EFFECT OF ASPECT RATIO ON SLIP FLOW IN RECTANGULAR MICROCHANNELS

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Abstract

Three dimensional numerical studies were carried out to investigate the effect of aspect ratio on gas slip flow in rectangular microchannels. We focused on aspect ratio effect on slip velocity, pressure distribution and mass flow rate. As aspect ratio decreases the wall slip velocity also decreases. As a result nonlinearity of pressure distribution increases. The slip velocities on sides and top/bottom walls are different and this difference decreases with increasing aspect ratio. These two velocities are equal when aspect ratio is 1. The ratios of slip mass flow rate over noslip mass flow rate increases with increasing aspect ratios.

INTRODUCTION

Recently micron-sized mechanical devices are becoming popular. The compactness and high surface-to-volume ratio make them alternative to conventional flow systems. The development of micromachining technology enables fabrication of micro-fluidic devices such as micro valves, micro-pumps, micro-nozzles, micro sensors, micro-heat exchangers. Microchannels and chambers are the essential part of such devices. It is already established that the flows in micro-devices behave differently from that of macro counterparts. Thus in order to design and fabricate micro-devices properly, insight characteristics of fluid flow and heat transfer in micro-geometries must be understood.

Gas flow in microchannels exhibits two different phenomena such as compressibility and rarefaction effects. Compressibility is due to large pressure difference in the microchannel. Pong et al.[6] investigated the pressure distribution along the microchannel and reported nonlinear pressure distribution which might be the cause of compressibility effect.

Nomenclature

a	speed of sound, m/s	**Greek symbols**	
f	Darcy friction factor	μ	dynamic viscosity (Ns/m^2)
H	Channel height	ρ	density, kg/m^3
W	Width of the channel	τ	shear stress
L	Length of the channel	σ	Lennard-Jones characteristic length
Kn	Knudsen number	σ_T	energy accommodation coefficient
R	gas constant	σ_m	momentum accommodation coefficient
u, v	velocity components, m/s	λ	mean free path
$\bar{u}$	crosswise average velocity	μm	micrometer
Re	Reynolds number	**Subscript**	
n	normal coordinate on the channel wall	i	channel inlet
x, y	dimensional co-ordinates, m	o	channel outlet
AR	aspect ratio		

The deviation of the state of gas from continuum is measured by the Knudsen number which is defined as $Kn = \frac{\lambda}{l}$, where λ is the mean free path of the molecules and can be calculated using $\lambda = \frac{k_B T}{\sqrt{2}\pi\sigma^2 p}$. l is the characteristic length of the channel. A classification of different flow regime is given by Schaaff et. al. 1961, as follows:

i) For $Kn \leq 0.01$, the fluid can be considered as continuum.

ii) For $Kn \geq 10$, it is considered as free molecular flow.

A rarefied gas with Knudsen number between 0.01 and 10 can neither be considered as absolute continuum flow nor free molecular flow. In that region the flow is further classified into slip flow for $0.01 < Kn < 0.1$ and transition flow for $0.1 < Kn < 10$.

Wu and Little et. al.[7] performed experiments for both laminar and turbulent gas flow in microchannels. They used different trapezoidal cross sections with hydraulic diameters equal to 55.81, 55.92 and 72.38 μm. They observed that the measured friction factors were (10-30%) higher than those predicted by the conventional theory. Arkilic et al.[2] studied the flow of Helium through a rectangular silicon microchannel. They observed lower friction coefficient compared with the conventional result for long channels.

G. L Morini and M. Spiga et al.[3] performed analytical solution for slip flow in a rectangular microtube. They reported that the entry length increases with both the aspect ratio and the Knudsen number. C. Aubert and S. Colin et. al.[1] performed analytical solution for second order slip flow in rectangular microducts and observed underestimated mass flow rate when second order terms were not taken into account. For square cross section, the second order terms become more significant.

Though a lot of investigations have been performed there are discrepancies among the reported results. Also the effect of aspect ratio on wall slip has not been investigated rigorously. The objective of our investigation is to observe the influence of aspect ratio and Knudsen number on wall slip velocity, mass flow rate and pressure.

MODEL DEVELOPMENT

Problem statement

We considered Nitrogen gas flow through three dimensional straight rectangular microchannels in Cartesian co-ordinate system. Figure.1 shows the geometry and co-ordinates of the microchannel. The flow domain was bounded by $0 \le x \le L$, $-H/2 \le y \le H/2$ and $-W/2 \le z \le W/2$. We considered various aspect ratios (H/W, $H \le W$). The inlet gas and wall temperature was 310°K. Isothermal wall condition was considered.

