# **Robust Adaptive Control For the Friction System Using Backstepping Method** and Recurrent Neural Network

\* S. I. Han <sup>1</sup>(hansng@suncheon.ac.kr)

Key words : Backsstepping sliding mode control, Dynamic friction Friction state observer, Recurrent fuzzy neural network

### 1. Introdution

A nonlinear friction often worsens the performance of the servo control system. Specially, in the low velocity range, the effects of the friction on the performance of the servo system are greater than moderate or steady state velocity range. The classical friction model cannot capture the characteristics such as the Streibeck effect, stick-slip, pre-sliding hysteretic motion, break-away force, which play a significant role in application on high precise motion control.

LuGre friction model, proposed by Canudas de Wit et al<sup>1</sup>, can capture both dynamic friction of low velocity and steady state friction characteristic. As the LuGre friction model has the simple and nice structure mathematically, then many researchers have chosen it as a standard friction model to control the frictional servo mechanical system such as the robots, X-Y table, electromechanical actuator, tire/road friction and machine tools. However, since the state variable representing the bristle deformations of the LuGre friction model cannot be measured directly, the estimation process for it must be presented to obtain more precise information on the friction dynamics

The recurrent FNN (RFNN)<sup>2</sup> naturally involves dynamic elements in the form of feedback connections used as internal memories. Thus, the RFNN is a dynamic mapping and demonstrates good control performance in the presence of uncertainty such as parameter variations of the system, external load, unmodeled dynamics compared to the feedforward FNN.

In this paper, we propose the composite friction control system, which consists of the backstepping sliding mode controller (BSMC), the robust dynamic friction state observer, the RFNN and the error estimator. The electro-mechanical servo system assembled with ball-screw and DC servo motor is chosen to demonstrate the good performance of the proposed control scheme through the simulation and experiment.

#### 2. Design backstepping SMC and friction state observer

The dynamic model for the mechanical system in the presence of friction is

$$J\dot{\theta} + \sigma_2\dot{\theta} + T_f + T_d = u \tag{1}$$

The deflection of the elastic bristle is

$$\dot{z} = \theta - f(\theta)z$$

where

$$f(\dot{\theta}) = \left|\dot{\theta}\right| / g(\dot{\theta}) \tag{3}$$

$$\sigma_0 g(\dot{\theta}) = T_c + (T_s - T_c)e^{-(\theta/\theta_s)2}$$
(4)

where  $T_c$  is Coulomb friction,  $T_s$  is stiction level,  $\theta_s$  is Stribeck angular velocity. The dynamic friction term excluding viscous friction torque is described by

$$T_f = \sigma_0 z + \sigma_1 \dot{z}$$
(5)  
$$T_f \text{ can be rewritten by}$$

$$T_f = \Phi(\dot{\theta})z + \sigma_1\dot{\theta}$$

$$T_{f} = \Phi(\dot{\theta})z + \sigma_{I}\dot{\theta}$$
(6)  
where the auxiliary function  $\Phi(\dot{\theta})$  is defined as follows:

$$\Phi(\dot{\theta}) = \sigma_0 - \sigma_I f(\dot{\theta})$$
(7)  
Let us substitute Eq. (6) into Eq. (1), the previous dynamic model is

written by  

$$J\ddot{\theta} + \sigma_3\dot{\theta} + T_z + T_d = u$$
 (8)

where  $\sigma_3 = \sigma_1 + \sigma_2$ ,  $T_z = \Phi(\dot{\theta})z$ .

Now, let us Eq. (19) represent state equations as follows:

$$\dot{x}_1 = \dot{\theta} = x_2 \tag{9}$$

$$\dot{x}_2 = [-\sigma_3 x_2 - T_z + u] / J \tag{10}$$

The new states are defined as follows in order to design the backstepping controller:

$$z_1 = x_d - x = x_d - x_1 \tag{10}$$

$$z_2 = x_2 + \alpha \tag{11}$$

where  $x_d$  is the desired command input. From Eq. (10), the derivative of  $z_1$  can be described as

$$\dot{z}_1 = \dot{x}_d - z_2 + \alpha \tag{12}$$

where  $\alpha = -k_1 z_1 - \dot{x}_d$ . The Lyapunov function is defined as

$$V_I = \frac{1}{2}z_I^2 \tag{13}$$

Define the second Lyapunov function as follows:

