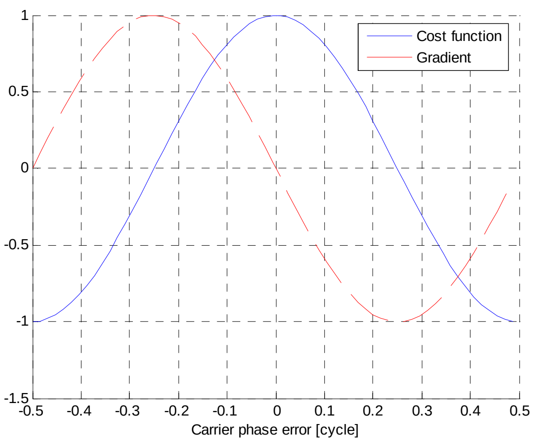

Figure 2 MLE cost function and gradient function for carrier phase error

<u>Doppler Frequency Tracking Loop</u>

Assuming the parameters of τ and ϕ are known, the gradient of log-likelihood cost function for f_d is given by

$$\left(\frac{\partial L}{\partial f_d}\right)_{f_d=\hat{f}_d} = -4\pi TA\sum_{k=0}^{N-1} r(k)C(k-\tau)k\sin\left(2\pi T(f_{IF}+\hat{f}_d)k+\phi\right) = 0 \tag{45}$$

Note that this is similar to the Q in the prompt arm of a normal FLL except for the use of k which is included for avoiding the use of a successive carrier phase measurements to generate the Doppler frequency error in FLL.

For an example Figure 3 depicts the normalized cost function and gradient function of Doppler frequency error in Eqs. (7) and (45) with assuming no error ($\tilde{\tau}=0$ and $\tilde{\phi}=0$) for noise free environments. It is also shown in the figure that the gradient function provides a linearly proportional value to Doppler frequency error which is similar to discriminator output of FLL.

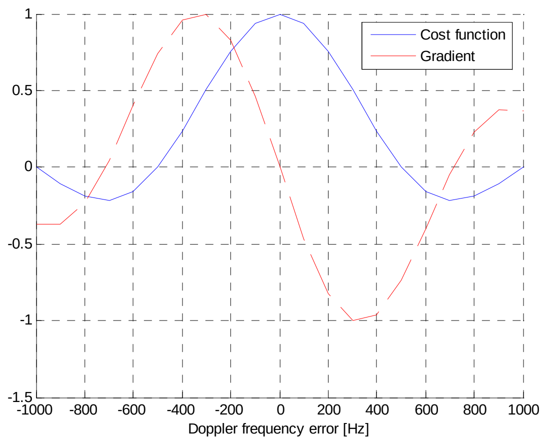

Figure 3 MLE cost function and gradient function of Doppler frequency error

<u>Code Phase Tracking Loop</u>

Assuming the parameters of ϕ and f_d are known, the gradient of log-likelihood cost function for τ is given by

$$\left(\frac{\partial L}{\partial \tau}\right)_{\tau=\hat{\tau}} = -2A\sum_{k=0}^{N-1} r(k)C'(k-\hat{\tau})\cos\left(2\pi T(f_{IF}+f_d)k+\phi\right) = 0 \tag{46}$$

This is equal to the I value in the Early-Minus-Late arm of a normal DLL.

For an example Figure 4 depicts the normalized cost function and gradient function with respect to code phase error in Eqs. (7) and (46) with assuming no error ($\tilde{\phi}=0$ and $\tilde{f}_d=0$) for noise free environments. This is the GPS L1 CA code case. It is also shown in the figure that the gradient function provides a linearly proportional value to code phase error which is similar to discriminator output of DLL. The nonlinearity in the figure comes from discrete sampling of PRN codes.

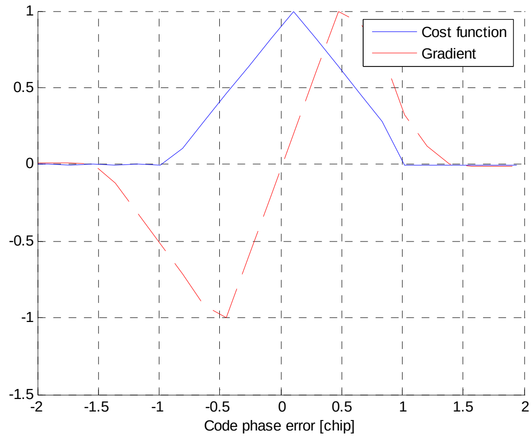

Figure 4 MLE cost function and gradient function for code phase error

Block Diagram Implementations

The block diagram implementation of the non-iterative solution of MLE for the carrier phase, the Doppler frequency and the code phase tracking in Eqs. (44), (45) and (46) is depicted in Figure 5. Note in the figure that the Doppler frequency tracking loop contains one more mixer for k of which integrated and dump value is noted as $\overline{Q}_p$. It seems the non-iterative solution for MLE requires one less integration and dump process than the conventional FLL/PLL/DLL where the I and Q values in prompt arm for carrier tracking and in Early-minus-Late arm for code tracking are used. But, in order to get signal amplitude A and to demodulate navigation data bits, the I value in the prompt arm should be used and the processing load of the MLE is still little bit larger than the conventional FLL/PLL/DLL.

