Design and comparative study of various Two-Dimensional Grain Configurations based on Optimization Method

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Abstract

Grain design has always been a vital and integral part of Solid Rocket Motor (SRM) design. Basing on the design objectives set by the system designer, the SRM designer has many options available for selecting the Grain configuration. Many of the available configurations may fulfill the required parameters of volumetric loading fraction, web fraction & Length to diameter ratios and produce internal ballistic results that may be in accordance to the design objectives. However, for any given set of design objectives, it is deemed necessary that best possible configuration be selected, designed and optimized. Hence optimal results of all applicable configurations are vital to be attained in order to compare and finalize the design that will produce most efficient performance.

Generally the engineers pay attention and have skills on a specific grain configuration. The designing methodologies and computer codes available usually focus on single grain configuration may it be Star, Wagon Wheel or slotted tube. Hardly one can find a software or a design methodology where all such configurations can be worked on jointly and not only adequate designs be found but optimal solutions reached by applying an optimization method to find final design best suited for any design objective.

In the present work design requirements have been set, grain configurations have been selected and their designing has been conducted. The internal ballistic parameters have been calculated and after finding the preliminary design solutions, the optimal solutions have been found.

In doing so, software has been developed comprising of computer programs for designing the 2D grains including Star, Wagon Wheel and Slotted Tube configurations.

The optimization toolbox of Matlab Fmincon has been used for getting optimal solutions. The affects of all the independent geometric design variables on the optimized solutions have been analyzed.

Based on results attained from Optimization Method, an in depth comparison of Grain Configurations and analysis of performance prediction outputs have been conducted to come to conclusion as to which grain configuration is ideal for the current design requirement under study.

Introduction

Grain design is essentially a geometrical constraint depending upon the Thrust~Time & Pressure ~ Time laws¹⁾. It is most imperative in completing the design of any Solid Rocket Motor. The essence is to evolve the burning surface and develop relation between web burnt and the burning surface²⁾. Design analysis of these grain configurations involving various independent geometric variables that define the different geometries has been conducted in detail. Considering large and limitless number of combinations of these variables that satisfy the vital requirements of thrust, pressure, mass of propellant and burning time in addition to basic needs of volumetric loading fraction and web fraction, sound approach has been found to ensure that the final design of each type of grain configuration is the optimal one based on performance.

Optimization of the same has been conducted by keeping thrust as the Objective function, burning time and propellant mass as major constraints and by defining upper and lower bounds for the independent design variables. Programs written during the course of studies are modifiable, user friendly and all the calculations have been carried out logically and sequentially.

The analysis applied in this paper identifies, for a given fixed length and outer diameter of the grain while remaining within constraints of burning time and mass of a particular propellant, the most efficient grain geometric configuration amongst those mentioned that delivers best possible internal ballistic design parameters.

This has been achieved by analyzing the effects of all the independent variables on the port area, burning surface, perimeter, sliver area, web fraction, volumetric loading fraction, neutrality, burning rate, thrust coefficient that in turn have been used to find chamber pressure and thrust for each grain configuration.

The various steps incorporated for completing the study are enumerated below:

- a. Evaluation of system requirements and design objectives.
- b. Geometric parameters of Star Grain³⁾, Wagon Wheel Grain and Slotted Tube Grain.
- c. Performance prediction.

- d. Design analysis of these grain configurations involving various independent geometric variables.
- e. Comparison of all optimal designs, analysis of performance prediction outputs and evaluation of best suited grain configuration for the current design requirements.

System requirements and design objectives

Requirements have been given for a given fixed length and outer diameter of the grain while remaining within constraints of burning time t_b and mass of propellant m_p . Maximum average thrust F_{av} has to be calculated fulfilling below mentioned design objectives. Moreover the propellant and nozzle parameters have also been dictated and have to be adhered.

