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Fuzzy Kalman filtering for a nonlinear system

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요 약

In this paper, we propose a fuzzy Kalman filtering to deal with a estimation error covariance. The T-S fuzzy model structure is further rearranged to give a set of linear model using standard Kalman filter theory. And then, to minimize the estimation error covariance, which is inferred using the fuzzy system. It can be used to find the exact Kalman gain. We utilize the genetic algorithm for optimizing fuzzy system. The proposed state estimator is demonstrated on a truck-trailer.

Key Words : Kalman filter, state estimation, T-S fuzzy model, error covariance, genetic algorithm

1. Introduction

The design of a Kalman filter relies on having an exact dynamic model of the system under consideration in order to provide optimal performance when the design contains relatively small modeling errors [1]. However, most dynamical systems in the world have severe nonlinear dynamics. It is a difficult work to design an efficient filter for nonlinear systems. Conventionally, known as the extended Kalman filter (EKF), has been proposed for state estimation by linearization of the nonlinear systems around the present estimate through application of linear filter[2]. But, the statistical properties of external disturbances and measurement noises are rarely known.

In order to design of nonlinear, few works have studied the estimation problem for nonlinear systems. In the last decade, there has been a rapidly growing interest in fuzzy control and fuzzy estimation of nonlinear systems[3,4]. Some of these published on fuzzy observer design. However these papers usually deal with the noise-free case. This fuzzy observers was designed for systems that was not affected by noise and they require a common solution to a set of Ricatti equations, which may be difficult or impossible to obtain [5].

This paper is concerned with the design of fuzzy Klman filter for the nonlinear system that is represented by T-S fuzzy model structure. Which is further rearranged to give a set of linear systems. First, we represent the fuzzy system as a local linear state space systems. Second, to find the exact Kalman gain, we design that a state estimator error covariance is represented by T-S fuzzy model. Third, we construct a global state estimator by combining the local state estimators. That is, this model is designed for a local linear state space model using standard Kalman filter theory. Finally, the proposed state estimator is demonstrate on a truck-trailer.

2. Problem formulation

Nonlinear system can be approximated as locally linear systems or represented by the T-S fuzzy model, which is composed of a set of fuzzy inference rules. The ith rule of the fuzzy linear model for nonlinear systems if of the form

$$\begin{aligned} &IF \quad z_1(t)\ is\ F_{i1}\ \text{and} \ \dots\ \text{and}\ z_n(t)\ is\ F_{in} \\ &THEN\, x(t+1) = A_i x(t) + B_i w(t) \\ &\quad y(t) = C_i x(t) + v(t), \qquad i = 1,2,3,\dots,L \end{aligned} \tag{1}$$

where F_{ij} is the fuzzy set, $y(t)$ is the measured output, A, B, and C are known constant matrices, the process noise $w(t)$ is white with PSD S_w, the measurement noise $v(t)$ is white with PSD S_v and the process noise and measurement noise are uncorrelated. Now we define L discrete time signals $x(t)$ and $y(t)$. The final output of the fuzzy system is inferred as follows:

$$\begin{aligned} x(t+1) &= \sum_{i=1}^{L}\mu_i(z(t))\left[A_i x(t) + B_i w(t)\right] \\ y(t) &= \sum_{i=1}^{L}\mu_i(z(t))\left[C_i x(t) + v(t)\right] \end{aligned} \tag{2}$$

where

$$w_i(z(t)) = \prod_{j=1}^{n} F_{ij}(z_j(t))$$

$F_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in F_{ij}, and

$$\mu_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{L} w_i(z(t))}$$

with $w_i(z(t)) \geq 0$ for all t, we get the following form:

$$\sum_{i=1}^{L}\mu_i(z(t)) = 1 \tag{3}$$

From these definitions, we define L discrete time signals and it can be defined as

$$x_i(t) = \mu_i(z(t))x(t), \qquad x(t) = \sum_{i=1}^{L} x_i(t) \tag{4}$$

We can derive from (2)

$$\begin{aligned} x(t+1) &= A(t)x(t) + \mu(t)B(t)w(t), \\ y(t) &= C(t)x(t) + \mu(t)v(t) \end{aligned} \tag{5}$$

where

$$\begin{aligned} A(t) &= \sum_{i=1}^{L}\mu_i(z(t))A_i, \qquad B(t) = \sum_{i=1}^{L}\mu_i(z(t))B_i, \\ C(t) &= \sum_{i=1}^{L}\mu_i(z(t))C_i \end{aligned} \tag{6}$$

