

From Theory to Implementation of a CPT-Based Probabilistic and Fuzzy Soil Classification

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ABSTRACT: This paper discusses the development of an up-to-date computerized CPT (Cone Penetration Test) based soil engineering classification system to provide geotechnical engineers with a handy tool for their daily design activities. Five CPT soil engineering classification systems are incorporated in this effort. They include the probabilistic region estimation and fuzzy classification methods, both developed by Zhang and Tumay, the Schmertmann, the Douglas and Olsen, and the Robertson et al. methods. In the probabilistic region estimation method, a conformal transformation is used to determine the soil classification index, U , from CPT cone tip resistance and friction ratio. A statistical correlation is established between U and the compositional soil type given by the Unified Soil Classification System (USCS). The soil classification index, U , provides a soil profile over depth with the probability of belonging to different soil types, which more realistically and continuously reflects the in-situ soil characterization, which includes the spatial variation of soil types. The CPT fuzzy classification on the other hand emphasizes the certainty of soil behavior. The advantage of combining these two classification methods is realized through implementing them into visual basic software with three other CPT soil classification methods for friendly use by geotechnical engineers. Three sites in Louisiana were selected for this study. For each site, CPT tests and the corresponding soil boring results were correlated. The soil classification results obtained using the probabilistic region estimation and fuzzy classification methods are cross-correlated with conventional soil classification from borings logs and three other established CPT soil classification methods.

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INTRODUCTION

During the past two decades, the cone penetration test (CPT) has gained wide popularity and acknowledgement as a preferred in situ tool for subsurface investigation and soil characterization. The CPT is a robust, simple, fast, reliable, and economical test that provides continuous sounding of subsurface sediments. The CPT test is basically conducted by advancing a cylindrical rod with a cone tip down into the soil. During penetration, the cone penetrometer simultaneously measures the cone tip resistance, q_c , and sleeve friction, f_s . When the piezocone penetration test (PCPT or CPTu) is used, the pore pressures generated during penetration can also be measured, depending on the location of the pressure transducer (at the cone face, u_1 , behind the cone base, u_2 , or behind the friction sleeve, u_3). The CPT/PCPT measurements can be effectively used in many geotechnical engineering applications, such as soil stratification and identification, and to evaluate different soil properties such as the strength and consolidation characteristics of the geomedia. This makes the CPT/PCPT technology valuable for a wide range of geotechnical engineering applications.

Due to the geometric design of the piezocone, pore water pressures generated behind the cone base (u_2) can influence the total stress measured by the cone tip. Therefore, the measured cone tip resistance (q_c) may have to be corrected for certain cone configurations to account for the effect of this pore water pressure developing behind the cone tip. Theoretically, the corrected cone tip resistance (q_t) is given by:

$$q_t = q_c + u_2 (1-a) \quad (1)$$

where $a = A_n/A_c$ is the effective area ratio of the cone, A_n = cross-sectional area of the load cell, and A_c = projected area of the cone.

However, the authors' experience in primarily Louisiana soil-types and other similar soil-types outside of Louisiana showed that using either q_c or q_t does not appreciably change the CPT-based soil classification results by utilizing methodologies depending on tip resistance and friction ratio. Therefore, tip resistance q_c has been used throughout this study for soil classification purposes.

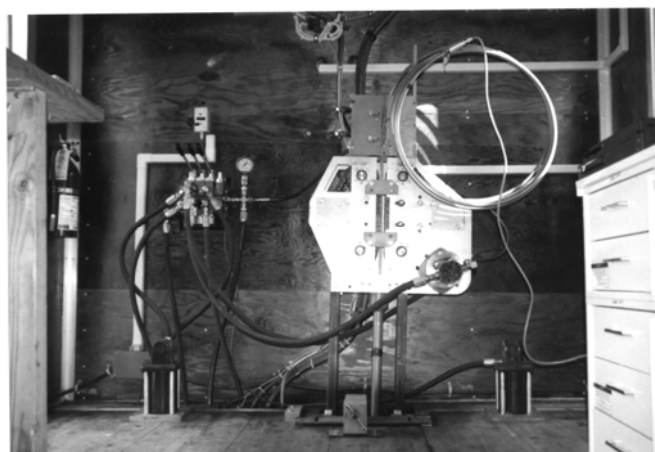
One important application of the CPT is its use in soil type identification and classification profiling. Several charts were proposed in the literature to classify the soil from the CPT (using q_c) or the PCPT (using q_t) data (e.g., Schmertmann 1978; Douglas and Olsen, 1981; Robertson et al., 1986; Robertson, 1990; Olsen and Mitchell, 1995). These charts were developed based on comparison/correlation between CPT/PCPT profiles and soil type data bases collected/evaluated from extensive soil borings. Thus the CPT soil classification depends on the physical response of the soil during cone penetration, which is directly related to the mechanical properties of the tested soils. According to Douglas and Olsen (1981), the CPT classification charts can not provide accurate prediction of soil type based on soil composition, but rather serve as a guide to the soil behavior type. The correlation between soil composition and mechanical properties is not simple, especially in transition zones of soil types, leading to probability of mis-classifying the soil type using the current CPT classification charts. To account for such probability of mis-classifying the soil, Zhang and Tumay (1999, 2000, 2003) developed a statistical-based probabilistic region estimation method to classify the soil from CPT data that



(a) Louisiana cone penetration test systems: CIMCPT on the right and REGEVITS on the left,



(b) The hydraulic segmental thrust system of REVEGITS,



(c) Continuous miniature cone/piezcone intrusion system of CIMCPT

Figure 1: CPT systems managed by LTRC

involves uncertainty in the correlation between soil composition and soil mechanical behavior. This method provides a profile of the probability or the chance of having each soil type (clayey, silty, and sandy) with depth.

Due to the soft nature of soil deposits in Louisiana, the CPT is considered a preferred tool for site characterization. The Louisiana Department of Transportation and Development (LADOTD) operates three CPT systems. These systems are Louisiana Electric Cone Penetration System, LECOPS (Tumay, 1994), Research Vehicle for Geotechnical In-situ Testing and Support, REVEGITS (Tumay, 1996, 1998), and Continuous Intrusion Miniature Cone Penetration Test system, CIMCPT, (Tumay et.al, 1998). Currently, the CIMCPT and REVEGITS are managed by the Louisiana Transportation Research Center (LTRC). Figure 1 depicts a photograph of the CIMCPT system and REVEGITS. In order to facilitate the use of CPT technology for soil classification, a Visual Basic MS-Windows program was developed in which five CPT classification methods were implemented. These methods include the probabilistic region estimation and fuzzy classification methods, both developed by Zhang and Tumay (1999), the Schmertmann (1978), the Douglas and Olsen (1981), and the Robertson et al. (1986) methods. The program (www.ltrc.lsu.edu/downloads.html) performs the analyses on the CPT soundings using the selected CPT classification method and provides the geotechnical engineers with soil classification profile with depth.