Design objective

a.	Grain Length :	3700 mm
b.	Mass of Propellant :	350± 10 kg
c.	Burning time :	4.5 ± 1 sec.
d.	Average Thrust :	>240 kN
e.	Total Impulse :	<u>≥</u> 800 kNs
f.	Grain Outer Radius :	147 mm

Propellant parameters

Considering, the mechanical properties and internal ballistics required, the following propellant parameters have been selected:

a.	Propellant :	HTPB/AP/AL
b.	Density :	1.74 g/cm^3
c.	C*:	1554

Nozzle parameters

a.	Exit diameter:	280mm
a.	Exit diameter:	280mm

Geometric Parameters

Star Grain

Star grain being well known for its simplicity, reliability, neutral burning and efficiency has been graded amongst the best and widely used grain configurations²). Certain disadvantages like higher sliver ratios and undesirable tail offs are objectionable and disadvantageous but affects of the same can be reduced by tapering and filleting the star points³.A star perforated grain has been defined in Fig 1.

a.	Number of star points	N
b.	Web thickness	е
c.	Grain outer radius	$\frac{D}{2}$
d.	Fillet radius	r
e.	Cusp Radius	r_1
f.	Star angle coefficient	$\zeta = \frac{\varepsilon \pi}{n}$
g.	Valley angle	θ / 2

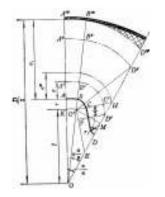


Fig 1. Star Perforated Grain

Wagon Wheel Grain

This configuration is best suited for motors requiring large burning surface areas. Wagon wheel design encompasses different webs along spokes that can be useful for dual thrust considerations. Likewise low volumetric loading fractions and web fractions are achievable with this configuration. Ballistic design analysis of the wagon wheel grain configuration involves parametric evaluation of the six independent geometric variables that defines the wagon wheel^{4),5)}. A Wagon Wheel perforated grain has been defined in Fig 2.

a.	Number of spokes	N
b.	Web thickness	e
c.	Grain outer radius	D/2
d.	Fillet radius	r
e.	Ray Side	h
f.	Wagon Wheel angle	$\zeta = \frac{\varepsilon \pi}{n}$
g.	Valley angle	a/2

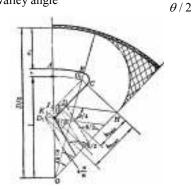


Fig. 2 Wagon Wheel perforated grain

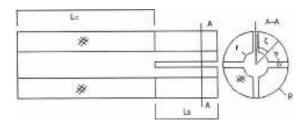
Slotted Tube Grain

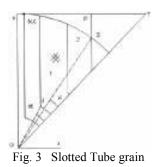
The Slotted Tube configuration consists of a cylindrical tube of propellant into which a number of slots have been cast. In tubular grain, to make A_b constant, the grain must be slotted⁴). In case of tube or cylindrical grains, for adjusting large values of burning area, long lengths of the grain may have to be incorporated that may not be feasible, hence slots can be put across the grain or in cross sections⁵). These slots connect the inner and outer surfaces of the tube and extend part of its length.

The configuration offers some significant advantages for the designers, the most obvious is its inherent lack of sliver in a non erosive situation since basically it is an internal burning cylinder. Being a very simple configuration, it remains free of stress concentration which is not so in case of Star and Wagon Wheel designs being more complicated^{6),7)}. Another feature of Slotted Tube grain designs is that they can contain thick webs, in some propellants the stresses arising from casting and curing of such webs are relieved by cracking although there are no stress concentration points. Moreover Slotted Tube grains, being relatively simple in design, require mandrels for casting propellant which are less costly and easy to be machined.

A discrete disadvantage of Slotted Tube grain design is in the exposure of the motor case in the slot region to hot, high velocity combustion gases that calls for an efficient insulation and liner. A slotted tube perforated grain has been defined in Fig 3.

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a.	Grain outer radius	R=D/2
b.	Cylindrical Radius	r_0
c.	Angular fraction	$\zeta = \varepsilon (\Pi/N)$
d.	Slot valley angle	$\eta = \Pi / N$
e.	Number of slot points	N
f.	Web thickness	e
g.	Half width of slot ,SW	Х
h.	Length of slot	LSLOT
i.	Length of cylinder	LCYL





Performance prediction

The ability to accurately predict the performance of SRMs has obvious importance in their designing. The main outputs of the performance predictions are the P_{av} and P_{max} . Performance requirements that satisfy the mission objectives are given in terms of impulse and time or thrust level. Therefore typical specification will include I_t , I_s , F_{av} , F_{max} and t_b .