3. Fuzzy Kalman filtering

We represent the nonlinear system into linear system by using fuzzy model. We combine the Kalman filter for the local systems given in (5) to obtain a state estimator for the T-S fuzzy model given in (1). The state $\hat{x}$ of the system can be derived by assuming a recursive estimator of the form. The predicted state is represented as

$$\hat{x}^{+}(t) = (I - K(k)C(t))\hat{x}^{-}(t) + K(t)y(t) \tag{7}$$

$$\hat{x}^{-}(t+1) = A\hat{x}^{+} + \mu(z(t))Bw(t) \tag{8}$$

where I is the identity matrix, "-"superscript is to indicate a quantity before the measurement is taken into account, and "+" superscript to indicate a quantity after the measurement is taken into account. Requiring the state estimate to be unbiased results in the constraint [ ]. We define the estimation error and its covariance P

$$e(t) := x(t) - \hat{x}(t) \tag{9}$$

$$P(t) := \varepsilon\left[e(t)e(t)^T\right] \tag{10}$$

where ε is the expected value operator.

However, we need to estimate exactly, the estimation error is inferred by a double-input single-output fuzzy system, for which the jth fuzzy IF-THEN rule is represented by

$$\begin{aligned} &IF \quad x_1\ is\ A_{1i}\ \text{and}\ x_2\ is\ A_{2i}, \\ &THEN\ y_j\ is\ \tilde{e}_j \end{aligned} \tag{11}$$

where two premise variables x_1 and x_2 are the measurement residual $e(k)$ and change rate $\dot{e}(k)$, respectively, consequent variable y_j is the estimation error, and A_{ij} are fuzzy set. It has the Gaussian membership function with center $\overline{x_{ij}}$ and standard deviation $\overline{\sigma_{ij}}$

$$f(x_i;\overline{\sigma_{ij}};\overline{x_{ij}}) = \exp\left[-\frac{1}{2}\left(\frac{x_i - \overline{x_{ij}}}{\sigma_{ij}}\right)^2\right] \tag{12}$$

In this paper, the GA methods will be

applied to optimize the parameters and the structure of the system, using the product-sum inference method, singleton fuzzifier, center average defuzzifier, and Gaussian membership function. That is, the defuzzified output of the fuzzy model based on the overall process noise with unknown uncertainty is given by

$$\overline{w_k} = \frac{\sum_{j=1}^{L} w_j A(x_{1j}) \times A(x_{2j})}{\sum_{j=1}^{L} A(x_{1j}) \times A(x_{2j})}$$

According to the approximation theorem by the GA, the overall process noise is optimized. Then the estimation error covariance in (10) can be expressed as follows

$$P := \left[\tilde{e}\tilde{e}^T\right] \tag{13}$$

Due to the estimated term $\tilde{e}$, the covariance matrix of $P^-(t+1)$ becomes

$$P^-(t+1) = A_i\left(P_i^-(t) - K_i(t) C_i P_i^-(t)\right) A_i^T + B_i \tilde{e}_i B_i^T \tag{14}$$

When the estimation error is employed, the conventional Kalman filter has to be modified. We can find the optimal Kalman gain by using (14).

$$K_i(t) = P_i^-(t) C_i^T \left(C_i P_i^-(t) C_i^T + S_v\right)^{-1} \tag{16}$$

The steady state Kalman filter presented can be used to estimate the states of each of the L dynamic systems. We can derive a global filter, which is a linear combination of the local steady state. And, to estimate exactly, we correct the estimation error covariance by using the fuzzy system.