SOIL CLASSIFICATION BY CPT

Soil identification and classification of soil stratigraphy can be achieved by analyzing the CPT data. The trends in CPT soil classification indicated that sandy soils usually have high cone tip resistance and low friction ratio; soft clay soils show low cone tip resistance and high friction ratio; organic soils such as peat have very low cone tip resistance and very high friction ratio; whereas overconsolidated soils tend to produce higher cone tip resistance and higher friction ratio.

Traditional CPT classification methods provide two-dimensional charts for soil classification based either on cone tip resistance (q_c or q_t), friction ratio (R_f), and pore pressure (u), or their normalization with respect to vertical overburden stress (σ_{vo}). These charts were developed through direct correlation between the CPT data (q_c , q_t , R_f , u) and the corresponding soil type determined from soil borings of the collected database. Several CPT charts have been proposed by investigators to classify the soil utilizing the CPT data (e.g., Schmertmann 1978; Douglas and Olsen, 1981; Robertson et al., 1986; Robertson, 1990; Olsen and Mitchell, 1995). While almost all the CPT methods (basically charts) give a specific classification to each soil layer along the penetrated depth; the probabilistic region estimation method proposed by Zhang and Tumay (1999) is unique in addressing the uncertainty in mis-classifying the soil. This statistical based method provides a profile of the probability or the chance of having each soil type (clayey, silty, and sandy) with depth. This method is similar to the classic soil classification methods which are based on soil composition. The following sections will summarize the CPT methods implemented and upgraded in the new Visual Basic software.

Probabilistic Region Estimation Method

The probability of incorrectly identifying soil type using the tradition CPT classification charts, especially in transition zones, motivated the development of the probabilistic region estimation method. This CPT classification method addresses the uncertainty of correlation between the soil composition and soil mechanical behavior.

In this method, conformal mapping was performed on the Douglas and Olsen (1981) chart to transfer the chart axis from the CPT data (q_c , R_f) to the soil classification index (U). The conformal transformation is accomplished using the following equations:

$$x = 0.1539 R_f + 0.8870 \log q_c - 3.35 \quad (2)$$

$$y = -0.2957 R_f + 0.4617 \log q_c - 0.37 \quad (3)$$

The soil classification index (U) is given as:

$$U = \frac{(a_1x - a_2y + b_1)(c_1x - c_2y + d_1)}{(c_1x - c_2y + d_1)^2 + (c_2x + c_1y + d_2)^2} - \frac{(a_2x + a_1y + b_2)(c_2x + c_1y + d_2)}{(c_1x - c_2y + d_1)^2 + (c_2x + c_1y + d_2)^2} \quad (4)$$

The coefficients in equation 4 are: $a_1 = -11.345$, $a_2 = -3.795$, $b_1 = 15.202$, $b_2 = 5.085$, $c_1 = -0.296$, $c_2 = -0.759$, $d_1 = 2.960$ and $d_2 = 2.477$.

A statistical correlation was then established between the U index and the compositional soil type given by the Unified Soil Classification System (USCS). A normal distribution of U was established for each reference USCS soil type (GP, SP, SM, SC, ML, CL, and CH). Each U value corresponds to several soil types with different probabilities. Boundary values were used to divide the U axis into seven regions as described in Figure 2a. Soil types were further rearranged into three groups: sandy and gravelly soils (GP, SP, and SM), silty soils (SC and ML) and clayey soils (CL and CH). Figure 2a also gives the probability of having each soil group within each region. The original method gives constant probability of each soil type (represented by the step lines) regardless of the U value within the same region (R_1 to R_7 in Figure 2a). This will allow for the sudden drop in the probabilities as the U value crosses the border from one region to another. This method was further modified from the origin to allow smooth transition of probability (curved lines) with U values, and hence to provide a continuous profile of the probability of soil constituents with depth. An example of a U profile is presented in Figure 3, which is compared with q_c and R_f profiles and the corresponding probabilistic region estimation on data obtained from Manwell Bridge located in Evangeline, LA is presented in Figure 3.

Fuzzy Classification Method

Most of existing CPT soil classification methods are based on statistical correlation between the CPT profile data and the USCS soil classification; hence leading to soil identification according to their mechanical behavior. In contrary to other methods, the CPT fuzzy soil classification approach is fundamentally different in releasing the constraint of soil composition, and instead is based on the certainty of soil behavior (i.e., cone tip resistance and local friction).

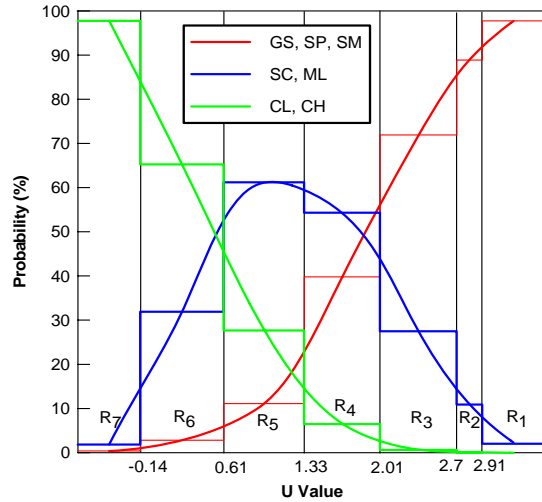


Figure 2a: Regions' boundaries and the corresponding probabilities of each soil group