Definition of the ballistic performance parameters is related to thrust - time and pressure - time curves for the motor. Other important parameters that influence the performance and have been calculated in this study are $A_{pb} A_{bb} A_{p}/A_{b} m_{p}$, BR, n, Cf, C*, $d_{b} A_{b} d_{e}$, A_{e} , V_{p} , V_{c}^{8} .

Optimization

The optimization toolbox of Matlab Fmincon for local optimization utilizing exact analysis has been used in which Fmincon attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. This is generally referred to as constrained nonlinear optimization or nonlinear programming. Medium-Scale Optimization has been used in which Fmincon uses a sequential quadratic programming (SQP) method. In this method, the function solves a quadratic programming (QP) subproblem at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration so a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function, is calculated. After choosing the direction of the search, the optimization function uses a line search procedure to determine how far to move.

Thurst has been computed as the objective function with constraints of burning time and mass of propellant. This has been done by keeping the diameter and length of grain fixed. Upper and lower limits have been defined for independent geometric variables and an initial guess provided.

Hence all the Burning Zones for Star Grain, Wagon Wheel Grain and Slotted Tube Grain have been calculated and then optimized. Value of Maximum attainable F_{av} has been calculated through this optimization method. m_p and t_b have been kept within given design ranges. Parametric study of all the independent design variables has also been conducted to analyze their affects of performance parameters during designing and optimization.

Results and Analysis

Results of Star Grain³⁾

Preliminary and optimal results of Star grain design are depicted in table 1, table 2 and Fig. 4.

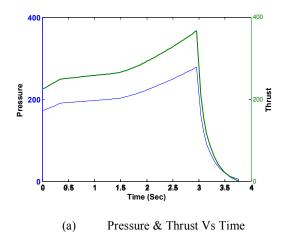
Table 1 Preliminary Design Results

Tuolo T Tremmary Design Resalts				
N	6	7	7	
e(mm)	70	63.21	63.21	
ε	0.8	0.99	0.9	
θ / 2 (deg)	30	35.56	40	
r(mm)	10.29	11.76	11.76	
$r_1(mm)$	13.32	11.76	11.76	
F_{av} (KN)	241.9	255.4	251.3	
$m_p(kg)$	351.7	346	340	
t_b (sec)	3.99	3.79	3.68	

S_{f}	3.48	4.6	3.35
S(m)	0.685	0.707	0.7
Burn rate mm/sec	20.28	20.57	20.48
$d_t(mm)$	101	101	101
I_t (kNsec)	966	968	926.7
P _{av} (MPa)	18.26	19.24	18.9
W_{f}	0.47	0.43	0.43
V_l	0.8	0.79	0.76
$d_{e}(m)$	0.28	0.28	0.28
$A e (m^2)$	0.06	0.06	0.06

Table 2 Optimal Design Results

Ν	8	$d_t(mm)$	101
e(mm)	65.76	$A_t(m^2)$	0.006
ε	0.99	I_t (kNsec)	971.9
θ / 2 (deg)	39.87	P _{av} (MPa)	19.44
r(mm)	7.35	W_{f}	0.447
r ₁ (mm)	7.35	V _l	0.8
F_{av} (KN)	258	$d_{e}(m)$	0.28
$m_p(kg)$	351.2	$A e (m^2)$	0.06
t _b (sec)	5.76	Burn rate mm/sec	20.72
S_f	3.55	$V_p (m^3)$	0.2
C_{f}	1.63	п	0.35
Neutrality	1.429	$V_c(m^3)$	0.251



