

# An analysis of GPS anti-jamming methods in spatial and temporal domain

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## Abstract

The GPS is widely used in various parts, therefore it is required higher integrity and continuity. These integrity and continuity are threatened by outer jamming signals which are intended or not. And various anti-jamming ways have been studied to remove these jamming signals. In this paper, we are going to test the efficiency of the anti-jamming algorithm in space and time-space domain, and analyze

**Keywords:** GPS, anti-jamming, STAP, array antenna.

## 1. Introduction

Nowadays the usage of the GPS is widely extended. It is used to find the position of planes, ships, cars and people. Also in military, it is used to guide missiles or smart bombs. Because of increased usage of the GPS, the system using the GPS is required integrity and continuity. These integrity and continuity are threatened by outer jamming signals which are intended or not. Especially used in military, the integrity is more important. Even though spread spectrum signal of the GPS offers its own anti-jamming powers, the GPS is able to be jammed by strong enough jamming signals because of its low signal power. Generally the jammer is closer to the receiver than a GPS satellite. Therefore the receiver easily lose the satellite signal by the jamming signal.

There are two jamming signals, a narrow-band jamming signal and a wide-band jamming signal. Each of these signals can be remove by time domain filtering and space domain filtering.

In this paper we are going to use the MVDR(Minimum Variance Distortionless Response) algorithm and the OPM(Output Power Minimization) algorithm to remove jamming signals in time and space domain, and analyze the accuracy of code/carrier tracking and the navigation efficiency.

## 2. Anti-jamming Algorithm

### 2.1 Spatial MVDR-Filtering

In spatial domain MVDR Filtering, only steering vector of satellite which is aiming to track the signal is necessary. Notion of MVDR beam-formation period is to minimize of the strength of array antenna's average output with maintaining the reply of the unit value about the direction of the satellite signal. This problem could be described as below

$$\begin{aligned} \hat{\mathbf{w}}_l &= \arg \min E \left[ |y_l(n)|^2 \right] \\ &\text{subject to} \\ \hat{\mathbf{w}}_l \mathbf{a}_l &= 1 \end{aligned} \quad (1)$$

Weight vector from optimization problem will minimize the entire noise by the above equation. So that MVDR beam-formation period will maximized output SINR

$$\mathbf{w}_l = \frac{\mathbf{R}^{-1} \mathbf{a}_l}{\mathbf{a}_l^H \mathbf{R}^{-1} \mathbf{a}_l} \quad (2)$$

$\mathbf{R} = E \{ \mathbf{u}(k) \mathbf{u}^H(k) \}$  is the matrix of covariance of  $N \times N$  input signal,  $\mathbf{a}_l = [\bar{a}_1 \ \bar{a}_2 \ \dots \ \bar{a}_L]$  is steering vector of each satellite.

To get a weight vector by using MVDR beam-formation, angle of arrival is needed against array antenna of satellite. This could get a input via Inertial Navigation System. And also, covariance matrix of the actual input signal is changed by jammer, weight vector via estimation of covariance should be updated continuously. For a optimized weight value, correlation matrix  $\mathbf{R}_{XX}$  which is put in should be sought. But in actual case,

$\mathbf{R}_{XX}$  couldn't be sought in advance. It should be sought via adaptive technique. Regarding problem about estimating the optimized weight equation, direct alternative of correlation matrix need multiplication as much as square of weight element number in each circulation. Complication is happen by inverse matrix of input correlation matrix. Adaptive algorithm requires store space and multiplication number to ratio times of element numbers. In a Constrained Gradient-Descent Optimization, weight value vector is initialized to the limited  $\mathbf{W}_0 = \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1}$  by limit vector and each repetition, weight value vector moves to negative direction of constrained gradient. the movement length is in proportion to the size of constrained gradient and is mediated by a constant  $\mu$ . After repetition of  $k$  times, weight value vector of the  $k+1$  times is as below

$$\begin{aligned} \mathbf{W}_{m+1} &= \mathbf{W}_m - \mu \nabla_{\mathbf{W}} H[\mathbf{W}_m] \\ &= \mathbf{W}_m - \mu [\mathbf{R}_{XX} \mathbf{W}_m + \mathbf{C} \lambda_m] \end{aligned} \quad (3)$$