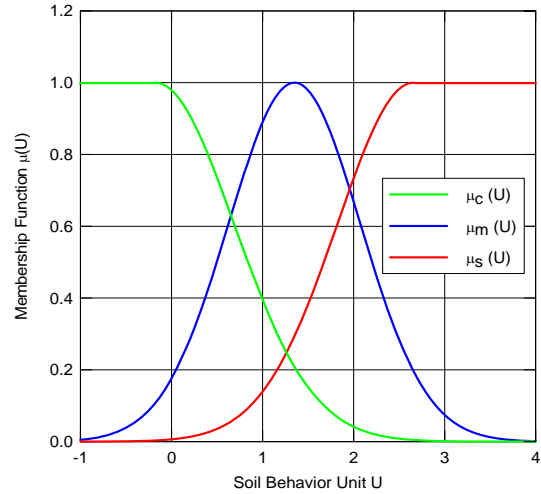


Figure 2b: CPT fuzzy soil classification chart

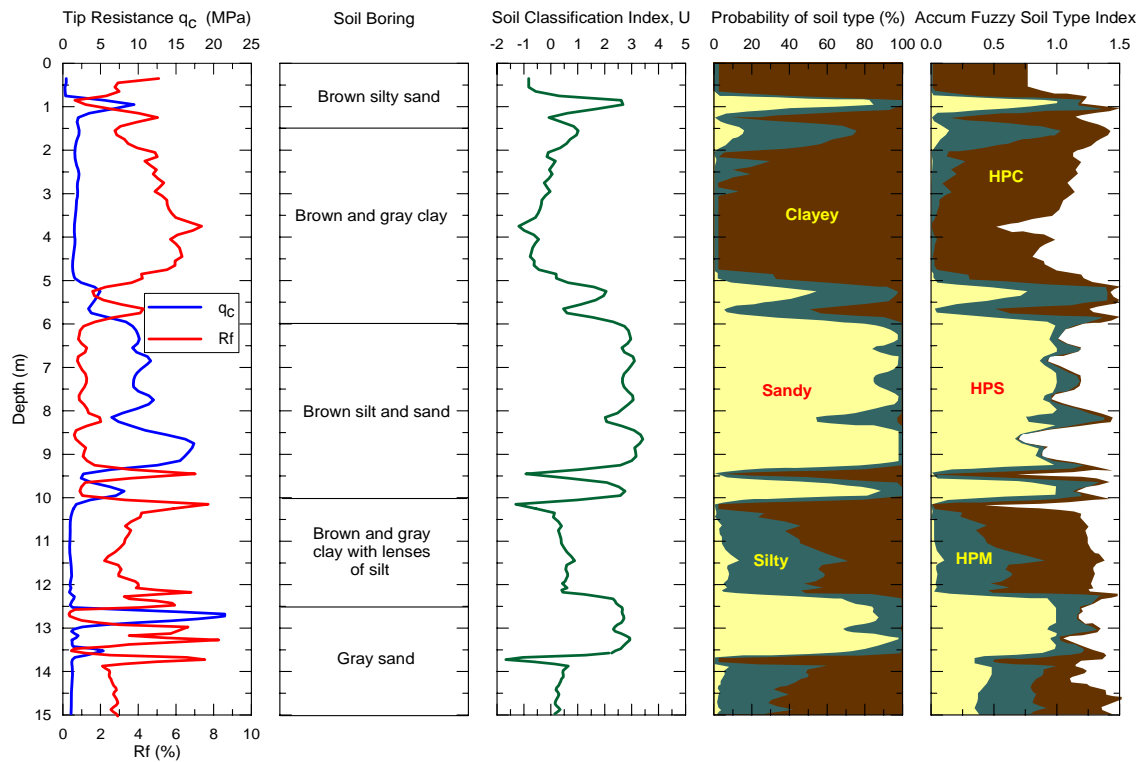


Figure 3: Probability and fuzzy soil type index profiles for Manwell Bridge, Evangeline

In CPT fuzzy soil classification, three soil types are defined: Highly Probable Clayey soil (HPC), Highly Probable Mixed soil (HPM), and Highly Probable Silty soil (HPS). The corresponding fuzzy membership functions of HPC, HPM, and HPS are given as (Zhang and Tumay, 1999):

$$\mu_c(U) = \begin{cases} \exp\left(-\frac{1}{2}\left(\frac{U + 0.1775}{0.86332}\right)^2\right) & U \geq -0.1775 \\ 1.0 & U < -0.1775 \end{cases} \quad (5)$$

$$\mu_m(U) = \exp\left(-\frac{1}{2}\left(\frac{U - 1.35}{0.724307}\right)^2\right) \quad -\infty < U < \infty \quad (6)$$

$$\mu_s(U) = \begin{cases} 1.0 & U > 2.6575 \\ \exp\left(-\frac{1}{2}\left(\frac{U - 2.6575}{0.834586}\right)^2\right) & U \leq 2.6575 \end{cases} \quad (7)$$

These empirical functions represent either “S” curve or “bell” curve types with a maximum membership value of one (1.0) for each soil type as depicted in Figure 2b. However, as seen in the figure, it is unlikely for all three membership values to have maximum values simultaneously, and that the accumulated sum depends on the U value. These empirical functions approximately relate the quantity change to quality change in soil composition and properties, reflecting an overall perspective of soil properties. The change is gradual from one soil type to another. The profile of fuzzy functions as compared to U profile and q_c and R_f profiles for Manwell Bridge, Evangeline Louisiana, are also shown in Figure 3.

Schmertmann Classification Method

The original CPT soil classification chart proposed by Schmertmann (1978) is shown in Figure 4a. Based upon CPT data taken from different sites in Louisiana, as well as CPT data taken from California, Oklahoma, Utah, Arizona and Nevada, as reported by Douglas and Olsen (1981), and comparison with soil borings, the original Schmertmann chart was modified by the first author as shown Figure 4b (Tumay, 1985). The chart depicts four distinct regions as identified by Douglas and Olsen (1981). Each region is further divided into sub-regions sorted out using Schmertmann classification modified slightly to reflect Louisiana research experience.

Douglas and Olsen Classification Method

Douglas and Olsen (1981) conducted comprehensive work correlating between the USCS soil classification and CPT data to develop a CPT-soil behavior type classification method. The development of this method was based on extensive data collected from sites in the western USA. The classification chart for the Douglas and Olsen method uses the cone tip resistance (q_c) and friction ratio (R_f) input parameters as shown in Figure 5. The chart shows the soil classification change (diagonally) from SP to SM to ML to CL to CH as the cone tip resistance decreases and friction ratio increases. Douglas and Olsen (1981) method demonstrates that the CPT classification charts can not provide an accurate prediction of soil type based on soil composition, but rather serve as a guide to soil behavior type (Lunne et al., 1997).

