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로델 도사노 송화창* 김태균
군산대학교 한전 전력연구원

NIPM-Based Optimal Power Flow Including Discrete Control Variables

Rodel D. Dosano Hwachang Song Tae-Kyun Kim
Kunsan National University KEPRI

Abstract - This paper proposes Nonlinear Interior Point Method (NIPM) including discrete control variables optimal power flow formulations. The algorithm utilizes the robustness in terms of starting point and fast convergence for large scale power system of NIPM and an introduction of rounding penalty function which is augmented in the Lagrangian function to handle discrete control variables. The derived formulation shows a simplified approach to deal with discrete control problems which is implementable in real large scale systems.

key words: nonlinear interior point method (NIPM), discrete control variables, rounding function, OPF

1. Introduction

The success of the evolution of the optimal power flow raised the expectations that they might be implemented for Energy Management System (EMS) real time applications. The effective use of OPF in EMS environment requires that the method selected must be accurate and stable, in that it produce solutions which change in a manner consistent with changes in network parameters [1]. This require that OPF must not be sensitive to (arbitrary or randomly) selected starting point. Another criteria for the real-time applications of OPF is to achieve acceptable performance in terms of execution time, CPU cycles and memory requirements.

Presence of discrete control variables in power system modeling such as variable transformer taps and switchable shunt capacitor and reactor banks increases the OPF computational time. This optimization problems including discrete control variables are very difficult to solve because of the inherit nature of integer in the solution and as well as with implemental difficulties. Inherent discreteness of control variables increases the complexity of the problem. Controls are implemented in several layers of continuous and discrete actions. Most control devices like switchable capacitor and reactor banks and transformer taps ratio have pre-specified discrete values. So no matter how accurate the optimization implementations on continuous value is, without engineering approximation, the application of optimal continuous values in physical control devices are impossible [2]. It is therefore necessary to employ most efficient optimization methods to take full advantage in simplifying the formulations and the implementations of the algorithm.

Due to the discrete nature of the optimization problem, a nonlinear optimization problem with mixed integer result were both the number and values of capacitor are treated as continuously differentiable. A decomposition technique was employed to decompose the problem into a continuous and integer problem. However, such procedures make the algorithm bulky and more complex. Recently, the introduction of new methods based on artificial intelligence (AI) such as Simulated Annealing (SA), Evolutionary Algorithms (EA's) and Artificial Neural Networks (ANN's) were utilized to overcome the difficulties of conventional optimal methods based on continuous values [3]. Furthermore, this advance methods where combined with the conventional method to produced a hybrid algorithm in order to deal with the existence of lots of local minima in the problem and as well as with the uncertainties in problem modeling. These hybrid algorithms produce good results for global optimization but excessive time consumption limits their application in large-scale power system , especially for real-time applications.

This paper deals with the application of nonlinear interior point method (NIPM) to circumvent the extant computational complexity and implementation difficulties power system optimal power flow including discrete control variables. The objective is to overcome the difficulties of NIPM in dealing with discrete control variables while utilizing its effectiveness in terms of fast convergence to handle large-scale power system.

2. Problem Formulation

The optimal power flow problem can be exactly formulated as:

$$\text{minimize} \quad f(x) \qquad (1)$$

subject to $h(x) = 0$

$$\underline{g} \leq g(x) \leq \overline{g} \qquad (2)$$

where $x = [Pg, Q(Qg, Qc), Vi, \theta i, T_B]^T \in R^{(n)}$,

$f(x)$ system loss function

$h(x) \equiv [h_1(x), \ldots, h_m(x)]^T$ equality constraints representing power flow equations

$g(x) = [g_1(x), \ldots, g_r(x)]^T$ inequality constraints representing operational limits and

Qc vector that consist of output of switchable shunt capacitor

Pg active power of generator

Qg vector that consist of reactive power outputs of dispatchable generators

Vi vector that consists of bus voltage magnitudes

θi voltage angles except for slack bus

n number of buses.