Lagrange Multipliers is selected by the things requiring  $\mathbf{W}_{m+1}$  which is satisfactory with limitation condition.

$$\begin{aligned}
1 &= C^T W_{m+1} = C^T W_m - \mu C^T R_{XX} W_m - \mu C^T C \lambda_m \\
&= C W_m - \mu C^T R_{XX} W_m - \mu C^T C \left( -[C^T R_{XX} C]^{-1} \right)
\end{aligned} \quad (4)$$

Formation below can be obtainable after getting Lagrange Multipliers  $\lambda_m$  and replace with weight repetition equation.

$$\begin{aligned}
W_{m+1} &= W_m - \mu [I - C(C^T C)^{-1} C^T] R_{XX} W_m \\
&\quad + C(C^T C)^{-1} [1 - C^T W_m]
\end{aligned} \quad (5)$$

$$F = C(C^T C)^{-1} \quad (6)$$

$$P = I - C(C^T C)^{-1} C^T = I - FC^T \quad (7)$$

Algorithm will be written as below

$$W_{m+1} = P[W_m - \mu R_{XX} W_m] + F \quad (8)$$

At the n times repetition, the simple estimation for  $R_{XX}$  is outer product between itself and tab voltage vector. The statistical limited Constrained Least Mean Square is as below.

$$W_0 = F \quad (9)$$

$$W_{m+1} = P[W_m - \mu y_m X_m] + F \quad (10)$$

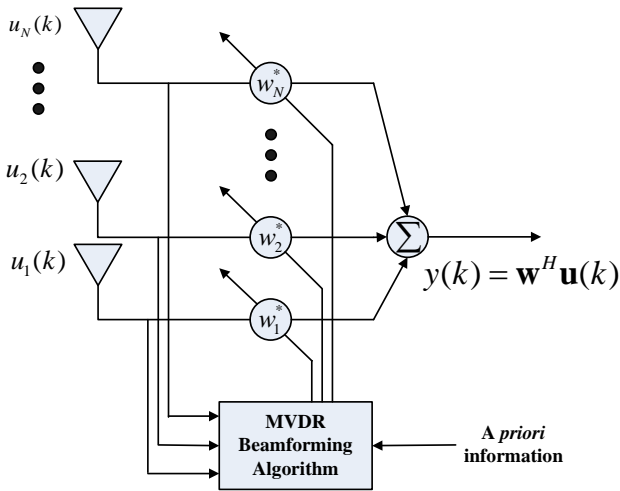


Figure 1. MVDR filter block of spatial domain

Because MVDR filter has different limitation vector in each satellite, every satellite needs its own MVDR filter.

## 2.2 STAP(Spatial-Temporal Adaptive Processing) Algorithm

OPM(Output Power Minimization) algorithm has delay tap in each array antenna. So weight value is multiplied by these delay tap which makes minimized output through adapting LMS algorithm in time and space area. It simply constraints the weight on the first tap of antenna 1(see figure 2), and then minimizes the output power, namely

$$\begin{aligned}
&\min_w w^H R w \\
&s.t. w^H a_{MN} = 1
\end{aligned} \quad (11)$$

Where  $a_{MN} = [1, 0, \dots, 0]^T$  is  $MN \times 1$  vector. Using the method of Lagrange multipliers, the solution to (11) is

$$W_{opt} = \frac{R^{-1} a_{MN}}{a_{MN}^H R^{-1} a_{MN}} \quad (12)$$

OPM filter could be described as below.