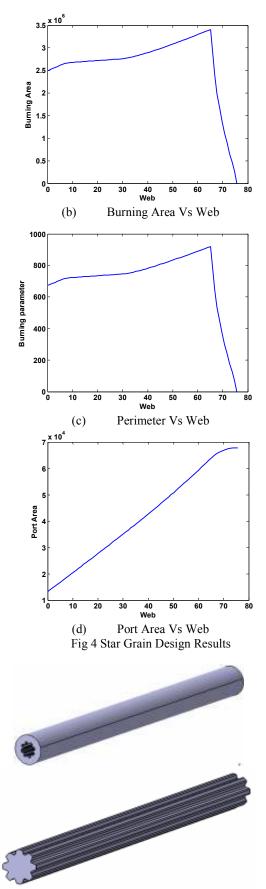


Fig 5 Optimized Star Grain

Analysis of Star Grain Results

With the diameter of the motor fixed, the star grain geometry is totally dependent upon six independent variables i.e. ε , N, θ , e, r and r_1 . Each of these variables has a bearing on explicit characteristic of star optimization. Effects of these six variables have also to be visualized while considering V_l , w_f , neutrality and S_{f} . r and r_1 are required to be kept as low as possible. F_{av} , V_l neutrality and S_f are not as sensitive to changes in r_1 as to changes in r. By increasing N, the perimeter and thereafter the burning surface area also increase and also F_{av} . Considering values of ε and $\theta/2$, the effects of N on neutrality and S_f are prominent. $\theta/2$ also affects the neutrality, S_f , m_p and F_{av} . By increasing the value of ε , the V_l and m_p can be increased and similarly F_{av} can be increased but with it S_f will also increase. The grain designer always prefers more e while remaining within desired limits of w_f and V_l . By increasing e the w_f and V_l will increase but this will also increase the m_p that may go out of the constrained range.

An analysis of a suitable range of ε , N, θ , e, r, r_1 satisfying the requirements of maximum F_{av} and ensuring sound values of w_f , V_{l_1} , S_f & neutrality while remaining within the design constraints of $m_p \& t_b$ has ensured an overall optimal design and value of optimized objective function F_{av} has been increased to maximum compared to the values attained during preliminary designs.

Results of Wagon Wheel grain

Preliminary and optimal results of Wagon Wheel grain design are depicted in table 3, table 4 and Fig. 6.

Table 3 Preliminary Design Results- Wagon Wheel

	5	0	0
N	7	7	7
e(mm)	30	25	26
ε	0.89	0.91	0.9
θ / 2 (deg)	35	34	35
r(mm)	6.8	7.2	7.3
h mm)	17	18	20
F_{AV} (Kn)	248	244.2	258.3
m_p (kg)	340	343.9	343.3
t_b (sec)	5.32	5.28	5.26
C_{f}	1.68	1.68	1.68
Neutrality	1.57	1.61	1.57
BR mm/sec	13.79	13.62	13.71
$d_t(mm)$	88.5	88.5	88.5
$A_t(m^2)$	0.006	0.006	0.006
I_t (kNsec)	1322	1290	1312
$P_{AV}(Mpa)$	24.5	24.1	24.6
W_f	0.4	0.37	0.37
V_l	0.77	0.78	0.78
$d_e(m)$	0.28	0.28	0.28

e 4 Optimized Design Results- Wagon Wheel				
	N	7	$d_t(mm)$	88.5
e(n	nm)	28	$A_t(m^2)$.0056
	Е	0.917	I_t (kNsec)	1367
$\theta/2$	(deg)	35.13	$P_{AV}(Mpa)$	25.3
r(n	nm)	7.54	W_f	0.37
h n	nm)	20.72	V_l	0.8
F_{AV} ((Kn)	256.6	$d_e(m)$	0.28
m_p (k	kg)	354	$A_e(m^2)$	0.06
t_b (sec)	5.33	$A_b(m^2)$	5.28
	S_{f}	1.76	R (mm)	147
(C_F	1.68	п	0.325
Neut	rality	1.64	BRmm/sec	13.88

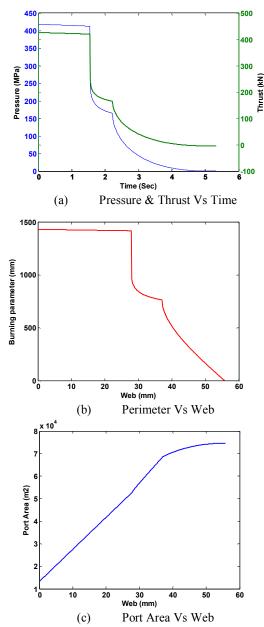


Table 4 Opt	imized Design	Results-	Wagon	Whee
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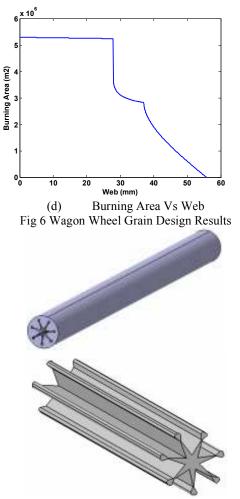