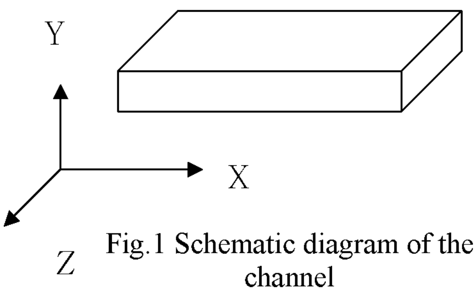

Fig.1 Schematic diagram of the channel

Governing equations

The three basic laws of conservation of mass, momentum and energy have been adapted for use in fluid motion. They are as follows:

$$\frac{\partial \rho}{\partial t} + div \rho \vec{v} = 0 \; (1), \quad \rho \frac{D\vec{v}}{Dt} = \nabla . \tau_{ij}^{'} - \nabla p \; (2), \quad \rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + div(k \nabla T) + \tau_{ij}^{'} \frac{\partial u_i}{\partial x_j} \; (3)$$

where, h is the enthalpy and $\tau_{ij}^{'} \frac{\partial u_i}{\partial x_j}$ is the viscous dissipation function. We solved the above equations considering the flow steady, compressible with ideal gas law $p = \rho RT$. We used Stoke's hypothesis $\lambda + \frac{2}{3}\mu = 0$, in momentum and energy equations to relate first and second kind of viscosity. We considered $\mu = 1.78e-05$ N.s/m^2.

The flow variables are normalized. Velocities (u, v) were normalized by mean outlet velocity $\bar{u}_0$, the stream wise co-ordinate x by the channel length L, the wall normal coordinate by the channel height H.

BOUNDARY CONDITIONS

Slip wall boundary conditions

We assumed the flow steady state and isothermal. The slip velocity boundary condition and the temperature jump boundary conditions for an isothermal flow are given as:

$$u_w - u_g = \frac{2-\sigma_m}{\sigma_m} Kn \frac{\partial u_s}{\partial n} \quad (4) \qquad T_w - T_g = 2\left(\frac{2-\sigma_T}{\sigma_T}\right) Kn \frac{\partial T}{\partial n} \quad (5)$$

where $\left(\frac{\partial u_s}{\partial n}\right)$ and $\left(\frac{\partial T}{\partial n}\right)$ shows the variation of tangential velocity and temperature normal to the wall. Here w and g represent wall and gas respectively. The slip conditions expressed in (4) and (5) are in normalized form. The coefficients σ_m and σ_T are the tangential momentum and energy accommodation coefficients respectively. In the present study we considered σ_m =1 and σ_T =1.

Numerical method

The governing equations were solved by finite volume method. Continuity and momentum equations were solved by the second order upwind implicit scheme. SIMPLEC algorithm was used for pressure-velocity coupling. The energy equation was also solved by second order upwind scheme. The domain was divided into small volume of hexahedral meshes for calculation.

Grid independency test

To evaluate the grid size effect, grid independency tests were carried out. Three different sizes of grid were tested for a typical channel. The difference of velocities with mesh number 17×33×1501 and 21×41×1801 is 2.28% and with mesh number

21×41×1801 and 25×49×2501 is 0.3%. For convenience we used mesh number 21×41×1801 or its multiple according to the length of the dimensions of the domain.

RESULTS AND DISCUSSION

To validate our simulation we first simulated compressible Nitrogen flow in a square conventional channel with hydraulic diameter 1cm. We used the pressure ratio (inlet/outlet) 1.0001. The outlet was atmospheric pressure condition. The wall and gas temperature was fixed to $310^{o}K$.

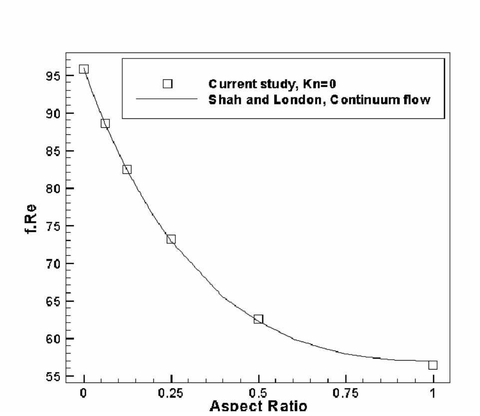

Fig.2 Comparison of friction coefficient with Shah and London et. al. [8]