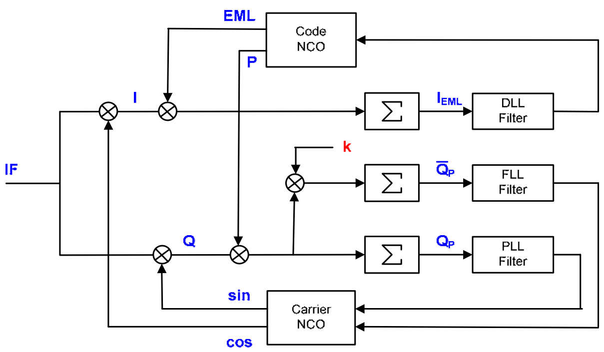

Figure 5 Block diagram of non-iterative solution for MLE: carrier phase, Doppler frequency and code phase tracking

A FLL discriminator output in a conventional FLL is calculated by using a 1st order approximation on the frequency, i.e., a difference between a successive carrier phase estimates divided by the coherent integration time given by

$$f_{d,k} = (\phi_k - \phi_{k-1}) / \Delta t \tag{47}$$

where Δt represents the coherent integration time equal to the sampling time of loop filters in seconds and a successive carrier phases ϕ_k and ϕ_{k+1} are obtained by using arctangent function of the I and Q values at k and k+1, respectively. Therefore, the use of the successive measurements for the Doppler frequency estimation in Eq. (47) may cause an approximation error and one step delay effect on the estimated value while the use of the gradient function in Eq. (45) provides the Doppler frequency error, which is similar to FLL discriminator output, directly to the estimator. This makes it possible to be a more robust tracking for GNSS signals.

5. Loop Filter Implementation

In order to produce an accurate estimate of the signal parameters from the noisy received signal at its output, a classical approach of designing a loop filter, e.g., FLL/PLL/DLL, which has a fixed update rate is implemented. For the robustness of carrier tracking loops, a combination of a 2nd order FLL and a 3rd order PLL is used and their bandwidths are set to accommodate for a given dynamic specification of receivers based on the minimum required signal-to-noise ratio. A carrier aided 1st order DLL with a quite narrow bandwidth is used to track code phase. A filter equation for FLL/PLL/DLL including code and carrier Numerical Controlled Oscillator (NCOs) is given in a state space form as:

$$\begin{bmatrix} \tau_{k+1} \\ \phi_{k+1} \\ f_{d,k+1} \\ \dot{f}_{d,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & r\cdot\Delta t & 0 \\ 0 & 1 & \Delta t & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_k \\ \phi_k \\ f_{d,k} \\ \dot{f}_{d,k} \end{bmatrix} + \begin{bmatrix} \omega_{DLL}\Delta t & 0 & 0 \\ 0 & 2.4\omega_{PLL}\Delta t & 0 \\ 0 & 1.1 r_{PLL}\omega_{PLL}^2\Delta t & \sqrt{2} r_{FLL}\omega_{FLL}\Delta t \\ 0 & r_{PLL}\omega_{PLL}^3\Delta t & r_{FLL}\omega_{FLL}^2\Delta t \end{bmatrix} \begin{bmatrix} e_\tau \\ e_\phi \\ e_{f_d} \end{bmatrix} \tag{48}$$

where $[\tau, \phi, f_d, \dot{f}_d]^T$ is the state vector for the code phase in chip, the carrier phase in cycles, the Doppler frequency in Hz and the rate of the Doppler frequency in Hz/s, respectively, $\omega_{0,PLL}$ and $\omega_{0,FLL}$ are the natural radian frequencies of the 3rd order PLL in rad/s and the 2nd order FLL in rad/s^2, respectively, r_{PLL} and r_{FLL} are the weighting factors of PLL and FLL, respectively ($r_{PLL} + r_{FLL}$ =1), r denotes the scale factor converting Doppler frequency to code chip rate which is related to the carrier aiding to DLL , and $[e_\tau, e_\phi, e_{f_d}]^T$ represents the DLL, PLL and FLL discriminator outputs which are proportional to gradients in Eqs. (46), (44) and (45), respectively .

Discriminator outputs of the fixed rate optimal filter in Eq. (48) are similar to the measurement innovation process in a Kalman filter. Thus, the Eq. (48) can be written as

$$\hat{\theta}_k = \hat{\theta}_k^- + K'e \tag{49}$$

where e denotes a vector form of discriminator outputs and K' represent a 4-by-3 fixed rate optimal filter gain matrix.

If we use a Kalman filter for signal tracking, it is possible to check the resultant design matrix of the Kalman filter by comparing the steady-state Kalman gain K_∞ and the fixed rate optimal filter gain K' in Eq. (49). This is because if the filter is well designed, these two gains should be same ($\overline{K} \approx K'$). That means the use of the state space form of tracking filters with fixed rate and bandwidths makes it possible to use an empirical approach for evaluating the performance of designed tracking loops by comparing the gain matrices of the Kalman filter and the fixed rate optimal filter.

6. Conclusion

In this paper we derived a novel signal tracking algorithm based on MLE approach. The MLE cost function for the GNSS signal parameters were reviewed and expanded to a various form of solution algorithms by a simple assumption on received signals or signal parameters. Two iterative solution methods were derived based on the maximizing (or minimizing) problem on the cost function. The Levenberg-Marquardt method provided an efficient solution algorithm but requires more computational time than conventional tracking methods. Assuming no coupling of signal parameters, i.e., SISO system, the non-iterative feed-forward solution method was derived based on the MLE cost function, where the gradient function of the cost function was used to correct errors similar to the discriminator output of conventional tracking loops. It was also shown that the use of non-iterative feed-forward manner to deal with received IF signal made it possible to save the processing time. The non-iterative feed-forward solution method was integrated with a fixed rate optimal filter to reduce the noise effect on the estimated signal parameters.

The presented methods will be ideally suited for improving the performance of the signal tracking module in severe signal environments such as high dynamics or indoor positioning.

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