Fig 7 Optimized Wagon wheel Grain

Analysis of Wagon Wheel Grain Results

The wagon wheel grain geometry is dependent upon six independent variables i.e. ε , *N*, θ , *e*, *r* and *h*

By increasing N, the perimeter and thereafter the burning surface area also increase and also F_{av} . By decreasing $\theta/2$, the F_{av} , A_b , m_p and V_l increase correspondingly while S_f decreases. By increasing the value of ε , the V_l and m_p can be increased and similarly F_{av} can be increased but with it S_f will also increase. . r is required to be kept low. The grain designer always prefers more e while remaining within desired limits of w_f and V_l . By increasing e the w_f and V_l will increase but this will also increase the m_p that may go out of the constrained range. By increasing h, value of F_{av} , $m_p \& V_l$ increases and S_f decreases while neutrality is negatively affected.

Due to these attributes of wagon wheel, compromises will result.

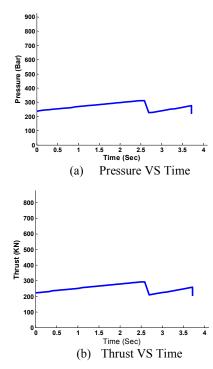
An analysis of a suitable range of ε , N, θ , e, r, h satisfying the requirements of maximum F_{av} and ensuring sound values of w_{f} , V_{l} , S_{f} & neutrality while remaining within the design constraints of m_{p} & t_{b} has ensured an overall optimal design and value of optimized objective function F_{av} has been increased to maximum compared to the values attained during

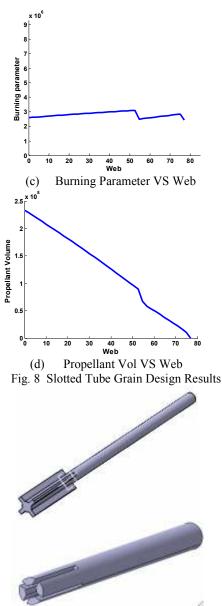
preliminary designs. This has been attained while remaining within the set design constraints.

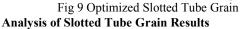
Results of Slotted Tube grain

Preliminary and optimal results of Slotted Tube grain design are depicted in table 5 and Fig. 8. Preliminary results for three different slotted tube configurations have been listed for analysis and comparison. Thrust and Pressure time histories have been shown along with burning parameters, propellant volume decay and web being burnt.

Table 5 Design Results – Stotleu Tube								
PR	OPTIMIZED							
	RESULTS							
N	4	4	4	4				
r (mm)	65	70	69	69.97				
ε	.8	.95	.89	0.99				
X (mm)	50	50	51	50.7				
LSLOT(mm)	450	925	875	925				
LCYL(mm)	3250	2775	2825	2775				
$F_{av}(KN)$	242	246	247	251.8				
$m_P(kg)$	349	348	348	359				
$t_b(sec)$	4.03	3.74	3.79	3.72				
BR(mm/sec)	19.85	19.96	20	20				
$d_t(mm)$	85	85	85	85				
I_t (kNsec)	980	922	937	937				
$P_{av}(MPa)$	26	26.2	26	26.8				
W_f	.55	.52	.53	0.52				
V_L	.80	.79	.79	0.80				
$d_e(m)$.28	.28	.28	.28				
$A_e(m^2)$.0057	.0057	.0057	.0056				
N	.34	.34	.34	.34				







The Slotted Tube grain geometry is dependent upon six independent variables i.e. *e*, *N*, *r*, *LSLOT*,*LCYL*, *X*.

Effects of these variables have been visualized while considering V_i , w_f and neutrality. By increasing the value of ε , the V_i and m_p has been increased and with it F_{av} has increased. By increasing N, the perimeter and thereafter the burning surface area also increases and also F_{av} but value of m_p gets out of the design constraint. Considering values of ε , the effects of N on neutrality are prominent. An increase in value of r increases F_{av} and decreases m_p that may get out of the constrained limits. Better neutrality is attainable at higher values of r but a compromise is required to be made between better neutrality and m_p while trying to achieve maximum F_{av} . Usually $L_S < 1/3 L_{CYL}$, F_{av} increases slightly by increasing the *LSLOT* but with it m_p also increases while better neutrality is attained

likewise by increasing value of X, F_{av} decreases slightly and neutrality improves.