4. Simulation results

In this section we consider state estimation for a discrete time model of a truck-trailer system. A noise-free representation of a truck-trailer system can be described as [4]

$$\alpha(t+1) = \alpha(t) + \frac{VT}{l} tan(u(t)),$$

$$\beta(t+1) = \beta(t) + \frac{VT}{L} sin(\alpha(t)),$$

$$N(t+1) = N(t) + VTcos(\alpha(t)) \sin\left(\frac{\beta(t+1)+\beta(t)}{2}\right)$$

$$E(t+1) = E(t) + VTcos(\alpha(t)) \cos\left(\frac{\beta(t+1)+\beta(t)}{2}\right)$$

where α is the angle of the truck, β is the angle of the trailer, N is northerly position of the rear of the trailer, and E is the easterly position of the rear of the trailer, l is the length of the truck, L is the length of the trailer, T is the sampling time, V is the constant speed of backward movement of the truck, and u is the controlled steering angle(measured counterclockwise with respect to the truck orientation). The following noisy fuzzy model, adapted from [6], can be used to represent the above system:

$IF\ z(t)\ is\ F_1$
$THEN\ x(t+1) = A_1 x(k) + B_1 w(k)$
$y(k) = C_1 x(k) + v(k)$
$IF\ z(t)\ is\ F_2$
$THEN\ x(t+1) = (A_2 x(t) + B_2 w(t))$
$y(t) = C_2 x(t) + v(t)$

The premise variable $z(k)$ is given as

$$z(t) = \beta(t) + \frac{\alpha(t)\,VT}{2/L}$$

The membership function are defined as $F_1 = \{0\}$ and $F_2 = \{\pm\pi\}$ and we use following system parameters:

$$A_1 = \begin{bmatrix} 1 - VT/L & 0 & 0 \\ VT/L & 1 & 0 \\ (VT)^2/(2/L) & VT & 1 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 1 - VT/L & 0 & 0 \\ VT/L & 1 & 0 \\ (VT)^2/(2/L)(\pi/100) & V/(\pi/100) & 1 \end{bmatrix},$$

$B_1 = B_2 = I_{3\times3}$, $C_1 = C_2 = I_{3\times3}$,
$l = 2.8m$, $L = 5.5m$, $V = -1m/s$ $T = 0.5s$

We will use the following matrices for the measurement noise covariance $S_v = 0.2^2$

<Table1>The initial parameters of the GA

Parameters	Values
Maximum Generation	200
Maximum Rule Number	50
Population Size	500
Crossover Rate	0.9
Mutation Rate	0.01
λ	0.8

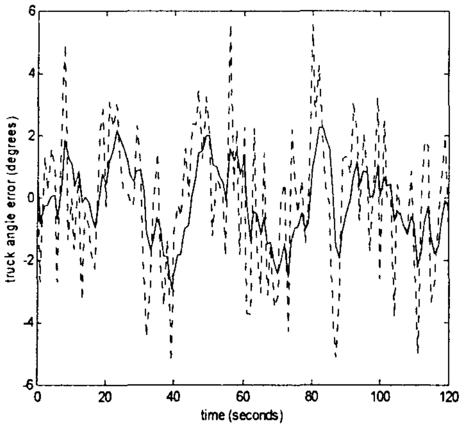

Fig. 1 **Truck angle error(degrees)**

Figure 1 shows that the simulation results of the proposed method. The dotted lines are measurement errors and the solid lines are estimation errors. And trailer position for a typical simulation with the initial conditions $\alpha[0]=-45°$, $\beta[0]=-45°$, and $N[0]=-5m$.

4. Conclusions

The nonlinear systems via the TS fuzzy system has been presented. The steady state was represented by the TS fuzzy model structure, which was further rearranged to give a set of linear model using standard Kalman filter theory. Then, to find the exact Kalman gain, the estimation error covariance was inferred by using fuzzy system. To optimize the employed fuzzy system, the genetic algorithm was utilized. The proposed state estimator was demonstrate on a truck-trailer.

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