THERMAL POSTBUCKLING CHARACTERISTICS OF STEP-FORMED FG PANELS WITH TEMPERATURE-DEPENDENT MATERIAL IN SUPERSONIC FLOW

Sang-Lae Lee *, Ji-Hwan Kim

Institute of Advanced Aerospace Technology, School of Mechanical and Aerospace Engineering, San 56-1, Sillim-Dong, Gwanak-Gu, Korea

Abstract

In this study, it is investigated the thermal post-buckling characteristics of step-formed FG panel under the heat and supersonic flow. Material properties are assumed to be temperature dependent as well as continuously varying in the thickness direction of the panel according to a simple power law distribution in terms of the volume fraction of the constituent. First-order shear deformation theory(FSDT) of plate is applied to model the panel, and the von Karman strain-displacement relations are adopted to consider the geometric nonlinearity due to large deformation. Also, the first-order piston theory is used to model the supersonic aerodynamic load acting on a panel. Numerical results are summarized to reveal the thermal post-buckling behaviors of FG panels with various volume fractions, temperature conditions and aerodynamic pressures in detail.

INTRODUCTION

Functionally graded materials (FGMs) have been developed by the concept of composition of material properties changing gradually over the volume of the structure, and thus the material ingredient varies continuously from one material to another. Ceramic and metal are widely used in FGMs for suitability of thermal resistance and durability. One of the applications is a skin panel for a spacecraft which suffers high temperature and aerodynamic pressure during launching and reentry status. During the supersonic flight, aerodynamic heating induces temperature rise to influence the behaviors of structures. Thus the analysis of thermal buckling and thermal post-buckling under the aerodynamic loads has been important topics for FG panels.

There have been many studies on the thermal buckling and thermal post-buckling behaviors of FG panels. Feldman and Aboudi [1] studied elastic bifurcation buckling of FG plates under in-plane compressive loading. Sohn and Kim [2] dealt with the thermal post-buckling of FG panels subjected to combined aerodynamic and thermal load in the supersonic region. Praveen and Reddy [3] investigated the static and dynamic thermo-elastic response of FG plates considering geometrical nonlinearity for a large deflection.

To reduce the high cost to fabricate the conventional type of FGMs, a step-formed gradation was proposed. Even though the material properties changes with discontinuity, step-formed FGMs have the more proper characteristics than the non-metalic composite materials in thermal environment. Now, thermal post-buckling behaviors of step-formed FG panels are examined by considering the influence of the aerodynamic loads. Material properties of the panel are assumed to be continuously varying in the thickness direction according to a simple power law distribution and temperature dependent. Von Karman strain-displacement relations are used to account for the geometric nonlinearity of the model. Numerical results are compared with the previous work, and the effect of supersonic airflow on the behavior of a thermally post-buckled FG panel is summarized in detail.

FORMULATIONS

Modelling of the FG Panels

Fig.1 depicts the dimensions and coordinate systems of a rectangular step-formed FG plate model under the influence of supersonic airflow and temperature rise. This model represents a skin panel of a spacecraft which suffers aerodynamic heating in supersonic flows. Top and bottom layers are composed of pure ceramic and metal, respectively. While the intermediate layers are mixture form of the two materials and layers are assumed to be homogeneously isotropic. Furthermore, the layers are assumed to be perfectly bonded. The volume fractions of ceramic and metal for i th layer are taken as the mid-plane values of the layer based on a simple power law distribution, which is given as

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k \qquad (0 \le k < \infty), \qquad V_{c\,i} + V_{m\,i} = 1 \tag{1.a}$$

$$\mathrm{V_C} = [(V_c)_1 \quad (V_c)_2 \quad \cdots \quad (V_c)_{n_l}] = [1-(V_m)_1 \quad 1-(V_m)_2 \quad \cdots \quad 1-(V_m)_{n_l}] \tag{1.b}$$

where, the superscript k , V_m and $(V_c)_i$ denote the volume fraction index , volume fraction of the metal, volume fraction of i-th layer ceramic, respectively. Furthermore, $V_c = 1$ and $V_c = 0$ stand for the bottom and top layer, respectively.

Fig.2 summarizes the typical distributions of the volume fraction of the ceramic through the plate thickness for various values of the volume fraction indices. The material properties of FG plates can be obtained by the linear rule of mixture as follows:

$$P_{\mathrm{eff}}(T,z) = P_m(T)V_m(z) + P_c(T)V_c(z) = P_c(T) + (P_m(T) - P_c(T))\left(1 - \frac{z}{h}\right)^k \tag{2}$$

where, P_{eff} , P_m and P_c are the effective material property, the material properties of the metal and ceramic, respectively.