$$e(n) = d(n) - y(n) \quad (13)$$

$$y(n) = w_0 x(n) + w_1 x(n-1) + \dots + w_N x(n-N) \quad (14)$$

$$w(n+1) = w(n) + \mu e(n) u(n) \quad (15)$$

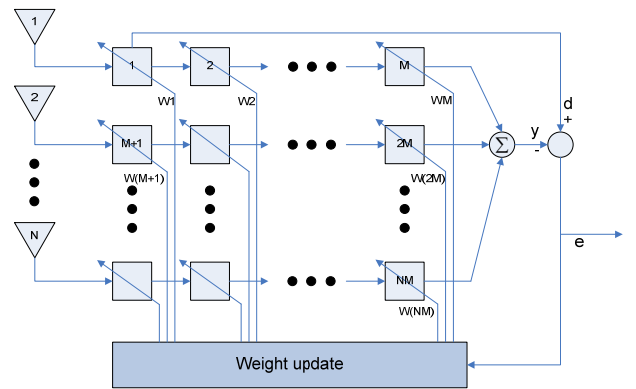


Figure 2. OPM(Output Power minimization) algorithm block

## 3. Simulation environment

To evaluate anti-jamming efficiency of GPS in real environment, a high efficient, array antenna based GPS receiver platform which is able to put on the jammer and anti-jamming algorithm is required. However it is not easy to use this kind of hardware to analyze an efficiency of the anti-jamming algorithm. Therefore we used a simulation program which is based on software to analyze it.

Table 1. Jamming signal type, Azimuth, and Elevation

Jamming type	Azimuth	Elevation	Power
Wide band jamming	50	100	-113dBW
Narrow band jamming	20	60	-113dBW

GPS signal and jamming signal were generated that they were assumed fixed user position. Generated signals were simulated of process of figure 3. 2X2 array antenna was used.

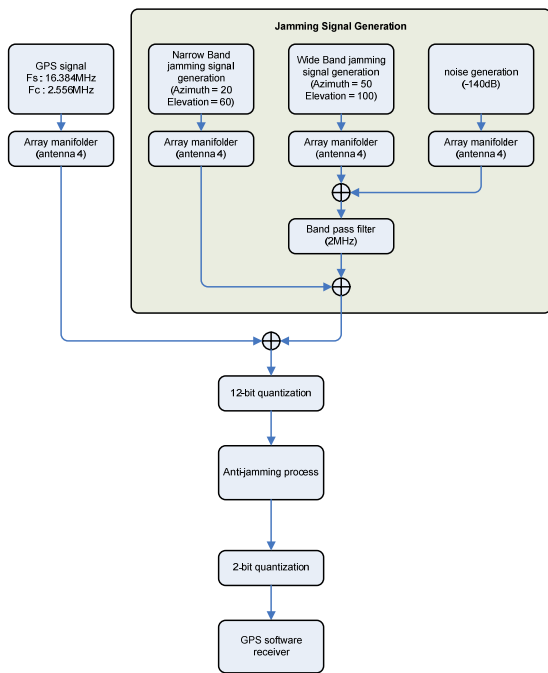


Figure 3. Simulation process block.

## 4. Simulation Result and Analysis

### 4.1 No Filter

Following figures are the results of simulations when the receiver without a filter gets a jamming signal. There is only a GPS signal between 0 and 1 second, and the jamming signal is added after 1 second.