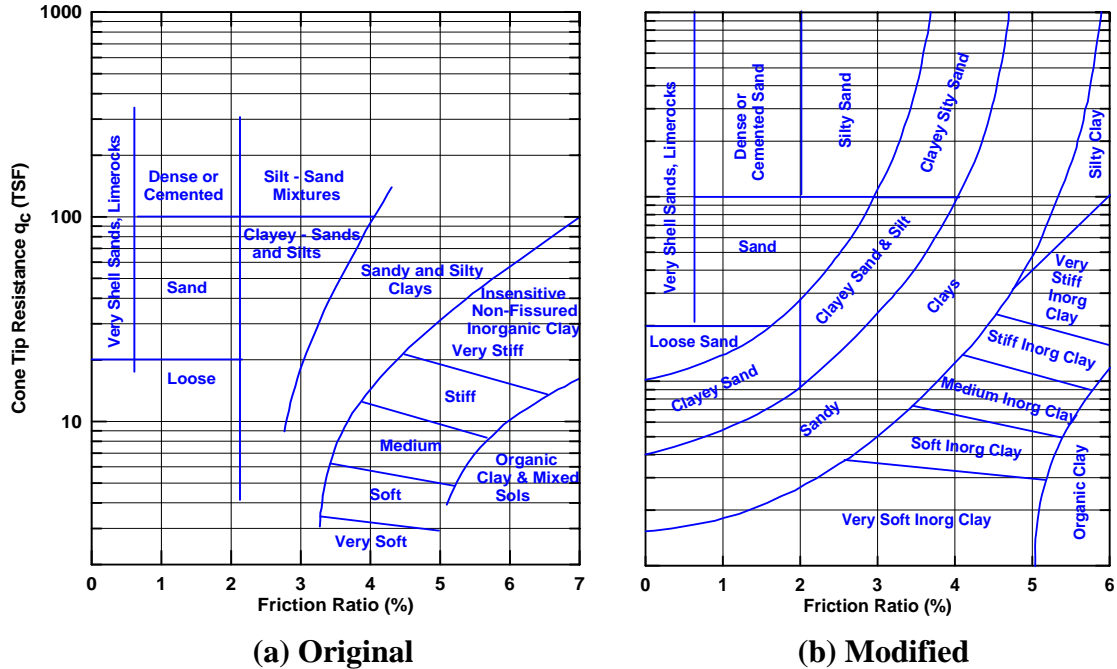


Figure 4: Original and modified Schmertmann classification charts

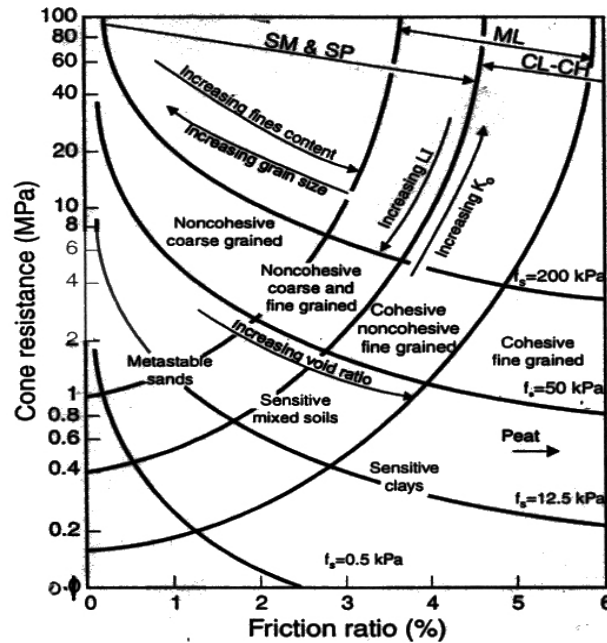
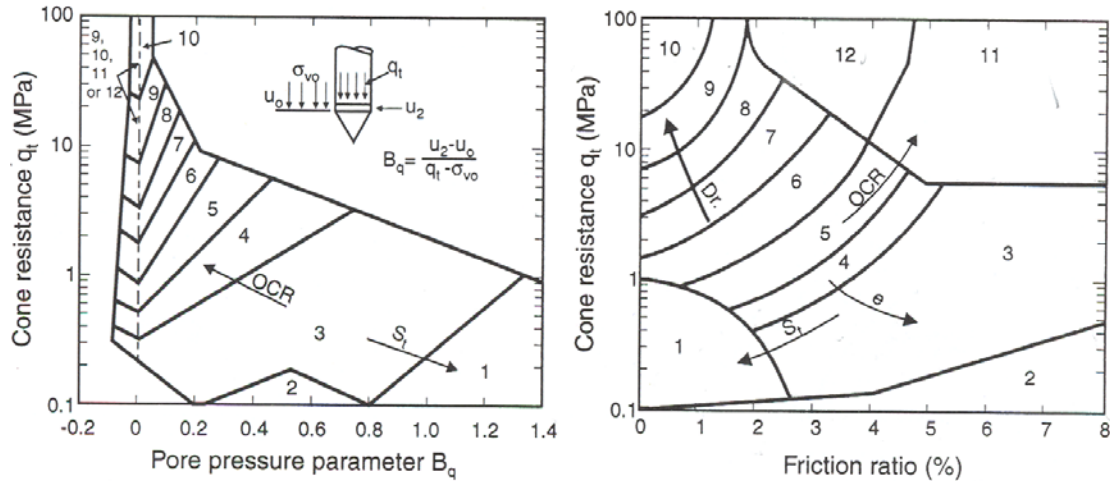


Figure 5: Douglas and Olsen (1981) soil classification chart

Robertson et al. Classification Method

Robertson et al. (1986) developed a soil behavior type classification method derived from PCPT data (q_c, f_s, u). They proposed two charts, one chart uses corrected tip resistance (q_t) and friction ratio (R_f) as input data; while the other chart uses q_t and

pore pressure parameter ($B_q = (u_2 - u_o)/(q_t - \sigma_{vo})$) as input data. They identified twelve different soil behavior types as shown in Figure 6. In case a soil falls within two different zones in respective charts, engineering judgment is required to classify the soil behavior correctly. Only the second chart was implemented in the visual basic soil classification software developed in this study.



1. Sensitive fine grained, 2. Organic material, 3. Clay, 4. Silty clay to clay, 5. Clayey silt to silty clay, 6. Sandy silt to clayey silt, 7. Silty sand to sandy silt, 8. Sand to silty sand, 9. Sand, 10. Gravelly sand to sand, 11. Very stiff fine grained, 12. Sand to clayey sand.

Figure 6: Robertson et al. (1986) soil classification charts

DEVELOPMENT OF SOIL CLASSIFICATION SOFTWARE

A visual basic soil engineering classification program, Louisiana Soil Classification by Cone Penetration Test Program (Figure 7), was developed utilizing the CPT data to provide geotechnical engineers with a user friendly methodology. Five CPT soil classification methods were implemented in this program. These include the probabilistic region estimation method and fuzzy classification method, both developed by Zhang and Tumay (1999), the Schmertmann (1978) method, the Douglas and Olsen (1981) method, and the Robertson et al. (1986) classification method. These methods use the cone tip resistance (q_c or q_t) and friction ratio (R_f) as input parameters.

The program is capable of reading CPT input data files of different units (SI, English, or millivolts raw data). Before running the program, the user can view the data file. The first step for the user is to input the project information. The program then plots the profiles of cone tip resistance, sleeve friction and friction ratio with depth. The user has the option to select the classification method and the corresponding display charts for output (graph and/or text). If the user selected a text chart for soil profile, the user can always change the layers manually. The program is available for free download from the LTRC Web site (www.ltrc.lsu.edu/downloads.html). Figure 7 describes the general features of the soil classification program.

