SELECTION OF DAMPING MODEL IN VIBRATION OF FLEXIBLE BEAMS

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Abstract

Many papers have studied computer-aided simulations of elastic bodies undergoing large deflections and large deformations. But there have been few attempts to validate their numerical formulations used in these studies. The main aim of this paper is to validate the absolute nodal coordinate formulation (ANCF) by comparing the results to experimental measurements on beams. Physical experiments with a high-speed camera were carried out to capture the large displacement of the beam and to verify the results of computer simulations. To consider the damping forces, the Rayleigh's damping and quadratic damping are employed and compared to the experimental results, respectively. Numerical results obtained from computer simulations were compared with the results from the physical experiments according to the 1st mode and the 2nd mode of the beam, respectively.

INTRODUCTION

The absolute nodal coordinate formulation (ANCF) is a new finite element technique proposed for modeling of large displacement and large displacement problems in flexible multibody dynamics [1, 3]. It produces finite elements that can represent arbitrary large displacements relative to a global frame of reference. The elements employ finite slopes as nodal variables and are generalizations of ordinary finite elements that use infinitesimal slopes.

In this paper, physical experiments with a high–speed camera were carried out to verify damping characteristics of a beam. Markers are attached to the end of the beam and the motions of the markers are traced by a data acquisition system. The beam was located on vertical direction and was excited horizontal direction. Numerical results of the first mode beam, using damping forces that were calculated from Rayleigh's proportional damping [2] and quadratic damping [4], were compared with experimental data. Also, Numerical results of the second mode beam, using damping force that was calculated from Rayleigh's proportional damping and quadratic damping, were compared with experimental data.

MEASUREMENT OF LARGE DEFORMATION

Experimental setup

In the large displacement test, the choice of the material is the most important thing. It must be very flexible and elastic but not plastic. A very thin beam used in this study has 500 mm length, 5 mm width and 0.5 mm thickness. A high-speed camera was used to capture the motion. The high-speed camera, REDLAKE Motion Scope 1000s, was set at 500fps for the purpose of this test. Figure 1 shows the high speed camera used in this study. Figure 2 shows the beam with a jig.

Figure 1 – High-speed camera

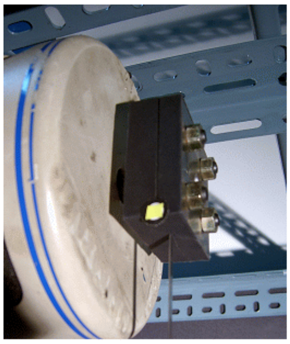


Figure 2 – Clamped beam with a jig

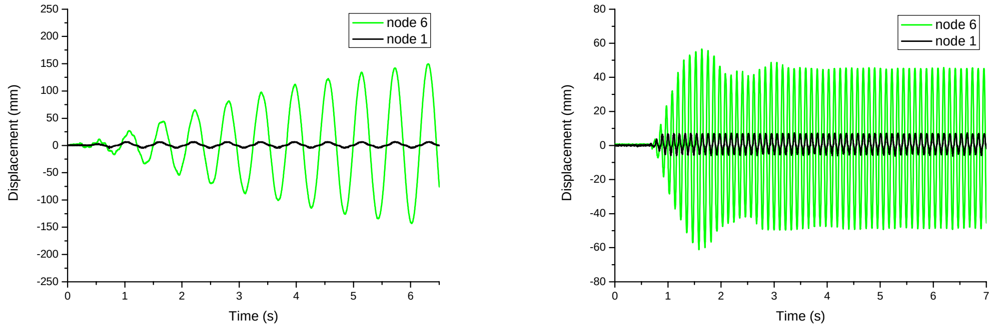
Experiments

The properties of the beams are shown in Table 1. The beam was located on the vertical direction and was excited along the horizontal direction. It was excited by the function generator near to the first natural frequency and second natural frequency.