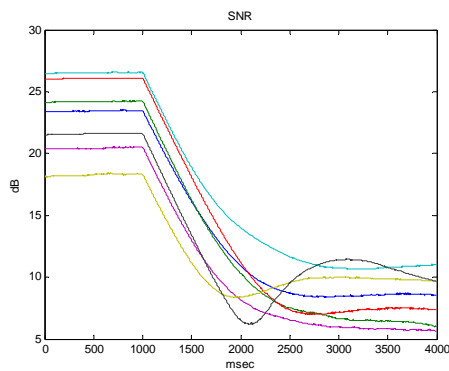


Figure 4. No filter SNR

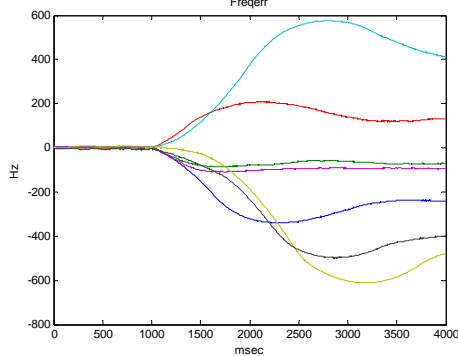


Figure 5. No filter Frequency error

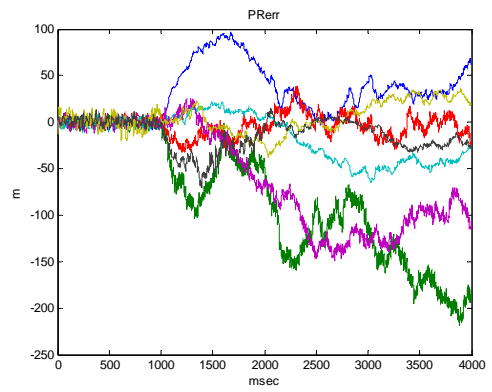


Figure 6. No filter Pseudo range error

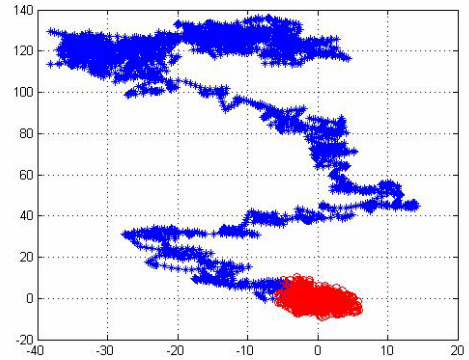


Figure 7. No filter Position error

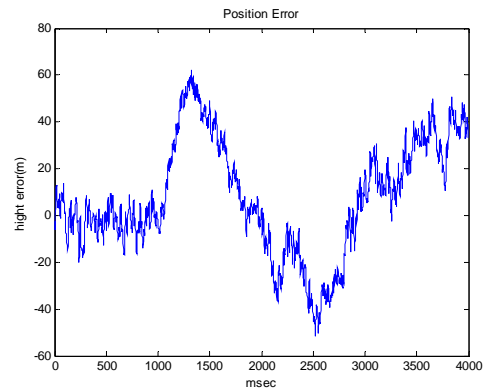


Figure 8. No filter Height error

If the receiver has no filter, pseudo range error and frequency error increase as you can see at Fig.5 and Fig 6. Therefore it is hard to track. At Fig.7 red is a position error when a only GPS signal is received and the blue is a position error when a jamming signal is added.

### 4.2 Using MVDR Filter

MVDR filter needs one filter in each satellite due to different limitation vector in each satellite. So it requires many calculations.