Open Cone Data File

Open Exit About

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Enter Project Information:

Date: 1/9/2007

Project Number: 450-18-0085 Parish: Jefferson

Project Title: 110 - WILLIAMS BOULEVARD

CPT Station: 110 Offset: +00

CPT No: Will-5

Ground Elevation (ft): 17

Ground Water Table (depth - ft): 3

GPS Data Latitude: Longitude:

Comments: CPT Test for Test Pile at 110 Williamson's Interchange

Cancel OK

View Data File

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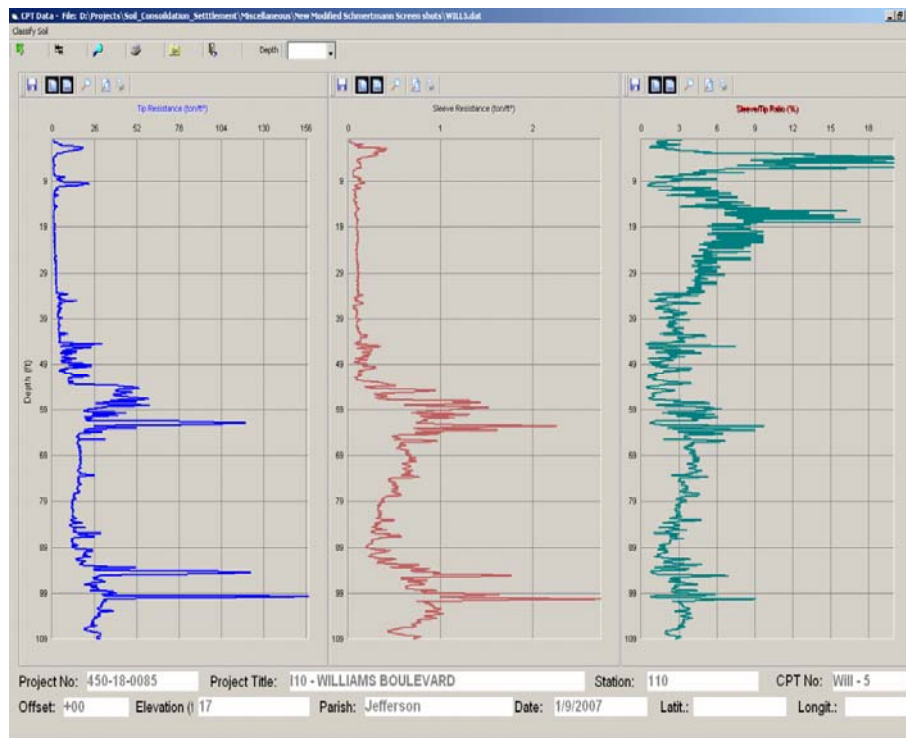
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SOIL_CPT Program

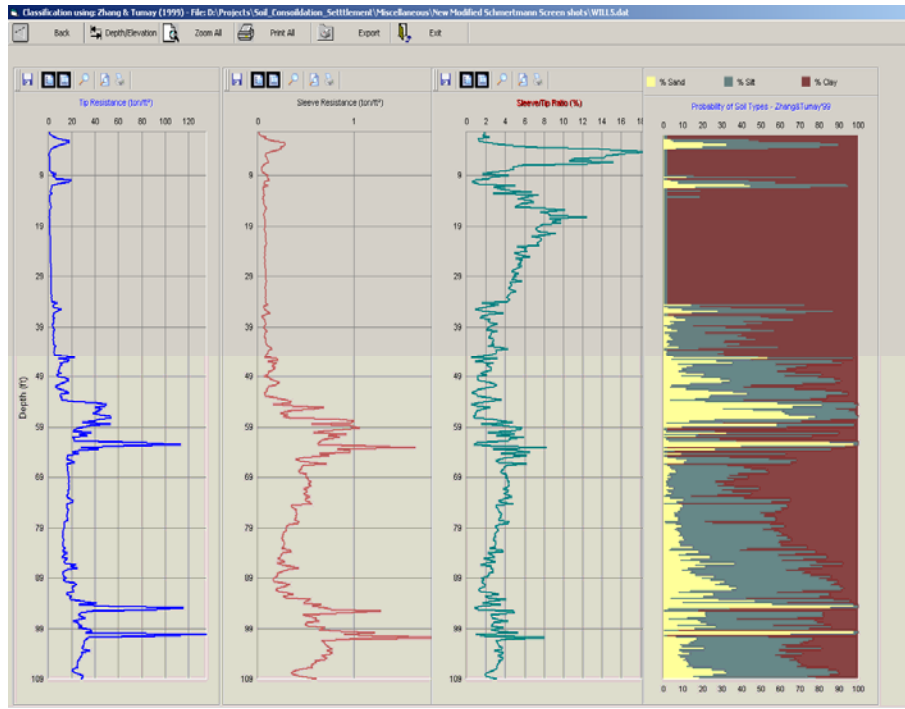
Louisiana Soil Classification by Cone Penetration Test Program

(a) Data and Information Input Screen

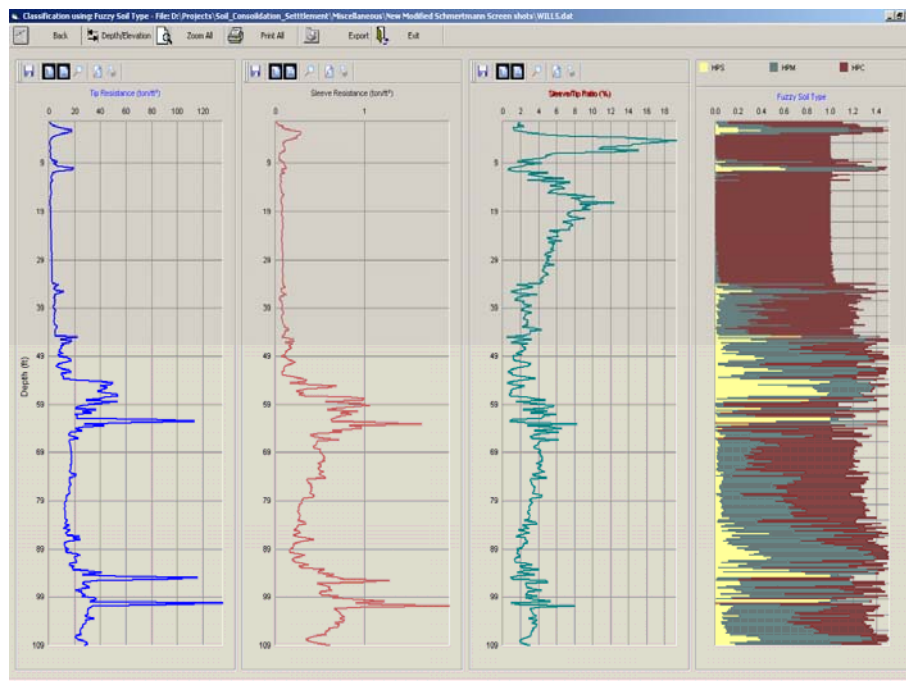


(b) CPT Profiles and Main Menu Screen

Figure 7: Features of Louisiana Soil Classification by Cone Penetration Test Program (www.ltrc.lsu.edu/downloads.html)