Table 1. Properties of beam

l (Length)	b (Width)	h (Thickness)	ρ (Density)	E (Young's modulus)
500 *mm*	5.0 *mm*	0.5 *mm*	7753.4 kg/m^3	167 *GPa*

Experimental results



(a) First mode *(b) Second mode*

Figure 3 – Horizontal position data

In the first mode, as time goes on, the displacement of end point was increased. The amplitude of excitation data is about 6 mm and the frequency is 1.7 Hz. And experiment of the second mode was the same method as that of the first mode. The amplitude of excitation data is about 6 mm and the frequency is 9.3 Hz. The horizontal displacement at end node was shown in Figure 3.

DAMPING MODELS

Linear damping

We used a linear model of damping forces:

$$\mathbf{Q}^{damp} = \mathbf{D}\dot{\mathbf{e}} \tag{1}$$

In this model, a proportional Rayleigh damping [2] is employed and the system damping matrix has the following form.

$$\mathbf{D} = \alpha \mathbf{M} + \beta \mathbf{C} \tag{2}$$

with mass matrix $\mathbf{M}$ and tangent stiffness matrix $\mathbf{C}$ (Jacobian matrix of forces) multiplied by coefficients that depend on frequencies ω_1 and ω_2 as well as on damping ratios ζ_1 and ζ_2 for the first two modes of the system. The ratios ζ_1 and ζ_2 should be identified from experimental data.

$$\alpha = \frac{2\omega_1\omega_2(\zeta_1\omega_2 - \zeta_2\omega_1)}{\omega_2^2 - \omega_1^2}, \quad \beta = \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2} \tag{3}$$

Table 2 represents the damping parameters for computer simulations.

Table 2. Proportional Rayleigh's damping parameters

ω_1 (rad/s)	ω_2 (rad/s)	$\zeta_1 = \zeta_2$	α	β
10.738	59.056	0.012	0.213	3.35e-4

Quadratic damping

Assume that the damping force has a linear and a quadratic part in velocities [4]

$$f(v) = \alpha_1 v + \alpha_2 v|v| \tag{4}$$

with unknown coefficients α_1 and α_2 that should be identified from experimental data. Equations of motion can be written as:

$$M\,\ddot{e} + C\,e + \beta_1\,M\,\dot{e} + \beta_2\,M\,\dot{e}\,|\dot{e}| = 0 \tag{5}$$

Following Van der Pol's method, we represent the solution in the form

$$e(t) \approx A(t)\cos\omega_0 t \quad , \quad A(t) = \frac{3\pi\beta_1 A_0 e^{-\beta_1 t/2}}{8\omega_0\beta_2 A_0(1 - e^{-\beta_1 t/2}) + 3\pi\beta_1} .$$

In order to identify the coefficients β_1 and β_2 we should measure the amplitudes A_k as well as time points t_k from an experimental curve as shown in Figure 4.

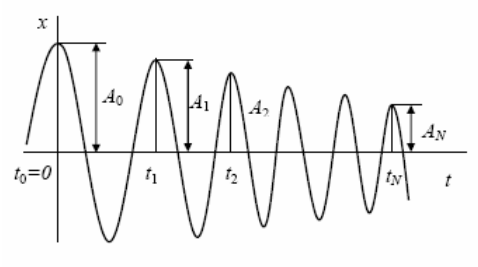


Figure 4 - Damped oscillations

Then we use the method of least squares

$$\Psi(\beta_1, \beta_2) = \sum_{k=0}^{N} \left(A(t_k) - A_k \right)^2 \to \min$$

and find the values β_1 and β_2.

Following this algorithm, parameters of the model of damping forces were obtained as shown in Table 3.