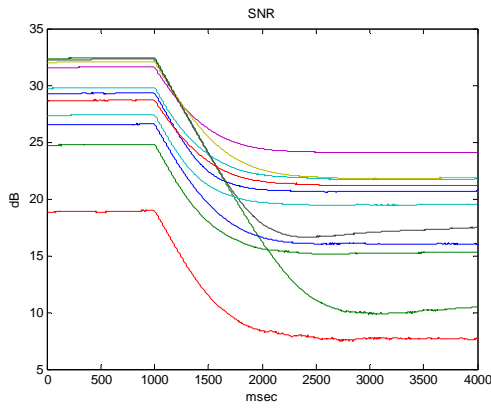


Figure 9. SNR after passing MVDR filter

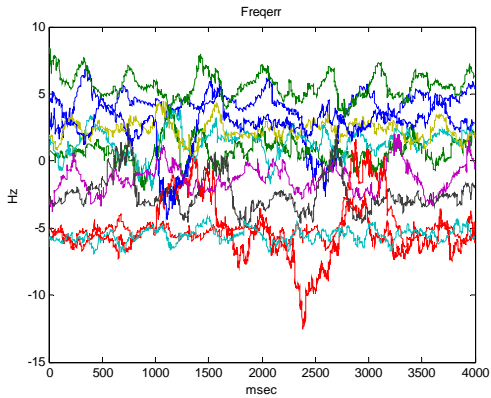


Figure 10. Frequency error after passing MVDR filter

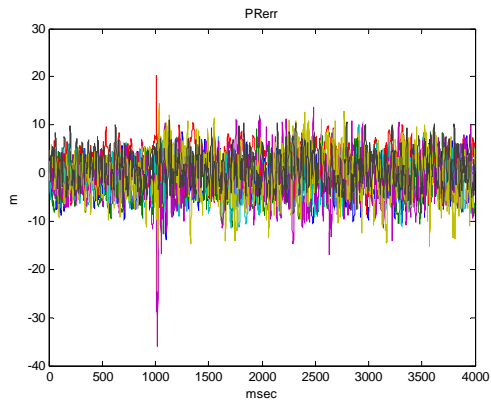


Figure 11. Pseudo range error after passing MVDR filter

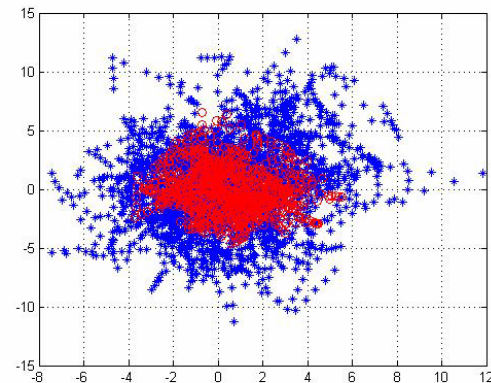


Figure 12. Position error after passing MVDR filter

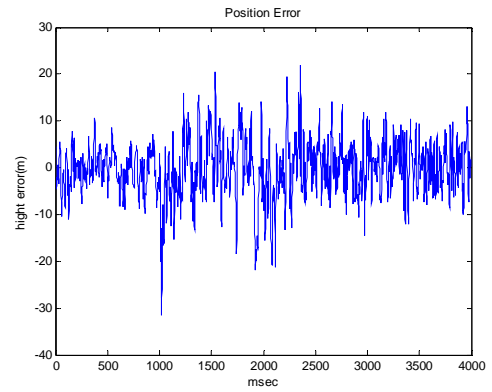


Figure 13. Height error after passing MVDR filter

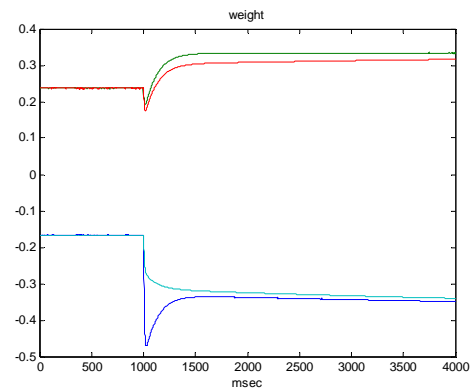


Figure 14. Weight value of MVDR filter

If a MVDR filter is used and the jamming signal is received after 1 second, the error increases at the time the jamming signal is received and then decreases soon. You can see it at Fig. 11 and 13. At Fig. 12 2-DRMS error is 3.1m with an only GPS signal and is 3.7m with an added jamming signal.

### 4.3 Using OPM Filter

The number of delay taps of an OPM filter is 20. The simulation is done in 8 seconds and the GPS signal is received until 1 second. The jamming signal is added after 1 second. The OPM filter needs more convergence time than the MVDR filter because it has no additional information of the input signal.