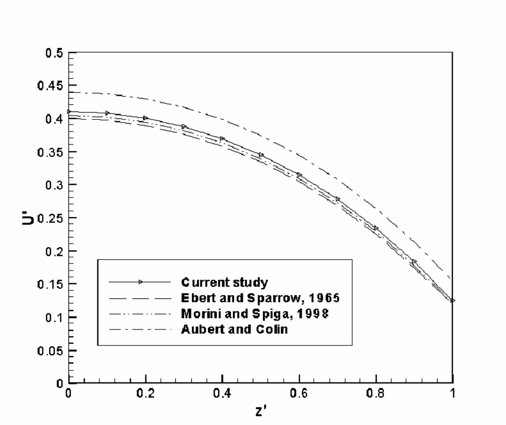

Fig.3. Comparison of normalized velocity, AR=1, Kno=0.1

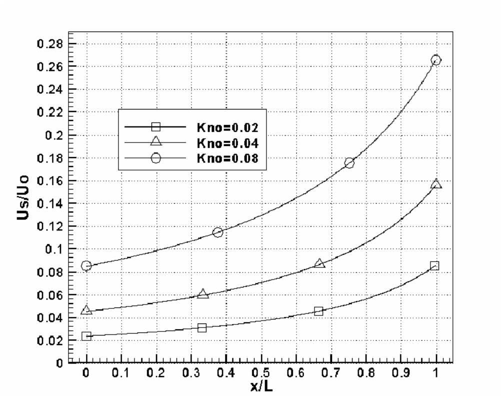

Fig.4 Wall slip velocity for different Knudsen numbers, AR=1.

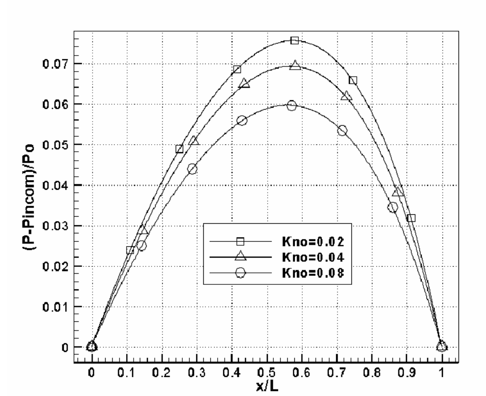

Fig.5 Nonlinearity of pressure distribution for different Knudsen numbers, AR=1.0, PR=2.0.

As the pressure ratio was small the flow was very near to incompressible flow. We computed friction coefficient (fRe). In our simulation it was 56.80 which were very near to theoretical value of 56.91.

We compared noslip (Kn=0) friction coefficients of current study with the noslip friction coefficients reported in Shah and London et. al.[8] for various aspect ratios in Fig.2. The figure shows that the friction coefficient decreases with the increase of aspect ratios. Fig.3 shows the comparison of cross sectional velocity distribution with the velocity distribution reported in different literatures. In all the comparisons the agreement is good. The slight discrepancy with the velocity distribution reported in this figure by Aubert and Colin is due to the use of Deissler second order slip boundary conditions. This model over predicts velocity.

The decrease of nonlinearity resulted from the increase of velocity as well as increase of Knudsen number. The slip velocity on the wall depends on Knudsen number. As the Knudsen number increases the slip velocity also increases. This is presented by the Fig.4. The figure shows the higher velocity for higher Knudsen number. As the Knudsen number decreases the velocity also decrease. The slip axial

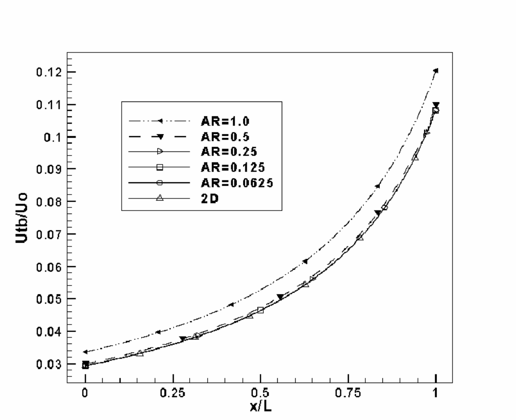

Fig.6 Normalized axial velocity distribution for different aspect ratio, Kn0=0.03

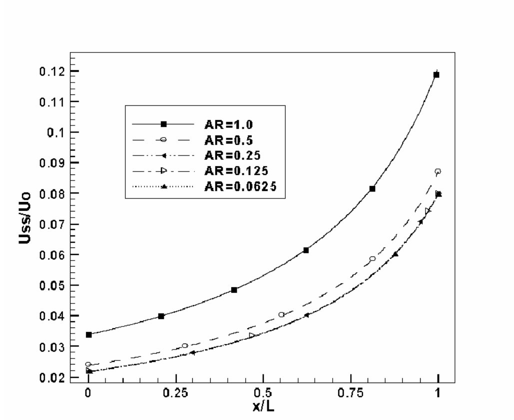

Fig.7 Normalized axial velocity distribution for different aspect ratios, Kn=0.03

Velocity along the lines z=W/2 on the top and bottom walls and along the lines y=H/2 on the side walls are denoted by Utb and Uss respectively. For the microchannel with aspect ratio 1.0, the slip velocities on the top, bottom and sidewalls become the same. We denoted axial velocity by Us. Uo is the center line outlet velocity.