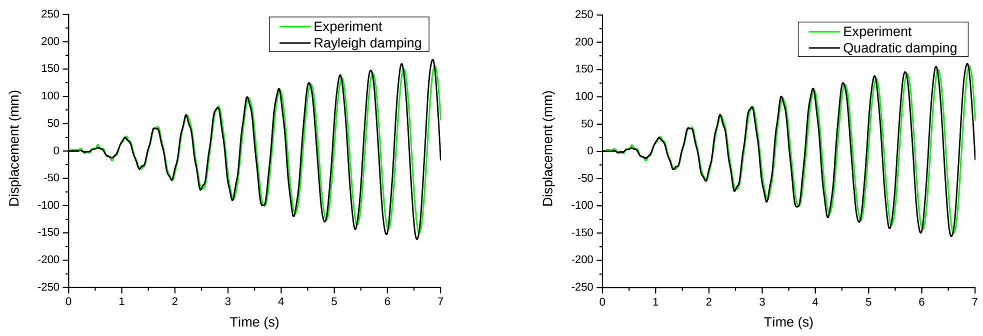
Table 3. Quadratic damping parameters

ω_0 (rad/s)	A_0 (mm)	β_1 (s^{-1})	β_2 (m^{-1})
10.738	182.267	0.108	0.158

NUMERICAL SIMULATIONS

The excitation input data of the beam was obtained from the experimental measurements. We imposed on the motion with the excitation input data at the node which is fixed by the jig.

Comparison for the first mode

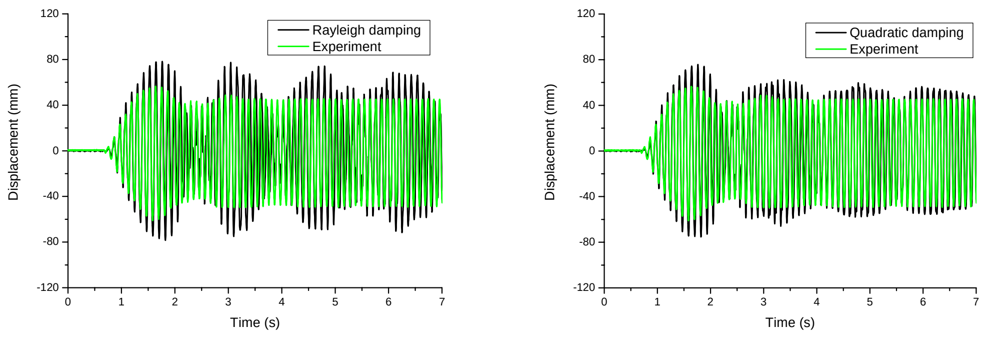


(a) Rayleigh damping *(b) Quadratic damping*

Figure 5 – Result of the first mode

The end displacements of the first mode with Rayleigh's damping and quadratic damping were shown in Figure 5, respectively. Both linear damping results and quadratic damping results showed good agreements with the experimental results.

Comparison for the second mode



(a) Rayleigh's damping *(b) Quadratic damping*

Figure 6 – Result of the second mode

In Figure 6, the displacements with Rayleigh's damping were oscillated and showed some differences compared to the experiments. This is due to the weak damping forces. On the other hand, the oscillations of the displacements with quadratic damping were decreased and the errors to the experiments were decreased. The numerical results with quadratic damping showed better result than that of Rayleigh's damping.

CONCLUSIONS

In this paper, we performed both physical experiments and computer simulations, which took into account for the resistance force as a proportional linear damping and quadratic damping. The experiment with a high speed camera well captures the large displacement of the beam. The excitation input data of the beam in the computer simulation was obtained from the experimental results. We imposed on the motion with the excitation input data at the node which is fixed by a jig.

The numerical results were compared to the experiments. For the first mode, the numerical results using Rayleigh's damping and quadratic damping were in accordance with the experimental results. On the other hand, for the second mode, the numerical results with quadratic damping showed better result than that of Rayleigh's damping.

Damping forces play an important role in the computer simulations of large deformation problems. According to this study, quadratic damping gives better results to the problems with the 1^{st} and the 2^{nd} mode simultaneously.

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