The extension of hierarchical covering location problem

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Abstract

The hierarchical covering location problem emphasizes the issue of locating of hierarchical facilities in order to maximize the number of customers that can be covered. In the classical HCLP(Hierarchical Covering Location Problem), it is assumed that the customers are covered completely if they are located within a specific distance from the facility, and not covered otherwise. The generalized HCLP is introduced that the coverage of customers is measured to be any real value rather than 0 or 1, where the service level may decrease according to the distance. Mixed integer programming formulation for the generalized HCLP is suggested with a partial coverage of service. Solutions are found using OPL Studio, and are evaluated for various cases.

Keywords: Hierarchical location; Partial coverage; Covering models.

1. Introduction

Location theory which is concerned with choosing the best location for facilities from a given set of potential locations so as to minimize total cost or to maximize total service while satisfying customer demand has been extensively extended since 1960s. Many problem types therefore have been identified and solution methods developed to solve them have been used to decide the location of facilities in many applications like hospitals, fire stations, schools, distribution centers, computer networks and so on[11]. In recent years, new problem types and methodologies to solve these problems have been designed extensively due to increasing competitiveness of companies and local public services. Brandeau and Chiu[3] surveyed typical problems that have been studied in location research. They identified numerous problems in location research as more than 50 problem types and indicated how those problem types relate to one another.

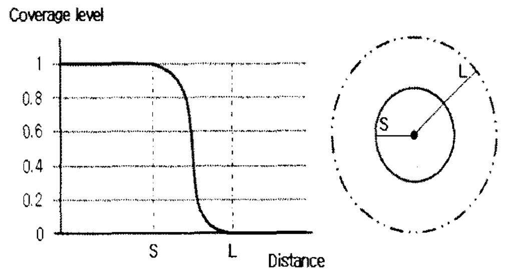
As one of many problems in location theory, hierarchical covering location problem (HCLP) was proposed to apply location problem to health service as extended concept of maximal covering location problem (MCLP) in 1982. The objective of MCLP study is to organize opened facilities in order to maximize the total demand of served customers, where a customer is considered served if customer located within a specified service distance from the closest facility[2]. Based on this concept, Moore and ReVelle[15] proposed a hierarchical problem, where the facility can provides several kinds of service for customers.

The original purpose of hierarchical covering location problem is to maximize the number of customers served within a specified critical distance or time by a fixed number of facilities. The hierarchical facility systems generally be composed of the one type of two or more obvious types of facility, and all the customers may not be covered. For example, health care systems may consist of clinics and hospitals; education systems may consist of elementary school, middle school, high school and university; and production - distribution systems may consist of factories and warehouses [7]. Rahman and Smith[17] reviewed the use of location-allocation models in health service development planning in the developing nations, in order to examine the suitability of these methods for designing health care systems. Mandell[13] addressed covering models for two-tiered emergency medical services systems, and Teixeira and Antunes[20] presented a discrete hierarchical location model for public facility planning like school network. When it comes to production-distribution systems, Lee[12] presented a hierarchical distribution problem with multi-level distribution centers, and Marianov *et al.*[14] addressed the issue of locating hierarchical facilities in the presence of congestion. Daskin[15] briefly discussed hierarchical facility location models and outlines interacting facility models where the relations between levels are considered explicitly. A classification of hierarchical problems was proposed by Narula[16], in which the hierarchical relationship among the different facility types are discussed. Sahin and Sural[19] divided the hierarchical facility location problems according to the characteristics of facility systems studied, which are based on the relation between customer and facility, service capacity at each level of facility, and spatial configuration of services in addition to the objectives to decide the location of facilities.

The hierarchical covering location problem emphasizes the issue of locating of hierarchical facilities in order to maximize the number of customers that can be served. A basic and key assumption in HCLP is that the customers are assumed to be covered completely if there is a customer located within specific distance from facility, and not covered at all if customer is located in the outside of the specific distance. In some actual situations, it may not be possible to guarantee the full coverage of the demand points depending on the distance between the customer and facility location. Covering models may focus on the worst case behavior of the system[5]. When the nature of the problem requires accounting for partial coverage, applying the classical HCLP might lead to unjustified outputs. In fact, most retail dealers or customers think of coverage level provided by the facility as a decreasing function of the distance from the facility to their locations[2].

Additionally, the facility level in many literature on HCLP is fixed as 2-level or 3-level and these fixed level problems have been primarily considered[6]. For example, Moore and ReVelle[15] proposed a hierarchical problem having two levels, Bumb[4] presented an approximation algorithm based on randomized rounding for solving the maximum problem of the 2-level uncapacitated facility location problem, and Ignacio *et al.*[10] presented two-level hierarchical location problem in computer network. Various examples of 2-level hierarchical location problem are found in Gao and Robinson[9], Ro and Tcha[18] and Aardal *et al.*[1]. Galvao *et al.*[8] proposed hierarchical model of 3-level facility for the location of perinatal facilities in the municipality of Rio de Janeiro, and Yeh and Parhami[21] presented parallel algorithms on three-level hierarchical cubic networks.