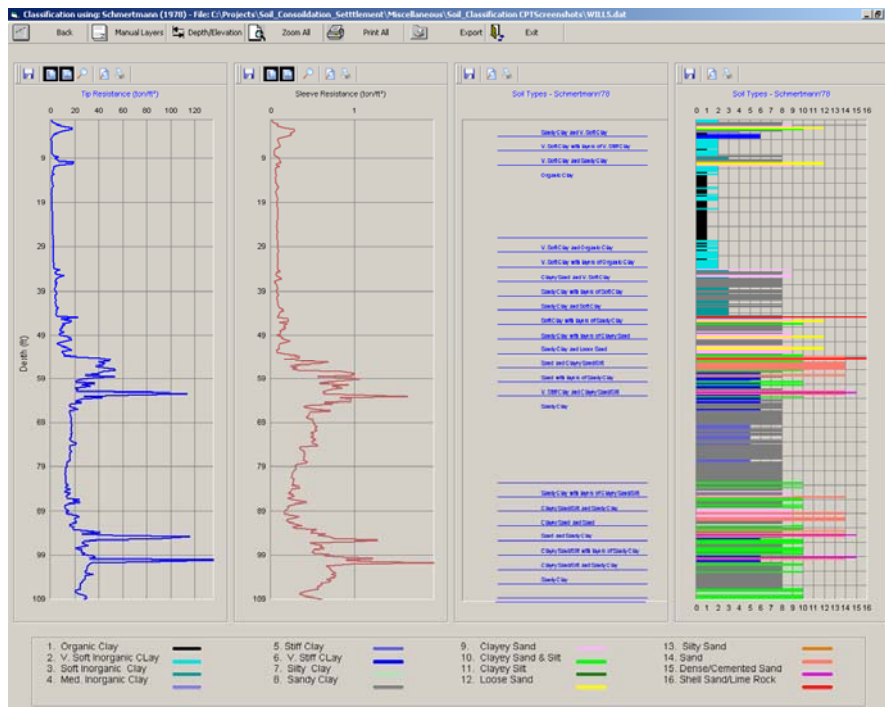


(c) Probabilistic Region Estimation Classification Method

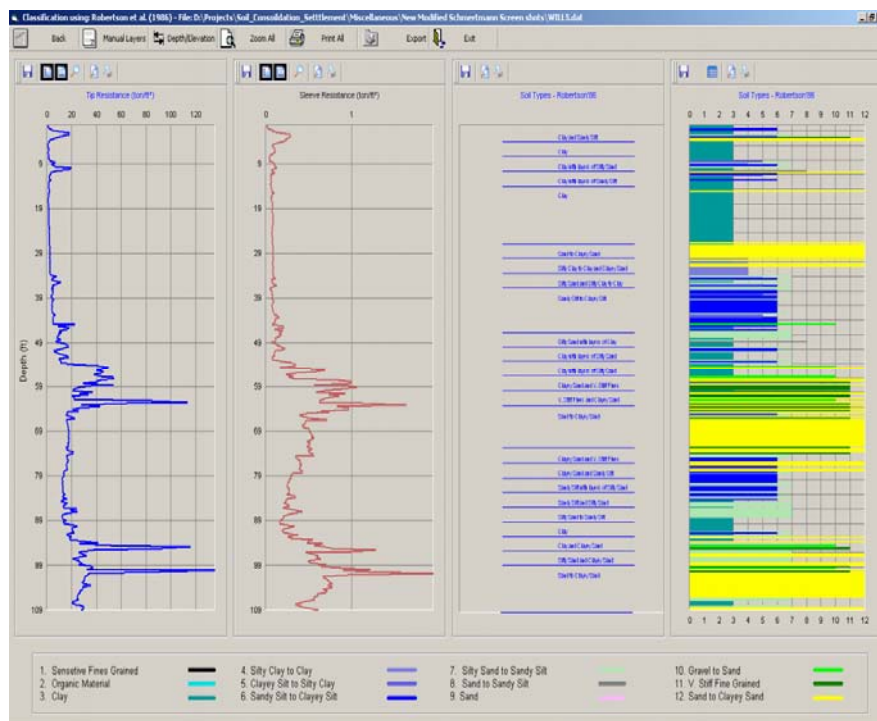


(d) Fuzzy Classification Method

Figure 7: Features of Louisiana Soil Classification by Cone Penetration Test Program (cont.) (www.ltrc.lsu.edu/downloads.html)