An analysis of a suitable range of ε , N, r, LSLOT,LCYL, X satisfying the requirements of maximum F_{av} and ensuring sound values of w_{β} V_l & neutrality while remaining within the design constraints of m_p & t_b has ensured overall optimal design and value of optimized objective function F_{av} has been increased to maximum compared to the values attained during preliminary designs. Like Star and Wagon Wheel designs, this has also been achieved while adhering to all the set constraints.

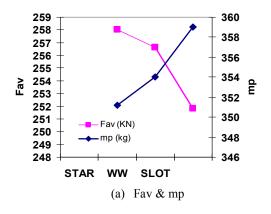
Comparison and Analysis of Optimal Designs

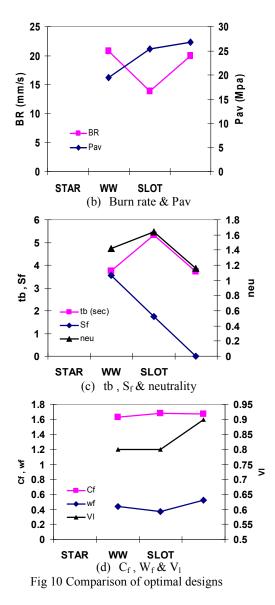
Comparison

Based on results attained from Optimization Method, an in depth comparison of Grain Configurations and analysis of performance prediction outputs has been conducted to come to conclusion as to which grain configuration is ideal for the current design requirement under study. Hence for a given fixed length and outer diameter of the grain while remaining within constraints of t_b and m_p , the most efficient grain geometric configuration that delivers best internal ballistic parameters has been identified.

Table 6 Comparison of optimal designs

	e eempa	ibon er epui	
	STAR	WAGON	SLOTTED
		WHEEL	TUBE
F_{av} (KN)	258	256.6	251.8
m_p (kg)	351.2	354	359
t_b (sec)	3.76	5.33	3.72
S_{f}	3.55	1.76	0
BRmm/sec	20.72	13.88	20
C_{f}	1.63	1.68	1.67
$P_{av}(MPa)$	19.44	25.3	26.8
W_f	0.44	0.37	0.52
V_l	0.8	0.8	0.9
пеи	1.42	1.64	1.16





Analysis

With the Grain length and diameter fixed, required F_{av} is >240 KN, m_p is 350± 10 kg & the desired limit of t_b is 4.5 ± 1 sec. All the above three grain configurations are fulfilling requirements of F_{av} , I_t , P_{av} , S_f , V_t , w_f , neutrality & *BR* while remaining within the required limits of t_b and m_p .

 F_{av} , m_p and t_b It is always desirable to attain required thrust in least t_b as practically it gives leverage to the system designer to adjust separation timings and thrust termination if required but typically from SRM design point of view, it is just important that this parameter be kept within the desired design limits. Least possible m_p is also appreciated as it reduces the overall system weight and size. From table 6 and figure 10(a) and (c), it is evident that max F_{av} is being achieved by Star configuration where average thrust of 258 kN is being attained in 3.76 sec and by utilizing 351.2 kg propellant, wagon wheel geometry also delivers 256.6 kN but in 5.33 sec and by using 354 kg whereas slotted tube configuration can give 251.8 kN in least time of 3.72 sec but requires maximum m_p i.e. 359 kg. So for the current objective requirements, Star geometry is delivering maximum thrust utilizing least m_p in relatively fine t_b . Hence considering F_{av} , m_p and t_b the Star grain geometry is finest of all the three configurations under study.

Sliver While designing any SRM grain, it is deemed necessary that the sliver should be kept low, because useful energy cannot be attained by the left over propellant at the end of burning. Considering the above three grain configurations from figure 10 (c), Slotted Tube geometrical grain operates with no sliver as it's slotted portion diminishes earlier and at the last zone of burning only tubular grain is left that is fully consumed during combustion. Maximum sliver is left in the star configuration and slightly lesser amount is left in wagon wheel grain. So comparing the sliver fraction, Slotted tube proves to be better between the three grain configurations under current situation.