Generally, FGMs are used in high temperature environments, the material properties of constituents must be dependent on temperature as well as position. Therefore, two main features of the materials are adopted. First, all material properties P of the common ceramics and metals can be expressed as

$$P(T) = P_0\left(\frac{P_{-1}}{T} + 1 + P_1 T + P_2 T^2 + P_3 T^3\right) \tag{3}$$

in which P_0, P_{-1}, P_1, P_2 and P_3 are the effective material properties and the coefficients of temperature. Therefore, a typical material property such as elastic modulus E, the mass density ρ, the Poisson ratio ν and thermal expansion coefficient α can be calculated using Eq.(3). The different kinds of material properties of FGMs are shown in [6]. In addition to this, we take Posson's ratio ν as 0.3.

Constitutive Equations

Based on the first-order shear deformation theory, the displacement fields for a FG panel can be expressed as:

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\phi_x(x,y,t) \\
v(x,y,z,t) &= v_0(x,y,t) + z\phi_y(x,y,t) \\
w(x,y,z,t) &= w_0(x,y,t)
\end{aligned}
\tag{4}
$$

Using the von Karman strain-displacement relations, the in-plane strain vector is expressed as:

$$\mathbf{e} = \boldsymbol{\varepsilon}^{\mathbf{0}} + z\boldsymbol{\kappa} \tag{5}$$

$$= \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + \frac{1}{2}\begin{Bmatrix} \left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ 2\dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} \end{Bmatrix} + z\begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \end{Bmatrix}$$

The constitutive equations for the FG plates can be written as,

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}\begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{Bmatrix} - \begin{Bmatrix} \mathbf{N}_{\Delta\mathbf{T}} \\ \mathbf{M}_{\Delta\mathbf{T}} \end{Bmatrix} \tag{6.a}$$

$$\mathbf{Q} = \mathbf{S}\boldsymbol{\gamma} \tag{6.b}$$

where $\mathbf{N}$, $\mathbf{M}$, and $\mathbf{Q}$ denote the in-plane force resultant, moment resultant and transverse shear force resultant vectors, respectively. $\mathbf{N}_{\Delta\mathbf{T}}$ and $\mathbf{M}_{\Delta\mathbf{T}}$ are the thermal in-plane force and thermal moment resultant vectors due to the temperature elevation, $\Delta T = T - T_0$, which are given as:

$$(\mathbf{N}_{\Delta\mathbf{T}}\ ,\ \mathbf{M}_{\Delta\mathbf{T}}) = \int_{-h/2}^{h/2} \mathbf{E}\ \{\alpha(z), \alpha(z), 0\}^T (1, z)\Delta T dz \tag{7}$$

Governing Equations

Using principle of virtual work, the governing equations for FG panel are obtained as follows.

$$\delta W = \delta W_{\text{int}} - \delta W_{\text{ext}} = 0 \tag{8}$$

where δW_{int} and δW_{ext} represent internal and external virtual work, and are given as,

$$\delta W_{\text{int}} = \int_A \left[\delta\boldsymbol{\varepsilon}^{\mathbf{T}}\mathbf{N} + \delta\boldsymbol{\kappa}^{\mathbf{T}}\mathbf{M} + \delta\boldsymbol{\gamma}^{\mathbf{T}}\mathbf{Q}\right] dA \tag{9.a}$$

$$
\begin{aligned}
\delta W_{\text{ext}} = \int_A [-I_0\left(\ddot{u}_0\delta u_0 + \ddot{v}_0\delta v_0 + \ddot{w}_0\delta w_0\right) - I_1\left(\ddot{u}_0\delta\phi_x + \ddot{\phi}_x\delta u_0 + \ddot{v}_0\delta\phi_y + \ddot{\phi}_y\delta v_0\right) \\
- I_2(\ddot{\phi}_x\delta\phi_x + \ddot{\phi}_y\delta\phi_y) + p_a\delta w] dA
\end{aligned}
\tag{9.b}
$$

in here $(I_0, I_1, I_2) = \int_{-h/2}^{h/2} \rho\cdot(1, z, z^2)dz$, while p_a is external load induced by aerodynamic pressure.