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{14}$$

Define the Lyapunov function as the following nonnegative function:

$$V_3 = V_2 + \frac{1}{2}Js^2$$
(15)

Since, however, the state variable z cannot be measured directly, we suggest the exponential stable observer with tunable transient performance to estimate z

$$\hat{z} = q + \frac{J}{\sigma_1} s + k_3 z_1 \tag{16}$$

and q is an auxiliary variable and satisfy the following equation:

$$\dot{q} = \frac{1}{\sigma_1} \left[ -\sigma_0 q - \sigma_2 x_2 - J \frac{\sigma_0}{\sigma_1} s + u_{RFNR} + \varPhi s - \sigma_0 k_3 z_1 \right]$$

$$-J((l-k_1^2)z_1 - (k_1 + k_2)z_2 + \ddot{x}_d) - \hat{T}_d - \hat{U}] - k_3 \dot{z}_1$$
(17)

The control input is

(2)

$$u_{RFNR} = J[(1 - k_1^2)z_1 - (k_1 + k_2)z_2 + \ddot{x}_d] + \sigma_3 x_2 + \beta \operatorname{sgn}(s) + \hat{T}_z + \hat{T}_d + \hat{U}$$
(18)

371

where  $\hat{U}$  is estimate of uncertainty estimation error  $U(=T_d - \hat{T}_d)$ .

Let us define the Lyapunov function as follows:

$$V_4 = V_3 + \frac{1}{2}\sigma_1 \tilde{z}^2 + \frac{1}{2\eta} \tilde{U}^2$$
(19)

where  $\tilde{U} = U - \hat{U}$ . The time derivative of Eq. (19) is

$$\dot{V}_{4} = \dot{V}_{4} + s[\Phi\tilde{z} - \beta s + \tilde{U}] + \sigma_{1}\tilde{z}\dot{\bar{z}} + \frac{1}{\eta}\tilde{U}\dot{\tilde{U}}$$
$$= -k_{1}z_{1}^{2} - k_{1}z_{2}^{2} - \beta|s| - \sigma_{0}\tilde{z}^{2} - \tilde{z}\tilde{U} + \tilde{U}(s - \frac{1}{\eta}\dot{\tilde{U}})$$
(20)

The adaptive robust estimation law for the estimate of uncertainty approximation error is chosen by

$$\hat{U} = \eta \cdot s \tag{21}$$
Then Eq. (61) can be written by

$$\dot{V}_{4} = -k_{1}z_{1}^{2} - k_{1}z_{2}^{2} - \beta|\mathbf{s}| - \sigma_{0}\tilde{z}^{2} - \tilde{z}\tilde{U}$$

$$\leq -\sigma_{0}\tilde{z}^{2} - \tilde{z}\tilde{U} = -\mathbf{Z}^{T}\mathbf{M}\mathbf{Z} \leq 0$$
(22)

Since  $\dot{V}_4 \leq 0$ , then,  $z_1 \rightarrow 0$ ,  $z_2 \rightarrow 0$ ,  $s \rightarrow 0$ ,  $\tilde{z} \rightarrow 0$  and  $\tilde{U} \rightarrow 0$  as  $t \rightarrow \infty$  by Barbalat lemma <sup>3</sup>.  $\hat{T}_d$  is the output of RFNN as shown in Fig. 1.



Fig. 1 Structure of the proposed RFNN

# 3. Results of the simulation and experiments







(b) Estimation results of the friction state of BSMC+OB



(c) Estimation results of the friction state of BSMC+OB+RFNN



Fig. 3 Simulation of the BSMC+OB & BSMC+OB RFNN system: robust case

# Fig. 4 Experimental results: tracking performance

For the servo system with the dynamic friction, simulation and experimental results are presented in Fig. 2 and Fig. 3. It is shown that the proposed method has the robustness and good estimation of



÷

#### References

1. Canudas de Wit C, Olsson H, Astrom K. J., "A New Model for Control of Systems with Friction," IEEE Trans Automat Control, Vol. 40, No. 3, pp. 419-425, 1995.

2. Leu Y. G, Lee T. T, and Wang W. Y., "On-line Tuning of Fuzzyneural Networks for Adaptive Control of Nonlinear Dynamic Systems," IEEE Trans System Man Cybern., Vol. 27, No. 6, pp. 1034-1043, 1997.

3. Slotine J. E., and Li W., Applied Nonlinear Control, Prentice-Hall, 1991.