By introducing slack variable vectors u, and l, the inequality constraints are transformed to the following equality constraints:

$$g(x) - l - \underline{g} = 0; \quad g(x) + u - \bar{g} = 0 \qquad (l, u) \geq 0 \tag{3}$$

where $x = [Pg, Q(Qg, Qc), Vi, \theta i, T_B]^T \in R^{(n)}$,

$f(x)$ system loss function

$h(x) \equiv [h_1(x), \ldots, h_m(x)]^T$

2.1 NIPM OPF with Continuous Variables

To eliminate the non-negativity constraints of slack variables, logarithmic barrier functions are introduced. Defining a Lagrangian function associated with equality constraints with the incorporated logarithmic barrier

$$L(\cdot) = f(x) - y^T h(x) - z^T (g(x) - l - \underline{g}) - w^T (g(x) - u - \bar{g}) - \mu \sum (\ln l_i + \ln u_i) \tag{4}$$

function as:

Then the following KKT equations for the system are derived as:

$$L'_x = \nabla f(x) - \nabla h(x) y - \nabla g(x)(z + w) = 0 \tag{5}$$
$$L'_y = h(x) = 0 \tag{6}$$
$$L'_z = g(x) - l - \underline{g} = 0 \tag{7}$$
$$L'_w = g(x) + u - \bar{g} = 0 \tag{8}$$
$$L'_l = Z - \mu L^{-1} = 0 \tag{9}$$
$$= ZLe - \mu e = 0$$
$$L'_u = -W - \mu U^{-1} = 0 \tag{10}$$
$$= WUe - \mu e = 0$$

By applying Newton's method to the perturbed KKT equations, the correction equations can be expressed as:

$$(\nabla^2 h(x) + \nabla^2 g(x)(z + w) - \nabla^2 f(x) y)\Delta x + \nabla h(x) \Delta x + \nabla g(x)(\Delta z + \Delta w) = L'_{x0} \tag{11}$$
$$\nabla h(x)^T \Delta x = -L'_{y0} \tag{12}$$
$$\nabla g(x)^T \Delta x - \Delta l = -L'_{z0} \tag{13}$$
$$\nabla g(x)^T \Delta x + \Delta u = -L'_{w0} \tag{14}$$
$$Z \Delta l + L \Delta z = -L'_{l0} \tag{15}$$
$$W \Delta u + U \Delta w = -L'_{u0} \tag{16}$$

where Lxo, Lyo, Lzo, Lwo, Llo and Luo denote the residuals of the perturbed KKT equations;

Using the equations above, the following relationship may be obtained:

Substituting the above equations and reducing the correction equations were the inequality constraints g(x) are eliminated. The size of the correction equations is determined only by that of equality constraints h(x) as:

$$\Delta l = \nabla g(x)^T \Delta x + L'_{z0} \tag{17}$$
$$\Delta u = -(\nabla g(x)^T \Delta x + L'_{w0}) \tag{18}$$
$$\Delta z = -L^{-1} Z \nabla g(x)^T \Delta x - L^{-1}(Z L'_{z0} + L'_{l0}) \tag{19}$$
$$\Delta w = -U^{-1} W \nabla g(x)^T \Delta x + U^{-1}(W L'_{w0} - L'_{u0}) \tag{20}$$

$$\begin{bmatrix} H'(\cdot) & J(x)^T \\ J(x) & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} \psi'(\cdot, \mu) \\ h(x) \end{bmatrix} \tag{21}$$

where

$$H'(\cdot) = H'_h + H'_g$$
$$= \nabla^2 h(x) y + \nabla^2 g(x)(z + w) - \nabla^2 f(x) + \nabla g(x) S' \nabla g(x)^T$$
$$S' = U^{-1} W - L^{-1} Z$$
$$J(x) = \nabla h(x)^T$$
$$\Psi'(\cdot, \mu) = -L_{x0} + \nabla g(x)(U^{-1} L_{u0} - L_{w0}) - L^{-1}(Z L_{z0} + L_{l0})$$
$$- \nabla h(x) y - \nabla f(x) + \nabla g(x)(U^{-1} W L_{w0} - L^{-1} Z L_{z0}) - \mu(U^{-1} - L^{-1}) \ldots)$$

H(·)consists of two terms: H'h is a linear combination of Hessian matrices of h(x), g(x) and f(x); H'g is regarded as the barrier matrix. The latter terms prevent inequality constraint g(x) for violating their two side limits [4].