(e) Schmertmann Classification Method



(f) Robertson et al. Classification Method

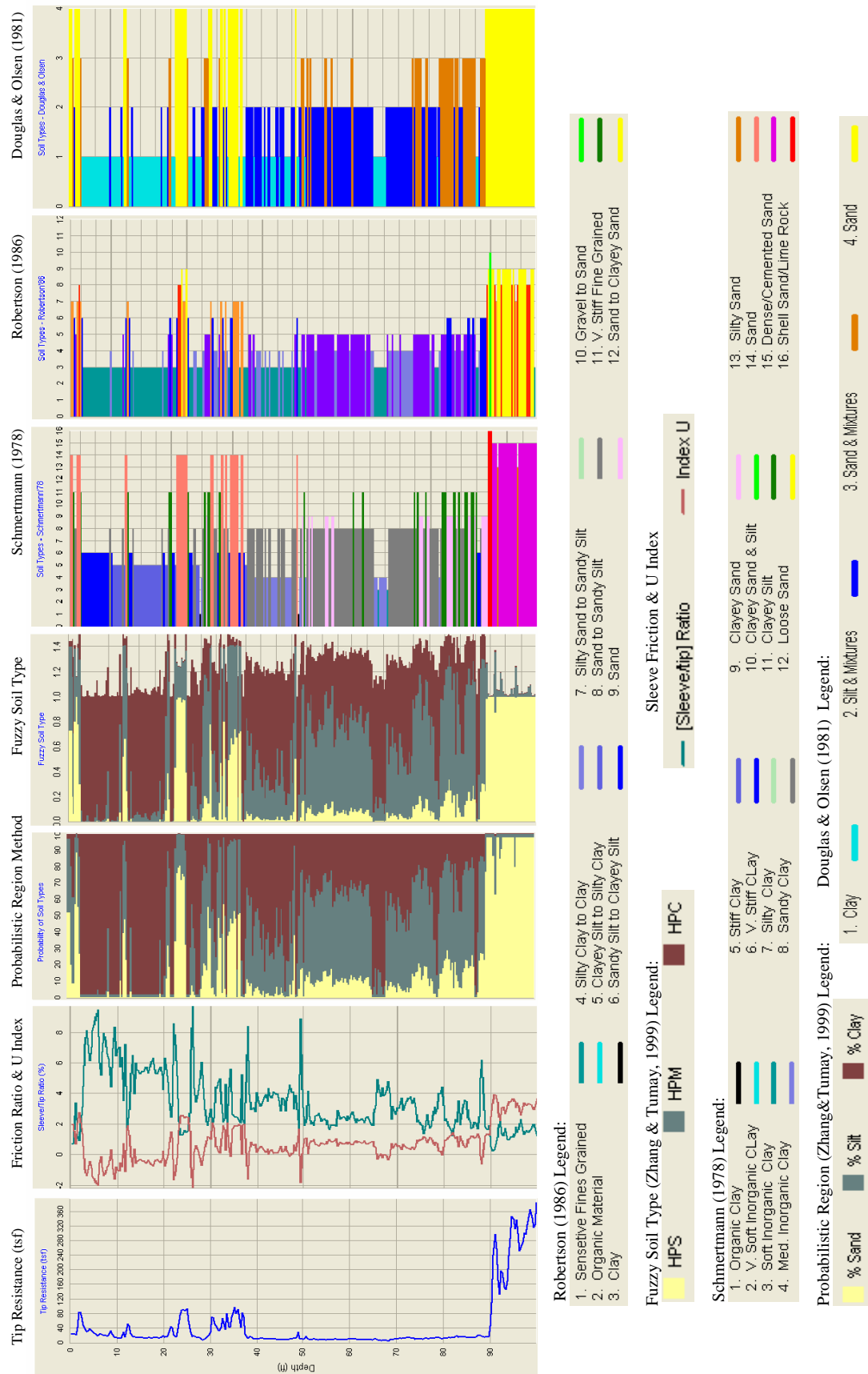
Figure 7: Features of Louisiana Soil Classification by Cone Penetration Test Program (cont.) (www.ltrc.lsu.edu/downloads.html)

COMPARISON OF CPT CLASSIFICATION METHODS

Three sites in Louisiana were selected to compare the soil classification obtained using the probabilistic region estimation method and the fuzzy classification method with those obtained from other established CPT classification charts. In each site, cone penetration tests (CPT) were performed around drilled boreholes using a 10 cm² cone penetrometer. The borings in the Jennings site showed that the soil profile consists of about 20 feet of brown to gray medium to stiff clay and silty clay soil, followed by brown fine silty sand from 20 feet to 38 feet. Underneath, there is a layer of gray soft to medium silty clay extended from 38 feet to about 90 feet followed by a gray medium sand to the maximum depth of boring. Figure 8 presents the CPT results and the comparison of soil classifications obtained using the different CPT classification methods. The soil profile at the New Iberia site consists of soft to medium tan and gray silty clay soils down to 52 feet, silt and clayey silt from 52 feet to 62 feet, and silty clay soil interbedded with terraces of silts and sand from 62 feet to the maximum depth of boring. The CPT results and the corresponding soil classifications obtained using the different CPT classification methods for New Iberia site are presented in Figure 9. The soil boring at the Jefferson site indicates that the soil deposit consists of silty clay soils down to 50 ft, silty sand and sandy soils from 50 feet to 65 feet interbedded with lenses of silty clay soil, and medium to stiff silty clay soil interbedded with thin layers of silts and sands from 65 feet to the maximum depth of boring. Figure 10 presents the CPT results and compares the soil classifications for the Jefferson site obtained using the different CPT classification methods. The comparisons demonstrate that the probabilistic region estimation method and the fuzzy CPT classification method are superior in predicting, with “continuous and detailed” accuracy, the soil-type profile with depth.

SUMMARY AND CONCLUSIONS

This paper describes the implementation and general features of five CPT soil engineering classification methods in a visual basic program (Louisiana Soil Classification by Cone Penetration Test, LSC-CPT) for friendly use by geotechnical engineers in their daily activities. These include the probabilistic region estimation method and fuzzy classification method, both developed by Zhang and Tumay (1999), the Schmertmann (1978) method, the Douglas and Olsen (1981) method, and the Robertson et al. (1986) classification method. In Zhang and Tumay’s two methods, a soil classification index, *U*, is determined and used to provide a continuous soil classification profile with gradual changes from one contiguous subsurface layer to another. The advantage of these two classification methods is demonstrated through comparison with soil borings in conjunction with three other established CPT classification charts. Three sites in Louisiana were selected for this comparison, which showed that the probabilistic region estimation and fuzzy CPT classification methods are capable of predicting, with good accuracy, continuous soil classification profile, including information on the probability of soil constituents in the layers encountered.