In this paper, the HCLP model is extended by adopting a concept of partial coverage, in which service can be provided as a decreasing function of service according to the distance from facility to customer, and a concept of generalization, in which the facility level can be used as all kinds of level types of hierarchical problem instead of fixed facility level like 2 or 3-level. When it comes to the concept of partial coverage, Figure 1 depicts the relation between the distance and the coverage level which is defined as a function of the distance of the demand point to the facility.



[Figure 1] Relation between distance and coverage.

There are two kinds of distances, named minimum specific distance(S) and maximum specific distance(L). It is assumed that the customer can be completely covered within the minimum specific distance, partially covered between minimum and maximum distance, and not covered in outside of the maximum specific distance.

This paper is organized as follows. In section 2, definitions and the mathematical formulation of the problem are suggested. The results of computational experiments are in section 3, and the last section contains some concluding remarks.

2. Formulation

The formulation for the generalized HCLP is suggested, which is based on Moore and ReVelle[15]. The sum of the coverage of customers is to be maximized in the objective function.

Indices

i : index for potential facility site.
j : index for customer.
k : index for service level.
l : index for facility level.

Decision variables

$$c_{ij} = \begin{cases} 1, \text{ if } R_{ij} \le S \\ f(R_{ij}), \text{ if } S < R_{ij} \le L \text{ , } (0 < f(R_{ij}) < 1) \\ 0, \text{ otherwise} \end{cases}$$

c_{ij} : the the coverage level provided by the facility i to the customer j

$$x_{ijk} = \begin{cases} 1, \text{ if a customer } j \text{ is covered with service level } k \text{ by a facility located at site } i \\ 0, \text{ otherwise} \end{cases}$$

$$y_{il} = \begin{cases} 1, \text{ if a facility of level } l \text{ is located at } i \\ 0, \text{ otherwise} \end{cases}$$

Data

n : the number of customer.
r : the number of facility level allowed.
f_j : the demand quantity of customer j.
t_i : the capacity of facility i opened
p_l : the number of facilities allowed to be opened with facility level l.
R_{ij} : the distance between the facility i and the customer j.
S : the full coverage distance (minimize specific distance).
L : the maximum partial coverage distance (maximum specific distance).

$$Maximize \quad \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} c_{ij} f_j x_{ijk} \quad (1)$$

Subject to

$$x_{ijk} \le \sum_{l=k}^{r} y_{il} \quad \text{for } \forall\, i, k \quad (2)$$

$$\sum_{i \in I} \sum_{k \in K} x_{ijk} \le 1 \quad \text{for } \forall\, j \quad (3)$$

$$\sum_{i \in I} y_{il} \le p_l \quad \text{for } \forall\, l \quad (4)$$

$$\sum_{j \in J} \sum_{k \in K} c_{ij} f_j x_{ijk} \le t_i \quad \text{for } \forall\, i \quad (5)$$

$$y_{il} \in \{0,1\} \quad \text{for } \forall\, i, l \quad (6)$$

$$x_{ijk} \in \{0,1\} \quad \text{for } \forall\, i, j, k \quad (7)$$

The objective function, (1) maximizes the total coverage level which is served by all levels of facilities. Constraint (2) states that customers can only be allocated to a facility at site i if there are facility which have capacity of providing the required service. For example, in the case of 2-level hierarchical problem, customer can be covered by low level service if there is at least either one lower level facility or one higher level facility and customer can be covered by high level service if there is at least one higher level facility. Moreover, constraint (2) means that this formulation can be applied to all kinds of level types of hierarchical problem instead of fixed facility level like 2 or 3-level. Constraint (3) requires that customer will be covered by at most one of the facility located. If there are one or more facilities that can cover the customer, only one facility that can give higher coverage will be selected because of the objective function. Constraint (4) ensures that the total number of facilities located will be below the permitted number of facilities in each facility level. Constraint (5) presents that the total service quantity has to be below the capacity of facility. The constraint sets (6) and (7) mean binary restriction on the decision variables.

When it comes to coverage level, Karasakal and Karasakal[11] suggested the general function for the partial coverage i.e, $c_{ij} = f(R_{ij})$, where c_{ij} is the coverage level provided by the facility i to the customer j, R_{ij} is the distance between the facility i and the customer j, and $f(R_{ij})$ is the function presenting relation between the coverage level and the distance from facility to customer. The shape of partial coverage function may be changed by the character of problem like linear or nonlinear. The partial coverage function which has decreasing shape according to the increase of the distance between facility and customer be used in this paper. The solution procedure and the results of computational experiments for the extended hierarchical covering location problem(E-HCLP) are introduced in the following sections.

3. Experimental results

In this section, the suggested formulation is tested by ILOG OPL Studio and the results of the experimental study are presented. In order to analyse the effect of partial coverage and generalization on the solutions, the optimal solutions of the classical HCLP and E-HCLP on example problems are compared. The ILOG OPL Studio for finding optimal solution runs on PentiumVI PC with 3.0 GHz CPU and 1GB RAM.