All controls are treated as continuous variables during the initial process, once the process is almost near the optimal solution; each discrete variable is discretized to its neighborhood discrete setting.

2.2 NIPM OPF including Discrete Control Variables

The Lagrangian function used in the conventional NIPM has fast convergence in large scale applications but suffer from the difficulties in dealing with discrete control variables like switchable shunt capacitor and reactor banks and transformer taps. Discrete control variables with NIPM can be implemented by introducing a rounding penalty function in the objective function.

The algorithm for NIPM mixed-integer problems is to run the optimal power flow with all variables as continuous values and after reaching specific value for duality gap and mismatch, an introduction of the rounding penalty function follows. Update of slack variables related to the discrete control variables within the upper and lower discretized values before the implementation of next stage of process are performed. The rounding penalty function is a positive-curvature quadratic function which is augmented on the Lagrangian handles the presence of discrete control variables in system. The aim is to penalize the continuous approximations of discrete control variables for moving away from its discrete steps and search for optimal discrete solution between the discretized upper and lower values. As shown in Fig. 1, the effect of the rounding function in the objective function have a gradual change in magnitude thereby not forcing process to jump away from the initial point which is near the optimal solution.

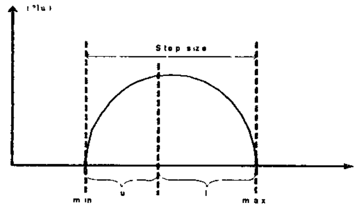

Fig. 1. Rounding penalty function

The new Lagrangian function after augmenting a quadratic penalty functions like the rounding function is:

$$L = L' + \gamma \sum l_i u_i \qquad (21)$$

where li is the lower slack variable which is 0 if it consider non-discrete variable; ui upper slack variables which is 0 if it consider non-discrete variable.

According to KKT stationary condition, the derived partial equations of the new Lagrangian function is:

$$L_x = \nabla f(x) - \nabla h(x)y - \nabla g(x)(z + w) = 0 \qquad (22)$$
$$L_y = h(x) = 0 \qquad (23)$$
$$L_z = g(x) - l - \underline{g} = 0 \qquad (24)$$
$$L_w = g(x) + u - \bar{g} = 0 \qquad (25)$$
$$L_l = Z - \mu L^{-1} + \gamma \mathbf{U} = 0$$
$$- ZLe - \mu e + \gamma \mathbf{U} L = 0 \qquad (26)$$
$$L_u = - W - \mu U^{-1} + \gamma \mathbf{L} = 0$$
$$WUe - \mu e + \gamma \mathbf{L} U = 0 \qquad (27)$$

where U and L are diagonal matrices which have elements consisting upper and lower slack discrete variables and zero if continuous variables.

By applying Newton's method to the perturbed KKT equations, the correction equations can be expressed as:

Additional simplification could further be done by making use of the relationships:

$$U\Delta \mathbf{l} = \mathbf{U}\Delta L \; ; \; L\Delta \mathbf{u} = \mathbf{L}\Delta u$$

Such that

$$Z\Delta l + L\Delta z + \gamma \mathbf{L}\Delta u + \gamma \mathbf{U}\Delta L = -L_{lo}$$
$$W\Delta u + U\Delta w - \gamma \mathbf{L}\Delta u - \gamma \mathbf{U}\Delta L = -L_{uo}$$

$$(\nabla^2 h(x) + \nabla^2 g(x)(z + w) - \nabla^2 f(x)y)\Delta x + \nabla h(x)\Delta x + \nabla g(x)(\Delta z + \Delta w) = L_x \qquad (28)$$
$$\nabla h(x)^T \Delta x = - L_y \qquad (29)$$
$$\nabla g(x)^T \Delta x - \Delta l = - L_z \qquad (30)$$
$$\nabla g(x)^T \Delta x + \Delta u = - L_w \qquad (31)$$
$$Z\Delta l + L\Delta z + \gamma L\Delta \mathbf{u} + \gamma \mathbf{U}\Delta l = - L_l \qquad (32)$$
$$W\Delta u + U\Delta w - \gamma \mathbf{L}\Delta u - \gamma U\Delta l = - L_u \qquad (33)$$