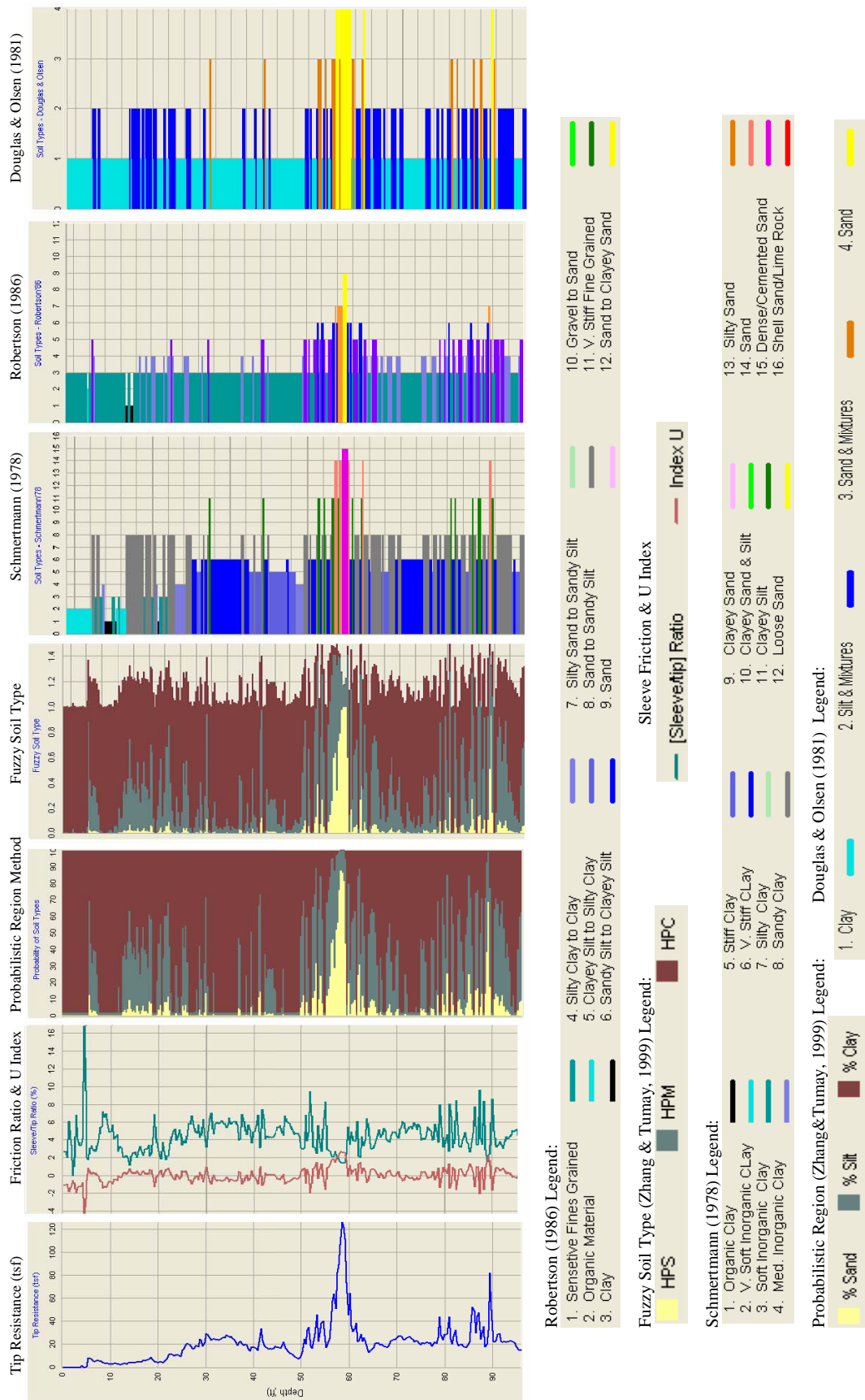


Figure 9: CPT data and corresponding CPT soil classification for New Iberia Site

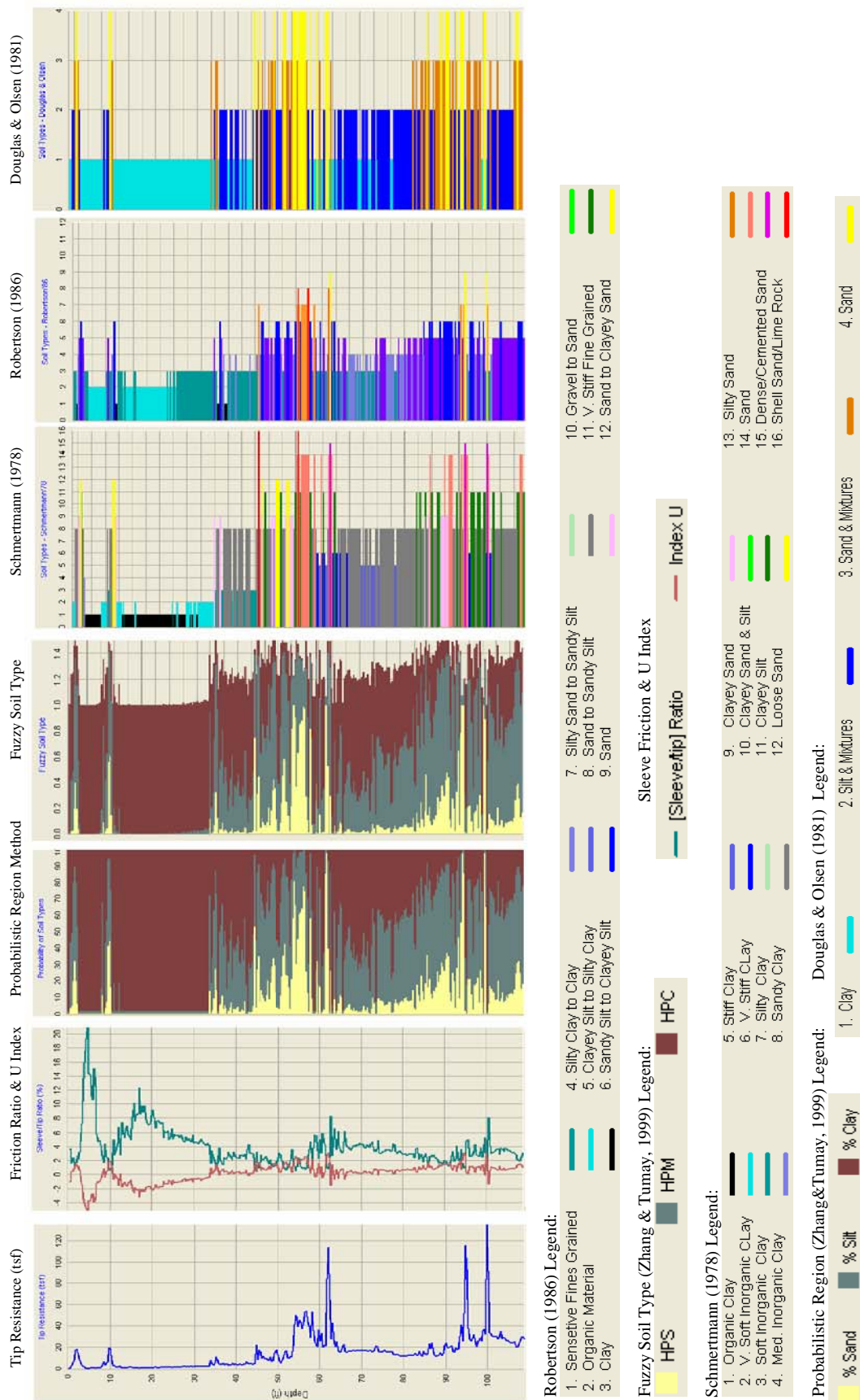


Figure 10: CPT data and corresponding CPT soil classification for Williams-Jefferson Site

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This paper can best be viewed in color. For a color pdf copy, please send an e-mail request to the first author.

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