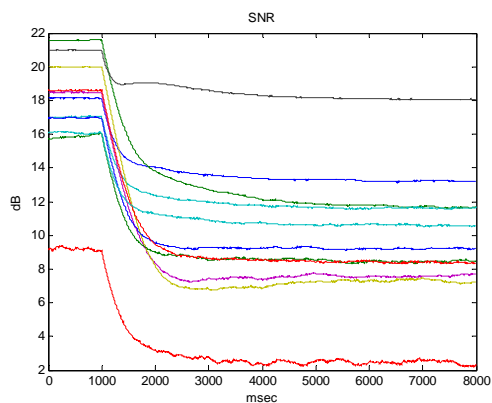


Figure 15. SNR after passing OPM filter

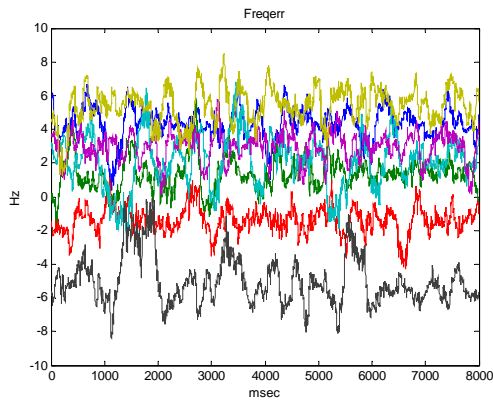


Figure 16. Frequency error after passing OPM filter

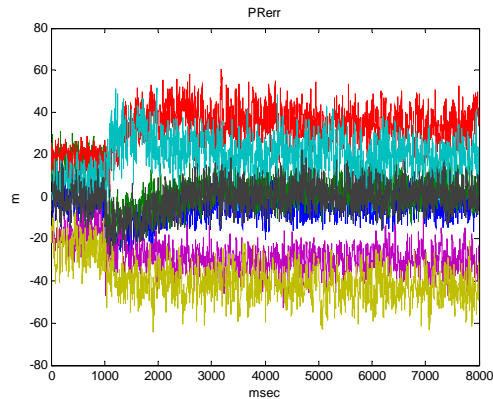


Figure 17. Pseudo range error after passing OPM filter

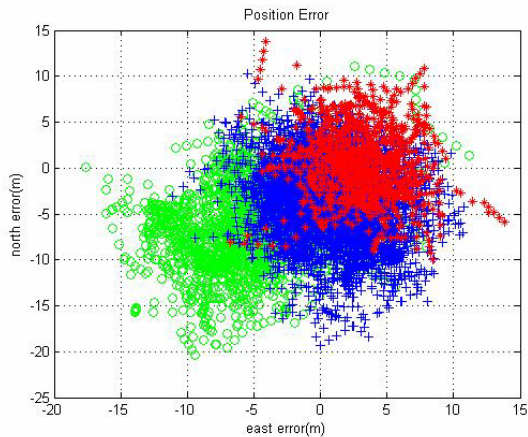


Figure 18. Position error after passing OPM filter

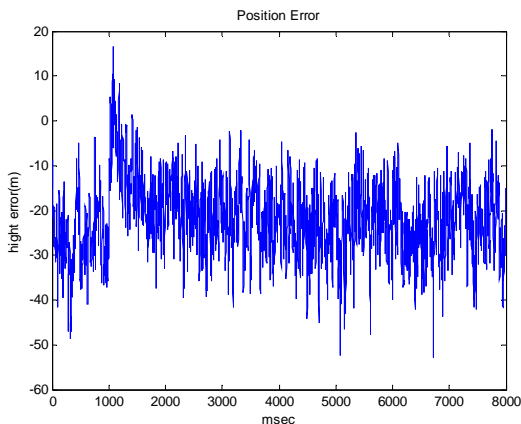


Figure 19. Height error after passing OPM filter

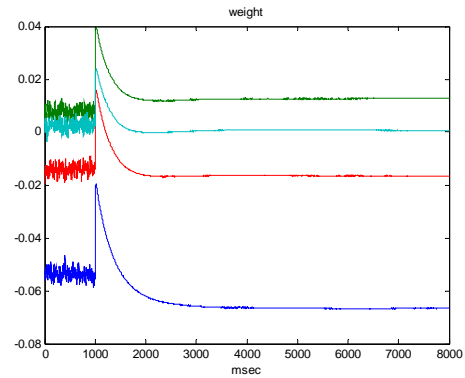


Figure 20. Weight value of OPM filter

OPM filter remove the jammer of space-time domain without the information about input signal which adds to filter. In OPM filter 20 delay tab were used in each antenna. You can see that the error increases at Fig. 17 and 19 when the jamming signal is received at 1 second. The error is decreases soon. At Fig. 17, 19 and 20 the filter needs more than 2 seconds to convergence when the jamming signal is inputted. At Fig 18 the red line shows the position error when the only GPS signal is received. The green line shows the position error during 2 seconds after the jamming signal is added and the blue line shows the position error after 2 seconds. 2-DRMS error of each color is following. The red is 5.2m, the green is 9.3m and the blue is 5.6m.

## 5. Conclusion

For implementing anti-jamming simulation, we produced simulation program replacing hardware and simulated removal of jamming in spatial-temporal domain by using OPM algorithm and MVDR algorithm. MVDR algorithm showed good performance for jamming removal. But it needs external additive information and too many calculations. On the other hand, OPM algorithm has low anti-jamming performance. But without external additive information, it removed jamming of space-time area effectively. More various anti-jamming algorithm study could be possible by using completed simulation program

## Acknowledgement

This research was supported by Agency for Defense Development.

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