Using the equations above, the following relationship can be obtained as:

$$(\nabla^2 h(x) + \nabla^2 g(x)(z + w) - \nabla^2 f(x)y)\Delta x + \nabla h(x)\Delta x + \nabla g(x)(\Delta z + \Delta w) = L_x \qquad (28)$$
$$\nabla h(x)^T \Delta x = - L_y \qquad (29)$$
$$\nabla g(x)^T \Delta x - \Delta l = - L_z \qquad (30)$$
$$\nabla g(x)^T \Delta x + \Delta u = - L_w \qquad (31)$$
$$Z\Delta l + L\Delta z + \gamma L\Delta \mathbf{u} + \gamma \mathbf{U}\Delta l = - L_l \qquad (32)$$
$$W\Delta u + U\Delta w - \gamma \mathbf{L}\Delta u - \gamma U\Delta l = - L_u \qquad (33)$$

Substituting the above equations and reducing the correction equations where the inequality constraints g(x) are eliminated. The size of the correction equations is determined only by that of equality constraints h(x) as:

$$\Delta l = \nabla g(x)^T \Delta x + L_z \qquad (34)$$
$$\Delta u = -(\nabla g(x)^T \Delta x + L_w) \qquad (35)$$
$$\Delta z = - (L^{-1} Z + \gamma L^{-1} \mathbf{U} - \gamma L^{-1} \mathbf{L}) \nabla g(x)^T \Delta x - L^{-1} L_{lo} + L_z (L^{-1} Z + \gamma L^{-1} \mathbf{U}) + L_w \gamma L^{-1} \mathbf{L} \qquad (36)$$
$$\Delta w = -(U^{-1} W - \gamma U^{-1} \mathbf{L} + \gamma U^{-1} \mathbf{U}) \nabla g(x)^T \Delta x - U^{-1} L_{uo} + L_w (U^{-1} W - \gamma U^{-1} \mathbf{L}) + L_z \gamma U^{-1} \mathbf{U} \qquad (37)$$

$$\begin{bmatrix} H(\cdot) & J(x)^T \\ J(x) & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} \psi(\cdot, \mu) \\ h(x) \end{bmatrix} \qquad (38)$$

where

$$H(\cdot) \equiv H_s + H_p = \nabla^2 h(x) y + \nabla^2 g(x)(z + w) - \nabla^2 f(x) + \nabla g(x) S \nabla g(x)^T$$
$$S = (U^{-1} W - \gamma U^{-1} \mathbf{L} + \gamma U^{-1} \mathbf{U}) - (L^{-1} Z + \gamma L^{-1} \mathbf{U} - \gamma L^{-1} \mathbf{L})$$
$$J(x) \equiv \nabla h(x)^T$$
$$\psi(\cdot, \mu) \equiv - L_x + \nabla g(x)(U^{-1}(W L_w \; L_{uo}) \; L^{-1}(Z L_z + L_l) - \gamma L_w (U^{-1} \mathbf{L} + L^{-1} \mathbf{L}) + \gamma L_z (L^{-1} \mathbf{U} - U^{-1} \mathbf{U}))$$
$$= h(x)y - \nabla f(x) + \nabla g(x)(U^{-1} W L_w \; L^{-1} Z L_z - \mu(U^{-1} - L^{-1}) \; (Z + W) \ldots)$$

By comparing this equations with that of the nonlinear interior point without augmented penalty function, it shows slight modifications on the Hessian matrix and right hand side matrices including terms for discrete controls variables which could be embedded in the program implementations.

3. Conclusions

This paper proposed a nonlinear interior point method (NIPM) optimal power flow including discrete control variables using a rounding penalty function to handle discrete controls. This augmented a positive-curvature penalty function causes only slight modifications of the conventional NIPM OPF thereby the difficulties regarding the robustness in the starting point of the method and computational time limitations can still be achieved. The formulation shows practicality of approach for large scale power system implementations.

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