The problem data used in these experiments were generated randomly. The number of customer locations ranges from 20 to 100 with 1 or 2 facilities to be located in 1st facility level(2 or 3 in 2nd level, 3 or 4 in 3rd level). The data representing the customer locations and the potential facilities were generated from a uniform distribution within the range [0, 200]. The Euclidean distances between customers and potential locations were used to define the coverage levels. We assume that the facility locations can fully serve the customers within the specific range(1st level:30, 2nd level:10, 3rd level:5) whereas the customers can not be served in outside of the maximum range. In order to display the decrease of service between minimum critical distance(S) and maximum critical distance (L), we use the sigmoid shaped function as follows.

$$f(R_{ij}) = 1/(1+\exp(P(R_{ij}-(S+L)/2)))$$

where

1st level facility : P=0.2, S=30< R_{ij} ≤L=80

2nd level facility : P=0.2, S=10< R_{ij} ≤L=40

3rd level facility : P=0.2, S= 5< R_{ij} ≤L=20

Table 1. Comparision results between classical HCLP and E-HCLP

Problem parameters			Classical HCLP		E-HCLP		Gap
n	i	p_l	Q	Time(sec)	Q	Time(sec)	(%)
20	3/4/5	1/2/3	290.0	0.38	179.2	1.19	38.21
20	5/7/10	2/3/4	339.0	4.26	263.5	3.23	22.27
40	3/4/5	1/2/3	457.0	0.78	316.6	1.59	30.72
40	5/7/10	2/3/4	602.0	11.59	446.3	47.94	25.86
60	3/4/5	1/2/3	698.0	1.91	460.3	2.64	34.05
60	5/7/10	2/3/4	757.5	21.16	550.8	791.53	27.29
80	3/4/5	1/2/3	976.0	2.59	585.4	4.09	40.02
80	5/7/10	2/3/4	1391.7	101.20	928.0(Best)	13357.02	33.32
100	3/4/5	1/2/3	1145	12.17	811.6	14.69	29.12
100	5/7/10	2/3/4	1754	169.98	1104.5(Best)	10330.20	37.03

n =the number of customers.
i=the number of potential facility locations.(1st level/2nd level/3rd level)
p_l =the number of facilities to be opened with facility level l (1st level/2nd level/3rd level)
Q= the quantity of service received by facilities.
Gap = 100*(Classical HCLP value - E-HCLP value)/Classical HCLP value

The solutions obtained by the E-HCLP were compared with the optimal solutions of classical HCLP having coverage range L. The result of computational experiments are summarized in Table 1. For each problem type, identified by the number of the customers(n), the number of the potential facility locations(i), and p_l=the number of facilities to be opened with facility level l(1st level/2nd level/3rd level), the total quantities of service received by facilities and the time are listed.

These results show that the using of partial coverage in the problems may change the optimal solution. For example, the average gap of 31.79% between E-HCLP and classical HCLP was happened. Also, these show the effect of the partial coverage on the optimal solution. Applying the classical HCLP for problems where partial coverage of demands point exits might be unrealistic and lead to unjustified outputs. In the comparison of E-HCLP and classical HCLP, if there is disagreement in at least one facility location in the solutions obtained, the optimal solutions are said to be different. The differences of facility location when the numbers of customer are 20, 40 and 60 are represented in table 2.

Table 2. The difference of facility location sited

Problem parameters			Classical HCLP (Sited Facility)	E-HCLP (Sited Facility)	No. of Difference
n	i	p_l			
20	5	2	y_{21}, y_{41}	y_{11}, y_{21}	4
	7	3	y_{12}, y_{32}, y_{42}	y_{22}, y_{32}, y_{62}	
	10	4	•	y_{23}	
40	5	2	y_{21}, y_{31}	y_{21}, y_{41}	4
	7	3	y_{12}, y_{32}, y_{62}	y_{12}, y_{22}, y_{62}	
	10	4	$y_{23}, y_{43}, y_{53}, y_{73}$	y_{23}, y_{43}	
60	5	2	y_{31}, y_{41}	y_{11}, y_{41}	3
	7	3	y_{32}, y_{42}, y_{72}	y_{32}, y_{42}, y_{62}	
	10	4	y_{23}, y_{83}, y_{93}	y_{23}, y_{93}	

The experimental result shows that the using ILOG OPL Studio to find optimal solutions is restricted in the big size problem. In the sample problems, OPL Studio couldn't find optimal solution when the number of customer is over 80. It is necessary to develop heuristic methodology in order to solve big size problems.

4. Conclusion

In this paper, we introduce the concept of partial coverage and hierarchical generalization for the HCLP. The E-HCLP can be used in more realistic problem because of these concept. The proposed formulation provides that the customer can be fully served within the minimum specific distance, partially served between minimum and maximum distance, and not served in outside of the maximum specific distance when it comes to the concept of partial coverage, and this formulation provides generalized HCLP formulation which be used in all distinct types of hierarchical problem. The suggested formulation is tested by ILOG OPL Studio and the optimal solutions of the classical HCLP and E-HCLP on example problems are compared in order to analyse the effect of partial coverage on the solutions. For the future research, it is necessary to develop heuristic methodology in order to solve big size problem in a reasonable computation time.

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