For supersonic speed, the aerodynamic pressure can be approximated by the first-order piston theory[2] and can be written as

$$p_a(x,y,t) = -\frac{\rho_a V_\infty^{\ 2}}{\sqrt{M_\infty - 1}}\left(\frac{\partial w}{\partial x} + \frac{1}{V_\infty}\frac{M_\infty - 2}{M_\infty - 1}\frac{\partial w}{\partial x}\right) = -\left(\lambda\frac{D_m}{a^3}\frac{\partial w}{\partial x} + g_a\frac{\partial w}{\partial t}\right) \tag{10}$$

Substitute Eqs. (9.a) and (9.b) into Eq. (8) and then assume solution as $\mathbf{d} = \mathbf{d_s} + \Delta\mathbf{d_t}$. For $\mathbf{d_s}$ and $\mathbf{d_t}$ represent the time independent and time dependent solutions, respectively.

The governing equations for the static solutions of the FG plate under combined aerodynamic and thermal loading can be expressed as

$$\left(\mathbf{K} - \mathbf{K}_{\Delta \mathbf{T}} + \lambda \mathbf{A}_{\mathbf{f}} + \frac{1}{2}\mathbf{N1}_{\mathbf{s}} + \frac{1}{3}\mathbf{N2}_{\mathbf{s}}\right)\mathbf{d}_{\mathbf{s}} = \mathbf{P}_{\Delta \mathbf{T}} \tag{11}$$

where, $\mathbf{M}$, $\mathbf{K}$, $\mathbf{K}_{\Delta \mathbf{T}}$, $\mathbf{N1}_{\mathbf{s}}$, $\mathbf{N2}_{\mathbf{s}}$, $\mathbf{A}_{\mathbf{f}}$, $P_{\Delta T}$ and subscript ‘s’ represent the mass matrix, the linear stiffness matrix, the thermal geometric stiffness matrix, the static first order nonlinear stiffness matrix, the static second order nonlinear stiffness matrix, the aerodynamic stiffness matrix, thermal load vector and the time independent solutions, respectively.

NUMERICAL RESULTS AND DISCUSSIONS

Code Verifications

To verify the code in this study, the thermal buckling analyses of square FG plates with various thickness ratios are studied. The material properties are assumed to be independent of the temperature. Young’s modulus and thermal expansion coefficients are $E_m = 70\text{GPa}$ and $\alpha_{\text{m}} = 23\times10^{-6}\,/\,°\text{C}$ for aluminum, while $E_C = 380\text{GPa}$ and $\alpha_{\text{c}} = 7.4\times10^{-6}\,/\,°\text{C}$ for alumina, respectively. As shown in Fig.3, the result for critical temperature change of an aluminum-alumina plate is compared with the analytical solution of Ref.[4], and shows good agreements with each other. Next, the static stability boundary of the square isotropic plate is compared with the previous work[5], and the results are favorably well as shown in Fig.4

Thermal Post-buckling in Supersonic Flow

The thermal post-buckling behaviors of square step-formed FG panels are investigated with various volume fractions and aerodynamic load for different boundary conditions. Silicon-Nitride (Si_3N_4) and stainless steal(SUS304) are used , and the material properties are assumed to be simultaneously dependent on the temperature and position. The reference temperature and the thickness ratio of the square plate(a/h) are taken 300K and 100,respectively.

Fig.5 depicts the center deflections of panel with all clamped edges for various volume fractions. When the temperature increases gradually from T_0 to 350 K, the panels are suddenly deflected at the critical temperatures either upward or downward. FG panels show similar characteristics with isotropic panels such as Nitride(Si_3N_4) and stainless steal(SUS304). If the aerodynamic loads are included, appearance of the bifurcation buckling is delayed and magnitude of the deflection is decreased due to the aerodynamic load, as shown in Fig.6

On the other hand, for the simply supported boundary condition of FG panels are shown in Fig.7, 8. At the lower temperature, the FG panels are deformed only downward direction as illustrated in Fig.7. However, at the higher temperatures, there is another static equilibrium state. In here, FG panels can be deflected both downward and upward direction. For instance, an FG panel with $k = 1$ at $T = 310K$ has just one static equilibrium position on downward direction, while at $T = 340K$, it has two equilibrium positions. Furthermore, if the aerodynamic loads are added to the panels, as shown in Fig.8, the outbreak of snap-through is delayed and the mount of deflection is decreased like clamped FG panels. These characteristics of a simply supported FG panel are due to the asymmetric material constitution through the thickness. When the temperature of an FG panel is increased uniformly throughout the volume, in-plane stress that is caused by thermal expansion through the thickness is neither uniform nor symmetric about the mid-plane of the panel.

CONCLUSIONS

Thermal post-buckling characteristics are investigated for step-formed FG panels subjected to be combined thermal and aerodynamic loads. Temperature-dependent material properties are assumed in this paper. In this study, the buckling temperatures of the FG panels are higher than those of the isotropic metal($\mathrm{SUS304}$) plates, but not ceramic(Si_3N_4). The volume fraction of the ceramic increases, the buckling temperature becomes higher. Furthermore, increase of aerodynamic loads results in a delay of buckling temperature of the FG panels and decreases the magnitude of the deflection.