As the Knudsen number increases the nonlinearity decreases. Figure 5 shows the highest nonlinearity for the case with outlet Knudsen number 0.02 and lowest for

Knudsen number 0.08. Here Pincom is the pressure for incompressible flow and Po is the outlet pressure. The peak position shifted to the right along stream wise direction with the decrease of Knudsen number. For Knudsen number 0.08, peak position occurred at 0.56125 which shifted to 0.566666 and 0.571334 for the Knudsen number 0.04 and 0.02 respectively. The difference between peak positions values along crosswise direction for the Knudsen number 0.08 and 0.04 is 16.17%.

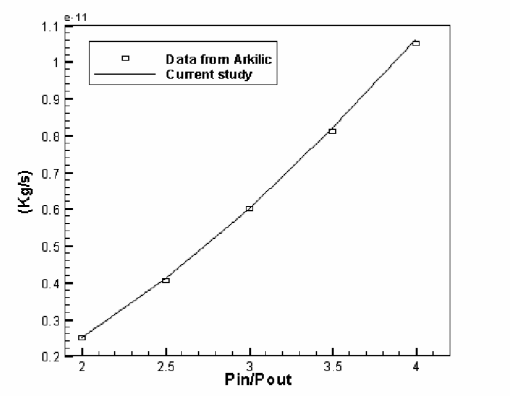

Fig.8 Comparison of mass flow rate with arkilic et. al.[4]

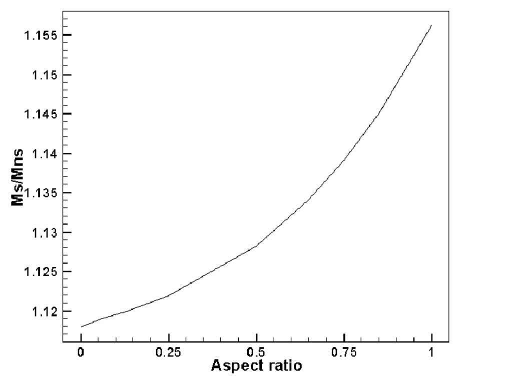

Fig.9 Plot of the value of M^* for different aspect ratios

As the Knudsen number decreases this difference also decreases. For the Knudsen numbers 0.04 and 0.02 this difference is 9.16%.

Figure 6 shows the normalized axial velocity distribution on the top and bottom walls for different aspect ratios. The velocity decreases with the decrease of aspect ratio but this decrease occurs up to aspect ratio 0.25. If we decrease the aspect ratio further the velocity does not decrease any more and coincides with the asymptotic inlet velocity to 0.0293 and outlet velocity to 0.108.The outlet velocity difference of aspect ratio 1.0 and 0.0625 is only 10.32%. Figure 7 exhibits the normalized wall axial slip velocity distribution on the side walls. The velocity gradually decreases with the decrease of aspect ratio. The outlet velocity difference of aspect ratio 1.0 and 0.0625 is 33.78%. Hence for different aspect ratios, the slip velocity difference on the side walls is much stronger unlike top and bottom walls. This is due to the weaker shear rate in the z direction.

Fig.8 shows the comparison of mass flow rate of the analytical result predicted by Arkilic et al.[2]. As the pressure ratio increases the mass flow rate also increases. In Fig.9 the value of M^* is plotted as a function of slip outlet mass flow rate

M_s over noslip outlet mass flow rate M_{ns} for different aspect ratios. We observe that as the aspect ratio increases the value of M^* also increases. The minimum value 1.117936 of M^* occurs at aspect ratio 0 or 2D channel and maximum value 1.156131 occurs at aspect ratio 1.0.

CONCLUSION

We investigated the effect of aspect ratio on gas slip flow in 3D rectangular microchannels. Different aspect ratios were used. As aspect ratio decreases the wall slip velocity also decreases. For top and bottom walls the velocity coincides to an asymptotic value for AR ≤ 0.25. For side walls the slip velocity decreases gradually. The difference of velocity between top/bottom and side walls decreases with the increase of aspect ratio. The Pressure nonlinearity decreases with the increase of aspect ratio. The ratio of slip over noslip mass flow rate increases with the increase of aspect ratio.

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