Neutrality The neutrality usually refers to flatness of the thrust-time curve and can also be defined as ratio of maximum pressure and the average pressure. The value of neutrality is desirable to remain near 1 thus a flat Thrust-time trace is desired. Considering the high F_{av} required in such short t_b , this motor has to act as a Booster Motor through out its burning time phase so totally flat thrust-time curve may not be possible to be attained. From table 6 and figure 10 (c), it is clear that slotted tube grain configuration is producing a more neutral behaviour compared to Star and Wagon Wheel configurations.

 w_f and v_i SRM grains are designed to satisfy w_f and v_i . Hence the two dependent parameters are of considerable importance in a grain design analysis. Usually the SRM design requires a highly loaded configuration. The impact of remaining sliver content on the design depends on the specific ballistic requirements, however in virtually all cases the sliver fraction will have a bearing on the required volumetric loading. If burning neutrality is not important, high v_i can be attained with small sliver fractions by increasing web fraction. Table 6 and figure 10 (d) indicates values of v_i , Slotted tube grain has highest loaded configuration.

 P_{av} In addition to the above referred design parameters, it is very important that P_{av} be kept as low as possible. P_{av} is mainly dependent upon grain burning surface areas A_b , propellant burning rate BRand throat diameter d_t . Normally with the d_t fixed, A_b and BR become crucial. BR dependents upon the propellant formulation and it is desired to be kept as low as possible. From figure 10 (b) it can be seen that Star grain is delivering the required thrust in P_{av} is 19.44 MPa whereas Wagon wheel and Slotted tube are operating at 25.3 MPa & 26.8 MPa respectively. Hence Star grain is fulfilling the design requirements at lowest pressure.

Conclusion

While designing any solid rocket motor grain, it is imperative that maximum F_{av} be attained within least allowable t_b and m_p . It is deemed necessary that the sliver should be kept low and a flat Thrust-time trace is desired. Usually the SRM design requires a highly loaded configuration and it is important that P_{av} be kept as low as possible.

For the current design objectives, while remaining within the essential constraints, the optimal designs of star, wagon wheel and slotted tube grain configurations are fulfilling all the design requirements.

Star grain is delivering the maximum F_{av} using least m_p while operating at lowest P_{av} but at relatively higher *BR* and S_f , the wagon wheel is delivering lower F_{av} at slightly higher m_p and relatively higher P_{av} but in least t_b and lowest required *BR*. Slotted tube is delivering lowest F_{av} by utilizing maximum m_p and at highest P_{av} but at lowest value of neutrality with no sliver.

It can be deduced that all the configurations are delivering the required design objectives but with certain compromises in different design parameters as compared with each other.

Considering the most vital design parameters of F_{av} , m_p and P_{av} and considering the current design requirements, the Star Grain design is the most viable and best suited option to be selected as final design amongst the three optimal grain design configurations mentioned in table 6.

By applying this method of design and optimization, various Two Dimensional Grains for SRM can be designed and optimized considering different design objectives like F_{av} , $m_p \& t_b$ and best possible configuration of all the available geometries can be finalized that will produce most efficient performance.

References

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Appendix

Nomenclature		
Burning area	=	A_b
Initial burning area	=	A_{bi}
Nozzle exit diameter	=	A_e
Initial port area	=	A_{pi}
Area of throat	=	A_t
Burning rate	=	BR
Characteristic velocity	=	C^*
Thrust coefficient	=	C_{f}
Nozzle exit diameter	=	d_e
Average thrust	=	F_{av}
Max thrust	=	F_{max}
Specific impulse	=	I_s
Total impulse	=	I_t
Grain outer radius	=	R
Mass of propellant	=	m_p
Burning Duration	=	t_b
Pressure exponent	=	n
Vol. chamber case	=	V_c
Avg. chamber pr	=	P_{av}
Vol. of propellant	=	V_p
Maximum pressure	=	$\dot{P_{max}}$
Volumetric Loading	=	V_l
Web Fraction	=	W_f