The thermal buckling characteristics of simply supported FG panels are different to the clamped FG panels. Due to the asymmetrical characteristics of the FG panels, bifurcation buckling does not occur for a simply supported FG panel. Additionally, thermally buckled shapes of simply supported FG panels depend on their deformed direction. Simply supported FG panels tend to only deflected downward at low temperatures.

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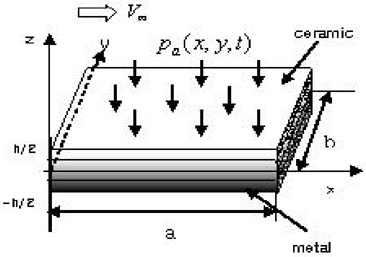

Figure 1 – Stepwise FG panel model under the air flow and thermal load

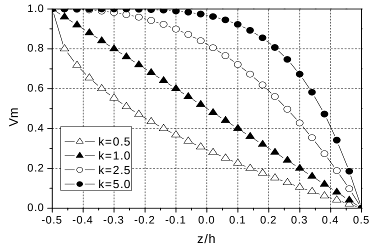

Figure 2 – Typical volume fraction distributions of the metal on FG panel

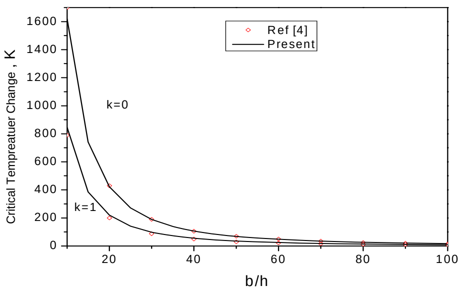

Figure 3 – Critical temperature of FG plate for buckling b/h

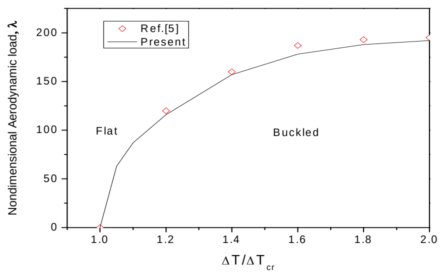

Figure 4 – Static stability boundary of a square isotropic plate.

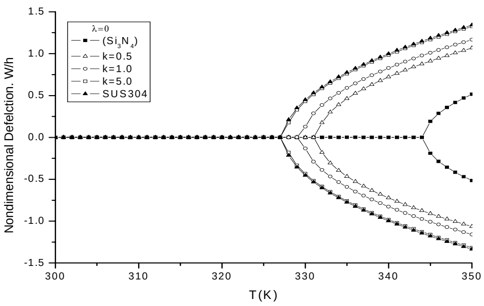

Figure 5 – Center deflection of clamped FG panel ($\lambda = 0$)

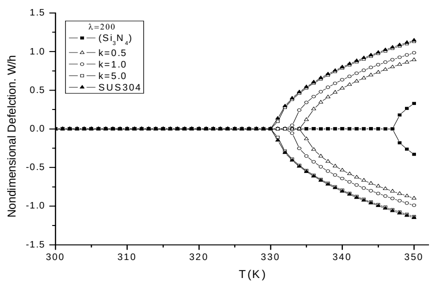

Figure 6 – Center deflection of clamped FG pane ($\lambda = 200$)

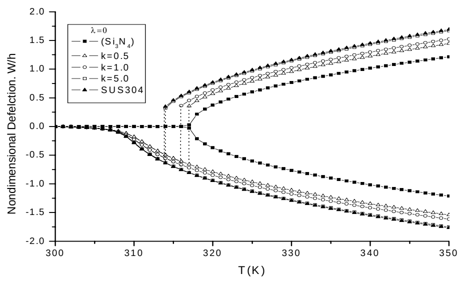

Figure 7 –Deflection of simply supported FG panel ($\lambda = 0$)

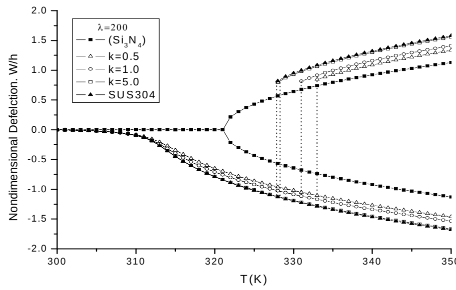

Figure 8 –Deflection of simply supported FG panel($\lambda = 200$)